**Liquid hold up**

Liquid holdup $H_L$, is defined as the fraction of an element of pipe which is occupied by liquid at same instant.

$$H_L = \frac{\text{Volume of liquid in a pipe element}}{\text{volume of the pipe element}}$$

It is necessary to be able to determine liquid holdup to calculate such things as mixture density, actual gas and liquid viscosities, effective viscosity and heat transfer.

The value of liquid holdup varies from zero for single-phase gas flow to one for single-phase liquid flow. Liquid holdup may be measured experimentally by several methods, such as resistivity or capacitance probes or by trapping a segment of the flow stream between quick closing valves and measuring the volume of liquid trapped.

The relative in-situ volume of liquid and gas is sometimes expressed in terms of the volume fraction occupied by gas, called gas holdup $H_g$, or void fraction. Gas holdup is expressed as:

$$H_g = 1 - H_L$$

A value for liquid holdup cannot be calculated analytically. It must be determined from empirical correlations and is a function of variables such as gas and liquid properties, flow pattern, pipe diameter and pipe inclination.

Liquid holdup equations are functions of dimensionless liquid and gas velocity numbers in addition to liquid viscosity number and angle of inclination.

When gas and liquid flow concurrently in a pipe, the gas normally travels faster than the liquid, causing a slippage between the phases. Because of this slippage, the in-situ liquid volume fraction at any given location in the pipe cannot be computed directly from input conditions.

An accurate prediction of liquid holdup is required to compute the hydrostatic head losses in two-phase. Many attempts to develop empirical correlations for predicting liquid holdup have been made. The liquid holdup of Hagedorn and Brown was not measured but was calculated to satisfy the measured pressure gradients after the pressure gradients due to friction and acceleration were accounted for. the data used are consisted of 51 field pressure profiles for vertical well

Other correlations was developed by Duns and Ros based on flow pattern map and function on the slip velocity, and Griffith presented a correlation as a gas void fraction for bubble flow.

With increases in exploration and production activity offshore during 1960’s resulted in the drilling of a large number of wells with large deviations in inclination angle from the
vertical. It soon became obvious that flow-pattern and liquid holdup prediction methods developed for vertical flow often failed in directional wells. The Beggs and Brill and the Mukherjee and Brill generalized correlations were developed to improve pressure drop prediction in directional wells and hilly-terrain pipelines based on experimental studies.

**Hagedorn and Brown**

Neither liquid holdup nor flow pattern was measured during the Hagedorn and Brown study, although a correlation for the calculated liquid holdup is presented. The correlation was developed by assuming that the two-phase friction factor would be obtained from the Moody diagram, based on two phase Reynolds number. This Reynolds number requires a value for $H_L$ in the viscosity term. The procedure used for obtaining the calculated $H_L$ is:

1. Measure $dP/dL$
2. Estimate a value for liquid holdup, $H_L^*$
3. Calculate $N_{Re TP}$ (Eq. 35-II) and find $f_{TP}$ from the Moody diagram
4. Calculate $(dP/dL)f$ and $(dP/dL)acc$.
5. Calculate $(dP/dL)el = dP/dL - (dP/dL)f - (dP/dL)acc$ and $\rho_s = (dP/dL)el \cdot gc/g$
6. Calculate $H_L = (\rho_s - \rho g)/(\rho L - \rho g)$ and compare with $H_L^*$
   - If not close, set $H_L^* = H_L$ and go to step 3. continue until convergence is obtained.

The value of $H_L$ obtained is not necessarily the actual liquid holdup, but it is the value required to balance the pressure losses once a friction factor has been selected. Several dimensionless numbers were used to correlate $HL$ and two secondary correction factors. These dimensionless numbers had been defined earlier by Ros and are given below.

\[ N_{Lv} = 1.938 \cdot v_{SL} \left( \frac{\rho_L}{\sigma_L} \right)^{1/4} \]
\[ N_{gv} = 1.938 \cdot v_{Sg} \left( \frac{\rho_L}{\sigma_L} \right)^{1/4} \]
\[ N_L = 0.15726 \cdot u_L \left( \frac{1}{\rho_L \sigma_L^3} \right)^{1/4} \]
\[ N_d = 120.872 \cdot d \left( \frac{\rho_L}{\sigma_L} \right)^{1/2} \]

Where

- $N_{Lv}$ = Liquid velocity number
- $N_{gv}$ = Gas velocity number
- $N_L$ = Liquid viscosity number
- $N_d$ = Pipe diameter number
- $d$ = Inside diameter, ft
- $V_{SL}$ = Superficial liquid velocity, ft/sec

\[ = (5.61 \cdot q_L / 86400 \cdot Ap) \left[ Bo \left( \frac{1}{1 + WOR} \right) + Bw \left( WOR / (1 + WOR) \right) \right] \]
V_{sg} = \text{Superficial gas velocity, ft/sec}
\quad = \left( \frac{q_L \left[ \text{GLR} - \text{Rs} \left( \frac{1}{1+WOR} \right) \right]}{86400 \text{Ap}} \right) \left( \frac{14.7}{P_{avg.}} \right) \left( \frac{T_{avg.}}{520} \right) \left( \frac{Z_{avg.}}{1} \right), \text{ft/sec}
q_L = \text{Liquid flow rate, bbl/day}
\text{Ap} = \text{Flow area, sq ft}
\text{WOR} = \text{Water oil ratio} = \frac{\text{BWPD}}{\text{BOPD}}
\text{GLR} = \text{Gas liquid ratio, scf/bbl}
P = \text{Average pressure, psia}
T = \text{Average temperature, F}
\text{Rs} = \text{Solution gas oil ratio, scf/ stk bbl}
B_o = \text{Oil formation volume factor, bbl/stb}
B_w = \text{Water formation volume factor, bbl/stb}
Z = \text{Gas compressibility factor}
\mu_L = \text{Liquid viscosity, cp}
\sigma_L = \text{Liquid mixture surface tension, dynes/cm}
\rho_L = \text{Liquid density, lb/cu ft}

The three empirical correlations required for obtaining a value of \( H_L \) are shown in the following figures.
Holdup factor correlation

Correlation for secondary correction factor
Three modifications have been made to the original Hagedorn and Brown method which have extended the range of application considerably. The Griffith correlation was used when bubble flow existed and the value of liquid hold up was checked to make sure that it is exceeded the holdup for no slippage and if not, the holdup of no slippage was used, since it was found that for some cases, the value calculated for HL was less than the no-slip holdup, this is physically impossible in upward two-phase flow. Another revised calculation for liquid holdup was made by Abdul-Majeed and Abu-Soof to revise the Hagedorn and Brown liquid holdup correlation using unpublished field data, since the modified Hagedorn and Brown correlation tended to underpredict pressure drops. Rossland, mentioned that the under prediction was mainly due to the underprediction of liquid holdup, The revised correlation gives the higher value of liquid holdup.

**Improved revision to Hagedorn and Brown liquid holdup correlation**

The liquid holdup of Hagedorn and Brown was not measured but was calculated to satisfy the measured pressure gradients after the pressure gradients due to friction and acceleration were accounted for. The resultant empirical correlation developed for predicting these pseudoholdup can give physically unrealistic values that suggested that liquid flows faster than gas.

Several two-phase flowing pressure loss evaluation studies were performed in which the modified Hagedorn and Brown correlation was included in addition to other existing correlations. The results of these studies showed that the modified Hagedorn and Brown (included Griffith method) correlation was the best overall predictor. However, it was found, based on the statistical results presented in these studies, that on average, the modified Hagedorn and Brown correlation tended to underpredict pressure drops. Rossland, mentioned that the under prediction was mainly due to the underprediction of liquid holdup.

An attempt was made by Abdul-Majeed and Abu-Soof to revise the Hagedorn and Brown liquid holdup correlation using unpublished field data. These data consisted of 51 field pressure profiles (for vertical well). In each profile, the well was divided into equal depth intervals and the pressure was measured at the end of each interval. The total number of pressure measurements in the collected profiles was 540 points. The ranges of flow variables covered by the measured data as follows:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value in field units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil flow rate, STBO/D</td>
<td>0 - 20000</td>
</tr>
<tr>
<td>Water flow rate, STBW/D</td>
<td>0 - 9000</td>
</tr>
<tr>
<td>Gas flow rate, Mcf/D</td>
<td>1 - 1200</td>
</tr>
<tr>
<td>Tubing diameter, inch</td>
<td>1 - 6.5</td>
</tr>
<tr>
<td>Oil gravity, API</td>
<td>9 - 55</td>
</tr>
<tr>
<td>Gas-Liquid ratio, SCF/STB</td>
<td>25 - 10000</td>
</tr>
<tr>
<td>Gas gravity (air=1)</td>
<td>0.6 - 1.25</td>
</tr>
<tr>
<td>Well head pressure, psig</td>
<td>15 - 1684</td>
</tr>
</tbody>
</table>
Bottom hole pressure, psig 130 - 6200
Well head temperature, degree F 60 - 150
Bottom hole temperature, degree F 85 - 320
Well depth, ft 7000 - 14070

The gradient equation adopted in the Abdul-Majeed and Abu-Soof work was the same as that developed by Hagedorn and Brown.

$$144 \frac{\Delta p}{\Delta h} = (\rho_{m}) + (\frac{fw^2}{2.9652 \times 10^{11}} d^5 \rho_{m}) + (\rho_{m} \Delta (\frac{v_m^2}{2g_c}) / \Delta h)$$

where:

$$\rho_{m} = \rho_L H_L + \rho_g (1 - H_L)$$

The two-phase friction factor (f) was calculated on the basis of a two-phase Reynolds number using the standard Moody diagram. The two-phase Reynolds number used was

$$\left( \frac{N_{Re}}{N_{Re}} \right)_{T.P.} = 2.2 \times 10^{-2} \frac{w}{d} \mu_L^{HL} \mu_g^{1-HL}$$

using this definition for Reynolds number, and the conventional relationship between “f” and “NRe” for single phase fluid. The values of liquid holdup in term of “ρ” were plotted vs. the correlating function (Cf) which suggested by Hagedorn and Brown.

Abscissa = Cf = $$(\frac{N_{LV}}{N_{GV}})^{0.575} \left( \frac{p}{pa} \right)^{0.1} \left( \frac{CN_L}{N_d} \right)$$

As for the comparison between the original and revised liquid holdup correlation, the two correlations were given.
Comparison of revised and original liquid holdup correlation

For any value of “Cf”, the revised correlation gives the higher value of liquid holdup.

Where

\[ H_L = \text{Liquid holdup} \]

\( (N_{Re})_{T,P.} = \text{Two phase Reynolds number.} \)

\[ W = \text{Mass flow rate, Ib/m/day} \]

\[ q = \text{Total liquid flow rate, STB/Day} \]

\[ m = \text{Mass associated with one bbl of stock tank liquid, Ib/STB bbl liquid} \]

\[ = \text{wt. of oil + wt. of water + wt. of gas} \]

\[ = \gamma_o (350) \left( \frac{1}{1 + \text{WOR}} \right) + \gamma_w (350) \left( \frac{\text{WOR}}{1 + \text{WOR}} \right) + 0.0764 \left( \text{GLR} \right) \gamma_g \]

\[ N_{Lv} = 1.938 v_{SL} \left( \frac{\rho_L}{\sigma_L} \right)^{1/4} \]

\[ N_{gv} = 1.938 v_{SG} \left( \frac{\rho_L}{\sigma_L} \right)^{1/4} \]

\[ N_d = \text{Pipe diameter number} \]
\[ V_{SL} = 120.872 \, d \left( \frac{\rho_L}{\sigma_L} \right)^{1/2} \]

\[ V_{SL} \text{ = Superficial liquid velocity, ft/sec} \]

\[ V_{SG} = \left( \frac{5.61 \, q_L}{86400 \, Ap} \right) \left[ Bo \left( \frac{1}{1+WOR} \right) + Bw \left( \frac{WOR}{1+WOR} \right) \right] \]

\[ V_{SG} \text{ = Superficial gas velocity, ft/sec} \]

\[ q_L = \text{Liquid flow rate, bbl/day} \]

\[ Ap = \text{Flow area, sq ft} \]

\[ WOR = \text{Water oil ratio} = \frac{BWPD}{BOPD} \]

\[ GLR = \text{Gas liquid ratio, scf/bbl} \]

\[ P = \text{Average pressure, psia} \]

\[ T = \text{Average temperature, F} \]

\[ d = \text{Inside diameter of the pipe, ft} \]

\[ 350 = 8.33 \times 42 \]

\[ 0.0764 = \text{Density of air, lb/cu ft} \]

\[ \gamma_o = \text{Oil gravity} \]

\[ \gamma_w = \text{Water gravity} \]

\[ \gamma_g = \text{Gas gravity} \]

\[ WOR = \text{Water oil ratio} \]

\[ GLR = \text{Gas liquid ratio, scf/stb} \]

\[ Rs = \text{Solution gas oil ratio, scf/stk bbl} \]

\[ Bo = \text{Oil formation volume factor, bbl/stb} \]

\[ Bw = \text{Water formation volume factor, bbl/stb} \]

\[ \rho_m = \text{Mixture density, lb/ cu ft} \]

\[ \sigma_L = \text{Liquid mixture surface tension, dynes/cm} \]

\[ \rho_L = \text{Liquid density, lb/cu ft} \]

\[ \mu_L = \text{Liquid viscosity, cp.} \]

\[ \mu_g = \text{Gas viscosity, cp.} \]

\[ Z = \text{Gas compressibility factor} \]