ECUACIÓN DE CONTINUIDAD $\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{v} = 0$

Coordenadas Rectangulares (x ; y ; z): $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho\, Vx) + \frac{\partial}{\partial y}(\rho\, Vy) + \frac{\partial}{\partial z}(\rho\, Vz) = 0$

Coordenadas Cilíndricas (r ; θ ; z): $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho\, r\, Vr) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho\ \ V\theta) + \frac{\partial}{\partial z}(\rho\, Vz) = 0$

Coordenadas Esféricas (r ; θ ; Φ): $\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho\, r^2\, Vr) + \frac{1}{r\, sen\theta}\frac{\partial}{\partial \theta}(\rho\ \ V\theta\ sen\theta) + \frac{1}{r\, sen\theta}\frac{\partial}{\partial \phi}(\rho\, V\phi) = 0$

ECUACIÓN DE MOVIMIENTO

$$\rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot (\underline{\nabla}\, \underline{V})\right) = -\underline{\nabla}\, p - \underline{\nabla}\, \tau + \rho \cdot \underline{g}$$

$$\rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V} \cdot (\underline{\nabla}\, \underline{V})\right) = -\underline{\nabla}\, p + \nabla^2 \underline{V} + \rho \cdot \underline{g}$$

1) RECTANGULARES (x ; y ; z)

1-1) En función de τ:

Componente x

$$\rho\left(\frac{\partial Vx}{\partial t} + Vx\frac{\partial Vx}{\partial x} + Vy\frac{\partial Vx}{\partial y} + Vz\frac{\partial Vx}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau xx}{\partial x} + \frac{\partial \tau yx}{\partial y} + \frac{\partial \tau zx}{\partial z}\right) + \rho\ gx$$

Componente y

$$\rho\left(\frac{\partial Vy}{\partial t} + Vx\frac{\partial Vy}{\partial x} + Vy\frac{\partial Vy}{\partial y} + Vz\frac{\partial Vy}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left(\frac{\partial \tau xy}{\partial x} + \frac{\partial \tau yy}{\partial y} + \frac{\partial \tau zy}{\partial z}\right) + \rho\ gy$$

Componente z

$$\rho\left(\frac{\partial Vz}{\partial t} + Vx\frac{\partial Vz}{\partial x} + Vy\frac{\partial Vz}{\partial y} + Vz\frac{\partial Vz}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{\partial \tau xz}{\partial x} + \frac{\partial \tau yz}{\partial y} + \frac{\partial \tau zz}{\partial z}\right) + \rho\ gz$$

1-2) En función de los gradientes de velocidad para fluidos newtoniano de ρ y μ ctes:

Componente x

$$\rho\left(\frac{\partial Vx}{\partial t} + Vx\frac{\partial Vx}{\partial x} + Vy\frac{\partial Vx}{\partial y} + Vz\frac{\partial Vx}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 Vx}{\partial x^2} + \frac{\partial^2 Vx}{\partial y^2} + \frac{\partial^2 Vx}{\partial z^2}\right) + \rho\ gx$$

Componente y

$$\rho\left(\frac{\partial Vy}{\partial t} + Vx\frac{\partial Vy}{\partial x} + Vy\frac{\partial Vy}{\partial y} + Vz\frac{\partial Vy}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 Vy}{\partial x^2} + \frac{\partial^2 Vy}{\partial y^2} + \frac{\partial^2 Vy}{\partial z^2}\right) + \rho\ gy$$

Componente z

$$\rho\left(\frac{\partial Vz}{\partial t} + Vx\frac{\partial Vz}{\partial x} + Vy\frac{\partial Vz}{\partial y} + Vz\frac{\partial Vz}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 Vz}{\partial x^2} + \frac{\partial^2 Vz}{\partial y^2} + \frac{\partial^2 Vz}{\partial z^2}\right) + \rho\ gz$$

2) CILÍNDRICAS (r ; θ ; z)

2-1) En función de τ:

Componente r $\rho\left(\frac{\partial Vr}{\partial t}+Vr\frac{\partial Vr}{\partial r}+\frac{V\theta}{r}\frac{\partial Vr}{\partial \theta}-\frac{V\theta^2}{r}+Vz\frac{\partial Vr}{\partial z}\right)=-\frac{\partial p}{\partial r}-\left(\frac{1}{r}\frac{\partial}{\partial r}(\tau rr)+\frac{1}{r}\frac{\partial \tau r\theta}{\partial \theta}-\frac{\tau\theta\theta}{r}+\frac{\partial \tau rz}{\partial z}\right)+\rho\, gr$

Componente θ $\rho\left(\frac{\partial V\theta}{\partial t}+Vr\frac{\partial V\theta}{\partial r}+\frac{V\theta}{r}\frac{\partial V\theta}{\partial \theta}+\frac{VrV\theta}{r}+Vz\frac{\partial V\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau r\theta)+\frac{1}{r}\frac{\partial \tau\theta\theta}{\partial \theta}+\frac{\partial \tau\theta z}{\partial z}\right)+\rho\, g\theta$

Componente z $\rho\left(\frac{\partial Vz}{\partial t}+Vr\frac{\partial Vz}{\partial r}+\frac{V\theta}{r}\frac{\partial Vz}{\partial \theta}+Vz\frac{\partial Vz}{\partial z}\right)=-\frac{\partial p}{\partial z}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r\,\tau rz)+\frac{1}{r}\frac{\partial \tau\theta z}{\partial \theta}+\frac{\partial \tau zz}{\partial z}\right)+\rho\, gz$

2-2) En función de los gradientes de velocidad para fluidos newtoniano de ρ y μ ctes:

Componente r $\rho\left(\frac{\partial Vr}{\partial t}+Vr\frac{\partial Vr}{\partial r}+\frac{V\theta}{r}\frac{\partial Vr}{\partial \theta}-\frac{V\theta^2}{r}+Vz\frac{\partial Vr}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rVr)\right)+\frac{1}{r^2}\frac{\partial^2 Vr}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V\theta}{\partial \theta}+\frac{\partial^2 Vr}{\partial z^2}\right)+\rho\, gr$

Componente θ $\rho\left(\frac{\partial V\theta}{\partial t}+Vr\frac{\partial V\theta}{\partial r}+\frac{V\theta}{r}\frac{\partial V\theta}{\partial \theta}+\frac{VrV\theta}{r}+Vz\frac{\partial V\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV\theta)\right)+\frac{1}{r^2}\frac{\partial^2 V\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial Vr}{\partial \theta}+\frac{\partial^2 V\theta}{\partial z^2}\right)+\rho\, g\theta$

Componente z $\rho\left(\frac{\partial Vz}{\partial t}+Vr\frac{\partial Vz}{\partial r}+\frac{V\theta}{r}\frac{\partial Vz}{\partial \theta}+Vz\frac{\partial Vz}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial Vz}{\partial r})+\frac{1}{r^2}\frac{\partial^2 Vz}{\partial \theta^2}+\frac{\partial^2 Vz}{\partial z^2}\right)+\rho\, gz$

3) ESFÉRICAS (r ; θ ; Φ)

3-1) En función de τ:

Componente r $\rho\left(\frac{\partial Vr}{\partial t}+Vr\frac{\partial Vr}{\partial r}+\frac{V\theta}{r}\frac{\partial Vr}{\partial \theta}+\frac{V\phi}{r\,sen\theta}\frac{\partial Vr}{\partial \phi}-\frac{V\theta^2+V\phi^2}{r}\right)=-\frac{\partial p}{\partial r}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau rr)+\frac{1}{r\,sen\theta}\frac{\partial}{\partial \theta}(\tau r\theta\, sen\theta)+\frac{1}{r\,sen\theta}\frac{\partial \tau r\phi}{\partial \phi}-\frac{\tau\theta\theta+\tau\phi\phi}{r}\right)+\rho\, gr$

Componente θ $\rho\left(\frac{\partial V\theta}{\partial t}+Vr\frac{\partial V\theta}{\partial r}+\frac{V\theta}{r}\frac{\partial V\theta}{\partial \theta}+\frac{V\phi}{r\,sen\theta}\frac{\partial V\theta}{\partial \phi}+\frac{VrV\theta}{r}-\frac{V\phi^2\cot\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial r}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau r\theta)+\frac{1}{r\,sen\theta}\frac{\partial}{\partial \theta}(\tau\theta\theta sen\theta)+\frac{1}{r\,sen\theta}\frac{\partial \tau\theta\phi}{\partial \phi}+\frac{\tau r\theta}{r}-\frac{\cot\theta}{r}\tau\phi\phi\right)+\rho\, g\theta$

Componente Φ $\rho\left(\frac{\partial V\phi}{\partial t}+Vr\frac{\partial V\phi}{\partial r}+\frac{V\theta}{r}\frac{\partial V\phi}{\partial \theta}+\frac{V\phi}{r\,sen\theta}\frac{\partial V\phi}{\partial \phi}+\frac{V\phi Vr}{r}+\frac{V\phi V\theta}{r}\cot\theta\right)=-\frac{1}{r sen\theta}\frac{\partial p}{\partial \phi}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau r\phi)+\frac{1}{r}\frac{\partial \tau\theta\phi}{\partial \theta}+\frac{1}{r\,sen\theta}\frac{\partial \tau\phi\phi}{\partial \phi}+\frac{\tau r\phi}{r}+\frac{2\cot\theta}{r}\tau\theta\phi\right)+\rho\, g\phi$

3-2) En función de los gradientes de velocidad para fluidos newtoniano de ρ y μ ctes::

Componente r $\rho\left(\frac{\partial Vr}{\partial t}+Vr\frac{\partial Vr}{\partial r}+\frac{V\theta}{r}\frac{\partial Vr}{\partial \theta}+\frac{V\phi}{r\,sen\theta}\frac{\partial Vr}{\partial \phi}-\frac{V\theta^2+V\phi^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left(\nabla^2 Vr-\frac{2}{r^2}Vr-\frac{2}{r^2}\frac{\partial V\theta}{\partial \theta}-\frac{2}{r^2}V\theta\cot\theta-\frac{2}{r^2 sen\theta}\frac{\partial V\phi}{\partial \phi}\right)+\rho\, gr$

Componente θ $\rho\left(\frac{\partial V\theta}{\partial t}+Vr\frac{\partial V\theta}{\partial r}+\frac{V\theta}{r}\frac{\partial V\theta}{\partial \theta}+\frac{V\phi}{r\,sen\theta}\frac{\partial V\theta}{\partial \phi}+\frac{VrV\theta}{r}-\frac{V\phi^2\cot\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\nabla^2 V\theta+\frac{2}{r^2}\frac{\partial Vr}{\partial \theta}-\frac{V\theta}{r^2 sen^2\theta}-\frac{2\cos\theta}{r^2 sen^2\theta}\frac{\partial V\phi}{\partial \phi}\right)+\rho\, g\theta$

Componente z $\rho\left(\frac{\partial V\phi}{\partial t}+Vr\frac{\partial V\phi}{\partial r}+\frac{V\theta}{r}\frac{\partial V\phi}{\partial \theta}+\frac{V\phi}{r\,sen\theta}\frac{\partial V\phi}{\partial \phi}+\frac{V\phi Vr}{r}+\frac{V\phi V\theta}{r}\cot\theta\right)=-\frac{1}{r sen\theta}\frac{\partial p}{\partial \phi}+\mu\left(\nabla^2 V\phi-\frac{V\phi}{r^2 sen^2\theta}+\frac{2}{r^2 sen\theta}\frac{\partial Vr}{\partial \phi}+\frac{2\cos\theta}{r^2 sen^2\theta}\frac{\partial V\theta}{\partial \phi}\right)+\rho\, g\phi$

En estas Ecuaciones

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 sen\theta}\frac{\partial}{\partial \theta}\left(sen\theta\frac{\partial}{\partial \theta}\right)+\frac{1}{r^2 sen\theta}\left(\frac{\partial^2}{\partial \phi^2}\right)$$

COMPONENTES DEL TENSOR ESFUERZO $\underline{\underline{\tau}}$

Coordenadas Rectangulares (x ; y ; z):

$\tau xx = -\mu\left[2\,\dfrac{\partial Vx}{\partial x} - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau xy = \tau yx = -\mu\left[\dfrac{\partial Vx}{\partial y} + \dfrac{\partial Vy}{\partial x}\right]$
$\tau yy = -\mu\left[2\,\dfrac{\partial Vy}{\partial y} - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau yz = \tau zy = -\mu\left[\dfrac{\partial Vy}{\partial z} + \dfrac{\partial Vz}{\partial y}\right]$
$\tau zz = -\mu\left[2\,\dfrac{\partial Vz}{\partial z} - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau zx = \tau xz = -\mu\left[\dfrac{\partial Vz}{\partial x} + \dfrac{\partial Vx}{\partial z}\right]$

$$(\underline{\nabla}\cdot\underline{V}) = \frac{\partial Vx}{\partial x} + \frac{\partial Vy}{\partial y} + \frac{\partial Vz}{\partial z}$$

Coordenadas Cilíndricas (r ; θ ; z):

$\tau rr = -\mu\left[2\,\dfrac{\partial Vr}{\partial r} - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau r\theta = \tau\theta r = -\mu\left[r\,\dfrac{\partial}{\partial r}\left(\dfrac{V\theta}{r}\right) + \dfrac{1}{r}\dfrac{\partial Vr}{\partial\theta}\right]$
$\tau\theta\theta = -\mu\left[2\left(\dfrac{1}{r}\dfrac{\partial V\theta}{\partial\theta} + \dfrac{Vr}{r}\right) - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau\theta z = \tau z\theta = -\mu\left[\dfrac{\partial V\theta}{\partial z} + \dfrac{1}{r}\dfrac{\partial Vz}{\partial\theta}\right]$
$\tau zz = -\mu\left[2\,\dfrac{\partial Vz}{\partial z} - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau zr = \tau rz = -\mu\left[\dfrac{\partial Vz}{\partial r} + \dfrac{\partial Vr}{\partial z}\right]$

$$(\underline{\nabla}\cdot\underline{V}) = \frac{1}{r}\frac{\partial}{\partial r}(rVr) + \frac{1}{r}\frac{\partial V\theta}{\partial\theta} + \frac{\partial Vz}{\partial z}$$

Coordenadas Esféricas (r ; θ ; Φ):

$\tau rr = -\mu\left[2\,\dfrac{\partial Vr}{\partial r} - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau r\theta = \tau\theta r = -\mu\left[r\,\dfrac{\partial}{\partial r}\left(\dfrac{V\theta}{r}\right) + \dfrac{1}{r}\dfrac{\partial Vr}{\partial\theta}\right]$
$\tau\theta\theta = -\mu\left[2\left(\dfrac{1}{r}\dfrac{\partial V\theta}{\partial\theta} + \dfrac{Vr}{r}\right) - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau\theta\phi = \tau\phi\theta = -\mu\left[\dfrac{sen\theta}{r}\dfrac{\partial}{\partial\theta}\left(\dfrac{V\phi}{sen\theta}\right) + \dfrac{1}{r\,sen\theta}\dfrac{\partial V\theta}{\partial\phi}\right]$
$\tau\phi\phi = -\mu\left[2\left(\dfrac{1}{r\,sen\theta}\dfrac{\partial V\phi}{\partial\phi} + \dfrac{Vr}{r} + \dfrac{V\theta\cot\theta}{r}\right) - \dfrac{2}{3}(\underline{\nabla}\cdot\underline{V})\right]$	$\tau\phi r = \tau r\phi = -\mu\left[\dfrac{1}{r\,sen\theta}\dfrac{\partial Vr}{\partial\phi} + \dfrac{\partial}{\partial r}\left(\dfrac{V\phi}{r}\right)\right]$

$$(\underline{\nabla}\cdot\underline{V}) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 Vr\right) + \frac{1}{r\,sen\theta}\frac{\partial}{\partial\theta}(V\theta\,sen\theta) + \frac{1}{r\,sen\theta}\frac{\partial V\phi}{\partial\phi}$$

RESUMEN DE OPERACIONES DIFERENCIALES CON EL OPERADOR ∇ RECTANGULARES (x ; y ; z)

$$(\underline{\nabla} \cdot \underline{V}) = \frac{\partial Vx}{\partial x} + \frac{\partial Vy}{\partial y} + \frac{\partial Vz}{\partial z}$$

$$(\nabla^2 s) = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\left(\underline{\underline{\tau}} : \underline{\nabla}\underline{V}\right) = \tau xx\left(\frac{\partial Vx}{\partial x}\right) + \tau yy\left(\frac{\partial Vy}{\partial y}\right) + \tau zz\left(\frac{\partial Vz}{\partial z}\right) + \tau xy\left(\frac{\partial Vx}{\partial y} + \frac{\partial Vy}{\partial x}\right) + \tau yz\left(\frac{\partial Vy}{\partial z} + \frac{\partial Vz}{\partial y}\right) + \tau zx\left(\frac{\partial Vz}{\partial x} + \frac{\partial Vx}{\partial z}\right)$$

$$[\underline{\nabla} s]\begin{cases} [\underline{\nabla} s]_x = \frac{\partial s}{\partial x} \\ [\underline{\nabla} s]_y = \frac{\partial s}{\partial y} \\ [\underline{\nabla} s]_z = \frac{\partial s}{\partial z} \end{cases}$$

$$[\underline{\nabla} \wedge \underline{V}]\begin{cases} [\nabla \wedge V]_x = \frac{\partial Vx}{\partial y} - \frac{\partial Vy}{\partial z} \\ [\nabla \wedge V]_y = \frac{\partial Vx}{\partial z} - \frac{\partial Vz}{\partial x} \\ [\nabla \wedge V]_z = \frac{\partial Vy}{\partial x} - \frac{\partial Vx}{\partial y} \end{cases}$$

$$[\underline{\nabla} \cdot \underline{\underline{\tau}}]\begin{cases} [\nabla \cdot \tau]_x = \frac{\partial \tau xx}{\partial x} + \frac{\partial \tau xy}{\partial y} + \frac{\partial \tau xz}{\partial z} \\ [\nabla \cdot \tau]_y = \frac{\partial \tau yx}{\partial x} + \frac{\partial \tau yy}{\partial y} + \frac{\partial \tau yz}{\partial z} \\ [\nabla \cdot \tau]_z = \frac{\partial \tau zx}{\partial x} + \frac{\partial \tau zy}{\partial y} + \frac{\partial \tau zz}{\partial z} \end{cases}$$

$$\nabla^2 \underline{V}\begin{cases} [\nabla^2 V]_x = \frac{\partial^2 Vx}{\partial x^2} + \frac{\partial^2 Vx}{\partial y^2} + \frac{\partial^2 Vx}{\partial z^2} \\ [\nabla^2 V]_y = \frac{\partial^2 Vy}{\partial x^2} + \frac{\partial^2 Vy}{\partial y^2} + \frac{\partial^2 Vy}{\partial z^2} \\ [\nabla^2 V]_z = \frac{\partial^2 Vz}{\partial x^2} + \frac{\partial^2 Vz}{\partial y^2} + \frac{\partial^2 Vz}{\partial z^2} \end{cases}$$

$$\underline{\nabla}\,\underline{V} = \begin{bmatrix} \frac{\partial Vx}{\partial x} & \frac{\partial Vy}{\partial x} & \frac{\partial Vz}{\partial x} \\ \frac{\partial Vx}{\partial y} & \frac{\partial Vy}{\partial y} & \frac{\partial Vz}{\partial y} \\ \frac{\partial Vx}{\partial z} & \frac{\partial Vy}{\partial z} & \frac{\partial Vz}{\partial z} \end{bmatrix}$$

	$[Vx$	Vy	$Vz]$
$\frac{\partial}{\partial x}$	$\frac{\partial Vx}{\partial x}$	$\frac{\partial Vy}{\partial x}$	$\frac{\partial Vz}{\partial x}$
$\frac{\partial}{\partial y}$	$\frac{\partial Vx}{\partial y}$	$\frac{\partial Vy}{\partial y}$	$\frac{\partial Vz}{\partial y}$
$\frac{\partial}{\partial z}$	$\frac{\partial Vx}{\partial z}$	$\frac{\partial Vy}{\partial z}$	$\frac{\partial Vz}{\partial z}$

$= [\underline{\nabla}\,\underline{V}]$

$$[\underline{V} \cdot \underline{\nabla}\,\underline{V}]\begin{cases} [V \cdot \nabla V]_x = Vx\frac{\partial Vx}{\partial x} + Vy\frac{\partial Vx}{\partial y} + Vz\frac{\partial Vx}{\partial z} \\ [V \cdot \nabla V]_y = Vx\frac{\partial Vy}{\partial x} + Vy\frac{\partial Vy}{\partial y} + Vz\frac{\partial Vy}{\partial z} \\ [V \cdot \nabla V]_z = Vx\frac{\partial Vz}{\partial x} + Vy\frac{\partial Vz}{\partial y} + Vz\frac{\partial Vz}{\partial z} \end{cases}$$

RESUMEN DE OPERACIONES DIFERENCIALES CON EL OPERADOR ∇ CILÍNDRICAS (r ; θ ; z):

$$(\underline{\nabla} \cdot \underline{V}) = \frac{1}{r}\frac{\partial}{\partial r}(rVr) + \frac{1}{r}\frac{\partial V\theta}{\partial \theta} + \frac{\partial Vz}{\partial z}$$

$$(\nabla^2 s) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$(\underline{\underline{\tau}} : \underline{\nabla}\underline{V}) =$$

$$\tau rr\left(\frac{\partial Vr}{\partial r}\right) + \tau\theta\theta\left(\frac{1}{r}\frac{\partial V\theta}{\partial \theta} + \frac{Vr}{r}\right) + \tau zz\left(\frac{\partial Vz}{\partial z}\right) + \tau r\theta\left(r\frac{\partial}{\partial r}\left(\frac{V\theta}{r}\right) + \frac{1}{r}\frac{\partial Vr}{\partial \theta}\right) + \tau\theta z\left(\frac{1}{r}\frac{\partial Vz}{\partial \theta} + \frac{\partial V\theta}{\partial z}\right) + \tau rz\left(\frac{\partial Vz}{\partial r} + \frac{\partial Vr}{\partial z}\right)$$

$$[\underline{\nabla} s]\begin{cases} [\underline{\nabla} s]_r = \dfrac{\partial s}{\partial r} \\ [\underline{\nabla} s]_\theta = \dfrac{1}{r}\dfrac{\partial s}{\partial \theta} \\ [\underline{\nabla} s]_z = \dfrac{\partial s}{\partial z} \end{cases}$$

$$[\underline{\nabla} \wedge \underline{V}]\begin{cases} [\nabla \wedge V]_r = \dfrac{1}{r}\dfrac{\partial Vz}{\partial \theta} - \dfrac{\partial V\theta}{\partial z} \\ [\nabla \wedge V]_\theta = \dfrac{\partial Vr}{\partial z} - \dfrac{\partial Vz}{\partial r} \\ [\nabla \wedge V]_z = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rV\theta) - \dfrac{1}{r}\dfrac{\partial Vr}{\partial \theta} \end{cases}$$

$$[\underline{\nabla} \cdot \underline{\underline{\tau}}]\begin{cases} [\nabla \cdot \tau]_r = \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\tau rr) + \dfrac{1}{r}\dfrac{\partial \tau r\theta}{\partial \theta} - \dfrac{1}{r}\tau\theta\theta + \dfrac{\partial \tau rz}{\partial z} \\ [\nabla \cdot \tau]_\theta = \dfrac{1}{r}\dfrac{\partial \tau\theta\theta}{\partial \theta} + \dfrac{\partial \tau r\theta}{\partial r} + \dfrac{2}{r}\tau r\theta + \dfrac{\partial \tau\theta z}{\partial z} \\ [\nabla \cdot \tau]_z = \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\tau rz) + \dfrac{1}{r}\dfrac{\partial \tau\theta z}{\partial \theta} + \dfrac{\partial \tau zz}{\partial z} \end{cases}$$

$$\nabla^2\underline{V}\begin{cases} [\nabla^2 V]_r = \dfrac{\partial}{\partial r}\left(\dfrac{1}{r}\dfrac{\partial}{\partial r}(rVr)\right) + \dfrac{1}{r^2}\dfrac{\partial^2 Vr}{\partial \theta^2} - \dfrac{2}{r^2}\dfrac{\partial V\theta}{\partial \theta} + \dfrac{\partial^2 Vr}{\partial z^2} \\ [\nabla^2 V]_\theta = \dfrac{\partial}{\partial r}\left(\dfrac{1}{r}\dfrac{\partial}{\partial r}(rV\theta)\right) + \dfrac{1}{r^2}\dfrac{\partial^2 V\theta}{\partial \theta^2} + \dfrac{2}{r^2}\dfrac{\partial Vr}{\partial \theta} + \dfrac{\partial^2 V\theta}{\partial z^2} \\ [\nabla^2 V]_z = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial Vz}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 Vz}{\partial \theta^2} + \dfrac{\partial^2 Vz}{\partial z^2} \end{cases}$$

$$[\underline{V} \cdot \underline{\nabla}\,\underline{V}]\begin{cases} [V \cdot \nabla V]_r = Vr\dfrac{\partial Vr}{\partial r} + \dfrac{V\theta}{r}\dfrac{\partial Vr}{\partial \theta} - \dfrac{V\theta^2}{r} + Vz\dfrac{\partial Vr}{\partial z} \\ [V \cdot \nabla V]_\theta = Vr\dfrac{\partial V\theta}{\partial r} + \dfrac{V\theta}{r}\dfrac{\partial V\theta}{\partial \theta} + \dfrac{VrV\theta}{r} + Vz\dfrac{\partial V\theta}{\partial z} \\ [V \cdot \nabla V]_z = Vr\dfrac{\partial Vz}{\partial r} + \dfrac{V\theta}{r}\dfrac{\partial Vz}{\partial \theta} + Vz\dfrac{\partial Vz}{\partial z} \end{cases}$$

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RESUMEN DE OPERACIONES DIFERENCIALES CON EL OPERADOR ∇ ESFÉRICAS (r ; θ ; Φ):

$$(\underline{\nabla} \cdot \underline{V}) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, Vr\right) + \frac{1}{r\ sen\theta}\frac{\partial}{\partial\theta}\left(V\theta\ sen\theta\right) + \frac{1}{r\ sen\theta}\frac{\partial}{\partial\phi}\left(V\phi\right)$$

$$(\nabla^2 s) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2 sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial s}{\partial\theta}\right) + \frac{1}{r^2 sen^2\theta}\frac{\partial^2 s}{\partial\phi^2}$$

$$\left(\underline{\underline{\tau}} : \underline{\nabla}\underline{V}\right) = \tau rr\left(\frac{\partial Vr}{\partial r}\right) + \tau\theta\theta\left(\frac{1}{r}\frac{\partial V\theta}{\partial\theta} + \frac{Vr}{r}\right) + \tau\phi\phi\left(\frac{1}{rsen\theta}\frac{\partial V\phi}{\partial\phi} + \frac{Vr}{r} + \frac{V\theta\cot\theta}{r}\right) + \tau r\theta\left(\frac{\partial V\theta}{\partial r} + \frac{1}{r}\frac{\partial Vr}{\partial\theta} - \frac{V\theta}{r}\right) +$$
$$\tau r\phi\left(\frac{\partial V\phi}{\partial r} + \frac{1}{rsen\theta}\frac{\partial Vr}{\partial\phi} - \frac{V\phi}{r}\right) + \tau\theta\phi\left(\frac{1}{r}\frac{\partial V\phi}{\partial\theta} + \frac{1}{rsen\theta}\frac{\partial V\theta}{\partial\phi} - \frac{\cot\theta}{r}V\phi\right)$$

$$[\underline{\nabla}s]\begin{cases}[\underline{\nabla}s]_r = \dfrac{\partial s}{\partial r}\\ [\underline{\nabla}s]_\theta = \dfrac{1}{r}\dfrac{\partial s}{\partial\theta}\\ [\underline{\nabla}s]_\phi = \dfrac{1}{rsen\theta}\dfrac{\partial s}{\partial\phi}\end{cases}$$

$$[\underline{\nabla}\wedge\underline{V}]\begin{cases}[\nabla\wedge V]_r = \dfrac{1}{rsen\theta}\dfrac{\partial}{\partial\theta}\left(V\phi sen\theta\right) - \dfrac{1}{rsen\theta}\dfrac{\partial V\theta}{\partial\phi}\\ [\nabla\wedge V]_\theta = \dfrac{1}{rsen\theta}\dfrac{\partial Vr}{\partial\phi} - \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(rV\phi\right)\\ [\nabla\wedge V]_\phi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(rV\theta\right) - \dfrac{1}{r}\dfrac{\partial Vr}{\partial\theta}\end{cases}$$

$$[\underline{\nabla}\cdot\underline{\underline{\tau}}]\begin{cases}[\nabla\cdot\tau]_r = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\tau rr\right) + \dfrac{1}{rsen\theta}\dfrac{\partial}{\partial\theta}\left(\tau r\theta sen\theta\right) + \dfrac{1}{rsen\theta}\dfrac{\partial\tau r\phi}{\partial\phi} - \dfrac{\tau\theta\theta + \tau\phi\phi}{r}\\ [\nabla\cdot\tau]_\theta = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\tau r\theta\right) + \dfrac{1}{rsen\theta}\dfrac{\partial}{\partial\theta}\left(\tau\theta\theta sen\theta\right) + \dfrac{1}{rsen\theta}\dfrac{\partial\tau\theta\phi}{\partial\phi} + \dfrac{\tau r\theta}{r} - \dfrac{\cot\theta}{r}\tau\phi\phi\\ [\nabla\cdot\tau]_\phi = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r\tau r\phi\right) + \dfrac{1}{r}\dfrac{\partial\tau\theta\phi}{\partial\theta} + \dfrac{1}{rsen\theta}\dfrac{\partial\tau\phi\phi}{\partial\phi} + \dfrac{\tau r\phi}{r} + \dfrac{2\cot\theta}{r}\tau\theta\phi\end{cases}$$

$$\nabla^2\underline{V}\begin{cases}[\nabla^2 V]_r = \nabla^2 Vr - \dfrac{2Vr}{r^2} - \dfrac{2}{r^2}\dfrac{\partial V\theta}{\partial\theta} - \dfrac{2V\theta\cot\theta}{r^2} - \dfrac{2}{r^2 sen\theta}\dfrac{\partial V\phi}{\partial\phi}\\ [\nabla^2 V]_\theta = \nabla^2 V\theta - \dfrac{2}{r^2}\dfrac{\partial Vr}{\partial\theta} - \dfrac{V\theta}{r^2 sen^2\theta} - \dfrac{2\cos\theta}{r^2 sen^2\theta}\dfrac{\partial V\phi}{\partial\phi}\\ [\nabla^2 V]_\phi = \nabla^2 V\phi - \dfrac{V\phi}{r^2 sen^2\phi} + \dfrac{2}{r^2 sen^2\theta}\dfrac{\partial Vr}{\partial\phi} + \dfrac{2\cos\theta}{r^2 sen^2\theta}\dfrac{\partial V\theta}{\partial\phi}\end{cases}$$

$$[\underline{V}\cdot\underline{\nabla}\,\underline{V}]\begin{cases}[V\cdot\nabla V]_r = Vr\dfrac{\partial Vr}{\partial r} + \dfrac{V\theta}{r}\dfrac{\partial Vr}{\partial\theta} + \dfrac{V\phi}{rsen\theta}\dfrac{\partial Vr}{\partial\phi} - \dfrac{V\theta^2 + V\phi^2}{r}\\ [V\cdot\nabla V]_\theta = Vr\dfrac{\partial V\theta}{\partial r} + \dfrac{V\theta}{r}\dfrac{\partial V\theta}{\partial\theta} + \dfrac{V\phi}{rsen\theta}\dfrac{\partial V\theta}{\partial\phi} + \dfrac{VrV\theta}{r} - \dfrac{V\phi^2\cot\theta}{r}\\ [V\cdot\nabla V]_\phi = Vr\dfrac{\partial V\phi}{\partial r} + \dfrac{V\theta}{r}\dfrac{\partial V\phi}{\partial\theta} + \dfrac{V\phi}{rsen\theta}\dfrac{\partial V\phi}{\partial\phi} + \dfrac{V\phi Vr}{r} + \dfrac{V\theta V\phi\cot\theta}{r}\end{cases}$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}$$

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