The Art of the Relativistic Rocket

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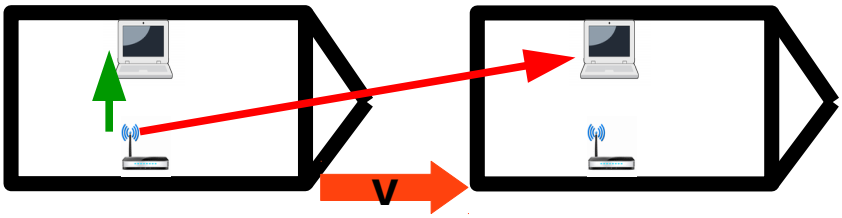
Problems

1.

Derive the formula for time dilation from the „wifi in spaceship“ example:



$$dt_R = \frac{dt_E}{\gamma} \qquad \gamma := \left(1-\beta^2\right)^{-1/2} \qquad \beta := \frac{v}{c}$$

Hint: Pythagoras!!

2.

Derive the relativistic rocket equation in its standard form:

$$\frac{\Delta v}{c} = \tanh\left(\frac{v_0}{c}\ln\frac{M_{wet}}{M_{dry}}\right)$$

from:

$$\Delta\delta = v_0 \cdot \frac{M_{wet}}{M_{dry}}$$

using

$$\delta(t_R) = \int_0^{t_R} a_R(t)\,dt \qquad dt_R = \frac{dt_E}{\gamma} \qquad a_R = \gamma^3 a_E \qquad a_E = \frac{dv}{dt_E}$$

3.

Which mass corresponds to the entire energy consumption of the US over one year? (320 mio people, 11 kW per person, 1 year ~ 3.16E7 s)

4.

A rocket has Δv = 10% c = 30.000 km/s and a mass ratio of 10.
Use the Tsiolkovski Equation to determine the necessary exhaust velocity.

5.

A rocket has Δv = 90% c = 270.000 km/s, M.r. = 10.
Use the Relativistic Rocket Equation to determine exhaust velocity.

6.

Which irradiation power (Watts) will exert a force of 1 N on a lightsail?
(Consider a reflecting as well as an absorbing light sail.)

$$p_{Light} = mc = E/c$$

(Photon momentum is equal to its energy divided by lightspeed.)

7.

Derive the power of a rocket motor
A.) as a function of exhaust velocity and thrust force.
B.) ...of thrust, Δv and mass ratio.

8.

Calculate velocities corresponding to:
A.) chemical binding energy densities: 1 eV/u.
B.) inter-hadron (nuclear) forces: 1 MeV/u.
C.) intra-hadron (rest mass) forces: 1 GeV/u.

Hints:
1 eV ~ 1.6E-19 J
1 u ~ 1.66E-27 kg
For **C.)**, use relativistic kinetic energy: $E_{kin} = mc^2 \cdot (\gamma - 1)$

9.

A power reactor has a thermal power of 3 GW. If all fission fragments were made to fly in a certain direction, how great would the thrust force be?
(Hint: Each Actinide nucleus, such as U235, delivers around 200 MeV per fission. Most of the energy goes into the kinetic energy of the fission fragments.)

10.

A.)

Derive these equations for thrust and Δv of an Orion Pulse Drive:

$$F_{thrust} = 300 \sqrt{\frac{W_{bomb}}{kt} \times \frac{m_{bomb}}{kg}} \text{ in t @ 1 g.}$$

$$\Delta v = 3000 \sqrt{\frac{W_{bomb}/kt}{m_{bomb}/kg}} \ \ln \frac{M_{wet}}{M_{dry}} \ \frac{km}{s}$$

where W_{bomb} = explosive power of each pulse unit, m_{bomb} = total mass of each.
1 kt TNT ~ 4.2E12 J.

Assume that the entire explosive energy is transformed into kinetic energy of the plasma which collides with the pusher plate inelastically.

In reality, a substantial amount of energy will be emitted as thermal radiation. Does the above assumption cause a massive overestimation of our results? Why/why not?

B.)

Design an Orion ship with Δv = 10% c, Thrust = 1 mio t @ 1 g.

11.

We have a 1 mio t ship fueled by D+3He fusion (18.3 MeV/reaction, v_0 = 9% c), accelerating at 1 g. Which mass of fuel will have to be burned per second? How great will the thermal power be? Compare with current world primary energy consumption: 18.000 GW.

12.

Calculate energy density of Fullerite with one positron in every tenth C_{60}-molecule. Compare with TNT. (1 t TNT ~ 4.2 GJ)
Energy of positron/electron annihilation: 1 MeV

13.

A Bussard ramjet ship leaves Earth at a constant acceleration of 1 g.
Calculate:
A.) ...distance covered (in Earth's reference frame) as a function of onboard time.
B.) ...Earth time passing as function of ship's time (distance).

14.

Write a short paragraph on why humanity should expand into space and become a multi-planet intelligence.