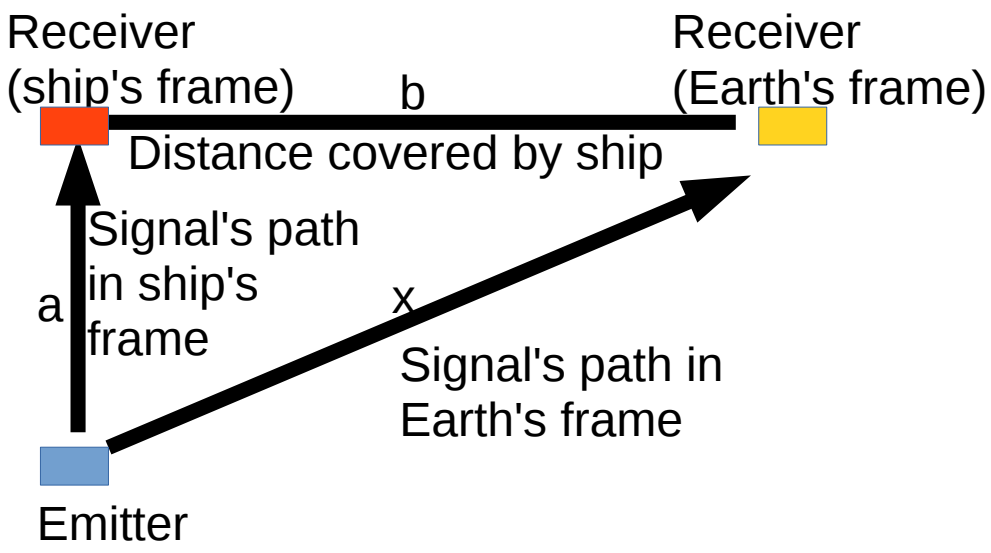
The Art of the Relativistic Rocket

Fabian Herrmann, 2016

Solutions

1.



Pythagoras:

$$a^2+b^2=x^2$$

Ship's velocity relative to Earth is v, t_E is the time between the signal leaving the emitter and reaching the receiver as measured from Earth:

$$b=v\cdot t_E$$

$$t_E=\frac{x}{c}$$

Lightspeed c is CONSTANT irrespective of reference frame!

t_R is the time between emission and reception of the signal in the ship's frame:

$$t_R=\frac{a}{c}$$

Put this into Pythagoras and solve for t_R!

$$(t_R\cdot c)^2+(v\cdot t_E)^2=(t_E\cdot c)^2$$

$$t_R=t_E\cdot\sqrt{1-\frac{v^2}{c^2}}=\frac{t_E}{\gamma}$$

2.

$$\delta(T_E) = \int_0^{T_E} \gamma^2 a_E dt_E = \int_0^{T_E} \gamma^2 dv = c \cdot \int_0^{\beta_E} \frac{d\beta}{1-\beta^2} = c \cdot \operatorname{artanh}(\beta_E)$$

→

$$\frac{\Delta v}{c} = \tanh\left(\frac{\Delta \delta}{c}\right) = \tanh\left(\frac{v_0}{c} \cdot \ln\left(\frac{M_{wet}}{M_{dry}}\right)\right)$$

3.

1236 kg.

618 kg of matter mixed with 618 kg of antimatter could power the entire US for one year, including industry, heating and transportation.

4.

30.000 km/s / ln(10) ~ 13.000 km/s.

5.

artanh(0.9) * c / ln(10) ~ 190.000 km/s.

6.

Reflecting sail:
F = 2 p / t = 2 E / ct = 2 P / c

(Force is two times – due to reflection – the light's momentum per time, that is, two times irradiated power divided by lightspeed.)

P = c F / 2 = 150 MW.

For absorbing sails, the „2“ must be dropped, so it is 300 MW.

That is, one municipal power plant for just one Newton!

7.

A.)

$$P = \frac{1}{2} \dot{m} v_0^2$$

with $\dot{m} = mass\ flow\ rate$

B.)

$$P = \frac{1}{2}\dot{m}\left(\frac{\Delta v}{\ln\left(M_{wet}/M_{dry}\right)}\right)^2$$

8.

A.)

v ~ 14 km/s

B.)

v ~ 14.000 km/s
(square root of 1E6 times the result from A.)

C.)

$$v = c\cdot\sqrt{1-\left(\frac{1}{\frac{E_{kin}}{mc^2}+1}\right)^2} = 0.88\ c$$

At around 90% c, the kinetic energy of any object becomes comparable to its rest mass energy. Space opera battle starships fighting at relativistic speeds wouldn't need antimatter torpedoes. Kinetic projectiles – pillows, beer bottles, kitty litter, science fiction paperbacks, etc. – hurled at the enemy ship would do the trick nicely.

9.

3 GW / 200 MeV = 9.4E19 fission reactions per second.
9.4E19 * 235 u * 14e6 m s^{-1} / 1 s ~ 512 N

This corresponds to the weight of ~50 kg at 1 g. By far not enough to lift the reactor.

10.

A.)

v_0 is the velocity with which the bomb plasma strikes the pusher plate. We assume that it is completely stopped by the plate (inelastic collision).

From the formula for kinetic energy $W_{bomb} = 0.5 * m_{bomb} * v_0^2$ (this is still in the non- or weakly relativistic realm), we get:

$$F_{thrust} = \frac{\sqrt{2W_{bomb}m_{bomb}}}{1s}$$

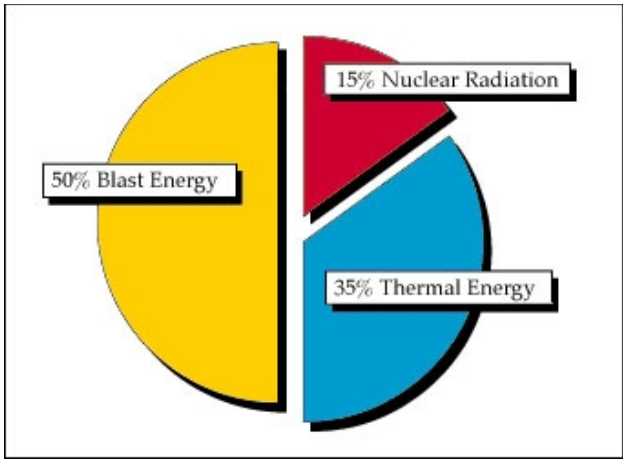
and thus:

$$\frac{F_{thrust}}{10^4\ N} = \sqrt{2}\cdot\sqrt{\frac{W_{bomb}}{4.2\ 10^{12}\ J}\cdot\frac{M_{bomb}}{1\ kg}}\cdot\frac{\sqrt{4.2\ 10^{12}}}{10^4} \approx 300\cdot\sqrt{\frac{W_{bomb}}{kt}\cdot\frac{m_{bomb}}{kg}}$$

$$\frac{v_0}{1\ km/s} = \sqrt{2} \cdot \sqrt{\frac{W_{bomb}/4.2\ 10^{12}}{m_{bomb}/1\ kg}} \cdot \frac{\sqrt{4.2\ 10^{12}\ J}}{1000} \approx 3000 \sqrt{\frac{W_{bomb}/kt}{M_{bomb}/kg}}$$

(Which then can be used in the Tsiolkovski Equation.)

Have we overestimated efficiency? A certain percentage of energy will not be released as kinetic energy but as thermal and nuclear radiation: (source: Atomic Archive)



The energy appears under the square root in our equations. Since only 50% of the energy released is blast (kinetic) energy, we should multiply the formulas by SQRT(0.5) = 0.7, changing the factor 300 to 212, 3000 to 2120 respectively (or, back-of-the-envelope-style: 200 and 2000). The square root reduces the effect of our overestimation.

B.)

$$300 \cdot \sqrt{\frac{W_{bomb}}{kt} \times \frac{m_{bomb}}{kg}} = 10^6$$

Chose e.g. W_{bomb} = 10 Mt = 10^4 kt (large thermonuclear device). We get m_{bomb} = 1000 kg.

Somewhat similar to US B53 warhead – 9 Mt, 4000 kg mass:

from

$$3000 \cdot \sqrt{\frac{10^4}{10^3}} \cdot \ln(Mr) = 30.000$$

we get: Mass ratio = 24.
So, if take-off mass is 10^6 t → 42.000 t payload, hull, etc. + 958.000 t bombs.

11.

$\dot{m} = F_{thrust}/v_0 = 10^9\ kg \cdot g\ /\ 0.9\ c \approx 370\ kg/s$

370 kg * 18.3 MeV / 5 u / s ~ 7250 x World Primary Energy Consumption (2016: 18.000 GW).

12.

1 MeV / (10 * 60 * 12 * 1.66E-27 kg) = 13.4 GJ / kg ~ 3190 x TNT

13.

A.)

Distance covered in Earth's reference frame:

$$x_E = \int_0^{T_E} v\, dt_E$$

Use results from problems 1.) and 2.) to express v as a function of t_E and dt_E as a function of dt_R:

$$x_E = c \cdot \int_0^{T_R} \frac{\tanh\left(\frac{\delta}{c}\right)}{\sqrt{1-\tanh^2\left(\frac{\delta}{c}\right)}} dt_R = c \cdot \int_0^{T_R} \frac{\tanh\left(\frac{g\,t_R}{c}\right)}{\sqrt{1-\tanh^2\left(\frac{g\,t_R}{c}\right)}} dt_R = \frac{c^2}{g} \cdot \left(\cosh\left(\frac{g\,T_R}{c}\right) - 1\right)$$

B.)

$$T_E = \int_0^{T_R} \frac{d\,t_R}{\sqrt{1-\frac{v^2}{c^2}}} = \int_0^{T_R} \frac{d\,t_R}{\sqrt{1-\tanh^2\left(\frac{g\,t}{c}\right)}} = \frac{c}{g} \cdot \sinh\left(\frac{g\,T_R}{c}\right)$$

to get Earth time as function of ship's distance, put in:

$$T_R = \frac{c}{g} \cdot acosh\left(\frac{x_E\,g}{c^2} + 1\right)$$

14.

If we assume spacetime to be flat – and a number of astronomical observations point to this conclusion –, then Nietzsche's eternal recurrence of the same is cosmological fact: Statistical physics dictates that there are infinitely many Earths, even infinitely many Hubble Volumes identical to our own – eternal recurrence in space –; and, due to the Ergotic Theorem, even far into the future, when the universe has been diluted into an evenly distributed sea of cold, thin photon gas, quantum fluctuations will bring about an ever-growing number of „Boltzmann Brains“ (intelligences arising out of vacuum as a thermodynamic „miracle“) and even infinitely many repetitions of the „big bang“ (which may very well have been a random product of thermodynamics itself). So, at a first glance it seems that nothing I do matters on a cosmological scale, as there always will be an infinite number of copies of me doing the same or the opposite or something else, now and for eternity. Why then embark on something as insanely complicated and dangerous as interstellar spaceflight? We could leave it to one of humanity's copies far away to conquer the universe... place ourselves cozily in Earth's gravity hammock.

Nietzsche's Overman would be the first creature on Earth to be able to accept eternal recurrence, not needing any comforting mythology about salvation. But Nietzsche's abhorrence of technology and women, his ignorance of natural science and his glorification of war made him short-sighted.

The universe's energy density keeps falling as it expands, meaning that thermodynamical „miracles“ become ever rarer. An intelligence though may increase energy flux density to the point of being able to sculpt spacetime itself, re-shaping reality and even create custom universes. These spacetime constructions could be seeded with intelligence in turn, which would sometimes gain the ability to sculpt their own reality on a far smaller scale – and so on, ad infinitum: This is the third kind of eternal recurrence – fractal recurrence in complexity, very different from Nietzsche's recurrence, as it is not passively experienced, but actively created. The higher the density of intelligences in the universe that gain spacetime sculpting ability, and the greater the area of spacetime each of them controls, the higher will the probability be that entropy can be overcome and the amount of intelligence that exists will grow infinitely, even asymptotically after one point in time, leading to a universal explosion of complexity, a final stage of cosmological evolution which is neither a „big crunch“ nor heat death, but the start of an epoch in which the abilities and knowledge of intelligence become infinite.

It could be asked whether this a „good“ or a „bad“ result. (Some people view intelligence as inherently evil.) But I think „good“ vs. „bad/evil“ are the wrong criterions. Instead the dialectic opposites we are dealing with are „intelligence“ vs. „entropy“. Why should we wish for intelligence to win? (E.g. Buddhism prefers merging into timeless, thoughtless, sensation-less Nirvana.) It is a question of „something“ vs. „nothing“. Intelligence is the most „something-like“ that can possibly exist, because it is able to actively perceive as well as shape reality, including itself. Where nothing exists, there is neither joy nor pain (which is why Buddhism prefers Nothingness). Where something exists, and is actively shaped by intelligence, joy can be maximized. The deep, powerful satisfaction felt by scientists, philosophers and artists when unravelling the secrets of reality points to the fact that in the long run, joy will become the dominant principle. We should therefore pursue spaceflight and the explosion of intelligence because of the happyness it brings us.

From my „Curiepolis“ book:
Baking pancakes in free fall is a bit of a challenge, but it is fun. Life in space is ultimately amusing, otherwise people wouldn't long for it. Space is a peaceful, amusing place. Quasars and supernovae are champagne-bubbling fireworks, not grim blows of devastation. The true revenge of monotheism consisted in its seduction of scholars to view the whole of reality als a dark, nefarious machinery, which doesn't know better than to constantly mangle organisms (and bring forth new ones in even greater number in order to mangle them too), evaporate, shatter, pulverize celestial bodies (and

bring forth new ones in order for suffering to be endless, until someday timeless entropy shall mercifully spread indifference over everything); it fell to a shiny-eyed nation of the warm, luminous, tempestous Pacific Ocean to help humanity learn that reality is, in fact, joy.

Eierkuchen im freien Fall backen ist eine kleine Herausforderung, aber es macht Spaß. Das Leben im Weltall macht Spaß, sonst hätten die Menschen keine Sehnsucht danach. Das Weltall ist ein friedlicher, amüsanter Ort. Quasare und Supernovae sind champagnerschäumende Feuerwerke, keine grimmigen Vernichtungsschläge. Die wahre Rache des Monotheismus bestand darin, dass er die Gelehrten dazu verleitete, die ganze Realität als finstere, bösartige Maschinerie aufzufassen, die nichts besseres zu tun hat, als pausenlos Organismen zu zerfleischen (und neue in noch höherer Anzahl hervorzubringen, damit diese auch zerfleischt werden können), Himmelskörper zu verdampfen, zersplittern, pulverisieren (und neue hervorzubringen, damit das Leid kein Ende hat, bis irgendwann die zeitlose Entropie gnädig Gleichgültigkeit über alles breitet); es sollte einen glanzäugigen Staat im warmen, leuchtenden, stürmischen Pazifik brauchen, um der Menschheit zu der Erkenntnis zu verhelfen, dass die Realität Vergnügen ist.

Somewhat similar thinking was pursued by Russian mystic Nikolai Fedorov, see: „The Philosophy of the Common Task“.