Effects of Bursty Crosstalk in DSL

Fernando Ramírez-Mireles, *Senior Member, IEEE*

***Abstract*—In this letter we propose a simple model for the bursty operation of digital subscriber line (DSL) modems. Based on this model, we derive the autocorrelation and the power spectrum density (PSD) of the bursty crosstalk. We analyze the effects of bursty operation in both the crosstalk PSD and the training of the DSL modem. This analysis suggests that both the duty cycle and the bursty activity rate must be optimized when bursty crosstalk is used to increase the performance of crosstalk-limited systems.**

***Index Terms*—Crosstalk, data communication, digital subscriber loops, modems.**

I. Introduction

DIGITAL subscriber line (DSL) allows high speed transmission using the twisted copper lines originally used for telephone services. Twisted pairs are deployed using telephone cables in which twisted pairs are packed closely together. Crosstalk electromagnetically coupled from neighboring twisted pairs also carrying DSL signals can severely limit the performance of DSL systems [1].

In general, DSL modems are designed to operate continuously, even in the periods of time in which there is no information to transmit. In order to reduce the crosstalk introduced in the telephone cable, it is desirable that DSL modems transmit in bursty mode, i.e., they stop transmitting when there is no information to send.

In this letter we propose a simple model for the bursty operation of DSL modems [2]. Based on this model, we derive the autocorrelation and the power spectrum density (PSD) for the bursty crosstalk. The PSD is used to analyze the effects of both the duty cycle and bursty activity rate.

II. Crosstalk Representation

A. Continuous Crosstalk

Continuous crosstalk is represented by

$$c(t) = \sum_i c_i(t)$$

where $c_i(t)$ are the individual components of the crosstalk produced by different types of disturbers, i.e., neighboring high speed data signals coupled electromagnetically. All $c_i(t)$ are assumed to be mean-zero, independent of each other, and wide sense stationary.

Manuscript received September 10, 2001. The associate editor coordinating the review of this letter and approving it for publication was Dr. N. Van Stralen.

The author is with Ikanos Communications, Fremont, CA 94538 USA (e-mail: ramirezm@ieee.org).

Publisher Item Identifier S 1089-7798(02)00983-3.

The correlation function of $c(t)$ is

$$R_c(\tau) = \mathcal{E}\{c(t)c(t+\tau)\} = \sum_i R_i(\tau)$$

where $R_i(\tau)$ is the autocorrelation of $c_i(t)$, and $\mathcal{E}\{\cdot\}$ is the expected value operator. The PSD of $c(t)$ is

$$S_c(f) = \mathcal{F}\{R_c(\tau)\} = \sum_i S_i(f)$$

where $S_i(f)$ is the PSD of $c_i(t)$, and $\mathcal{F}\{\cdot\}$ denotes the Fourier transform.

B. Bursty Function

Bursty operation is achieved multiplying each $c_i(t)$ by a bursty function $b_i(t)$ that takes the value 1 or 0 depending if there is information to transmit or not, respectively. Hence, the 1 and 0 values appears in a random fashion. We assume that the functions $b_i(t)$ are all independent of each other, identically distributed, and wide sense stationary. We further assume that $c_i(t)$ and $b_k(t)$ are independent for all values of i, k.

The expected value of $b_i(t)$ is $\overline{b}$ and the correlation function of $b_i(t)$ is

$$R_b(\tau) = \mathcal{E}\{b_i(t)b_i(t+\tau)\}$$

for all values of i. The PSD of $b_i(t)$ is

$$S_b(f) = \mathcal{F}\{R_b(\tau)\}.$$

C. Bursty Crosstalk

Bursty crosstalk is represented by

$$\tilde{c}(t) = \sum_i b_i(t)c_i(t).$$

The correlation function of $\tilde{c}(t)$ is

$$\tilde{R}_c(\tau) = \mathcal{E}\{\tilde{c}(t)\tilde{c}(t+\tau)\} = \sum_i \tilde{R}_i(\tau)$$

where $\tilde{R}_i(\tau)$ is the autocorrelation of $b_i(t) \cdot c_i(t)$. The PSD of $\tilde{c}(t)$ is

$$\tilde{S}_c(f) = \mathcal{F}\{\tilde{R}_c(\tau)\} = \sum_i \tilde{S}_i(f)$$

where $\tilde{S}_i(f)$ is the PSD of $b_i(t) \cdot c_i(t)$.

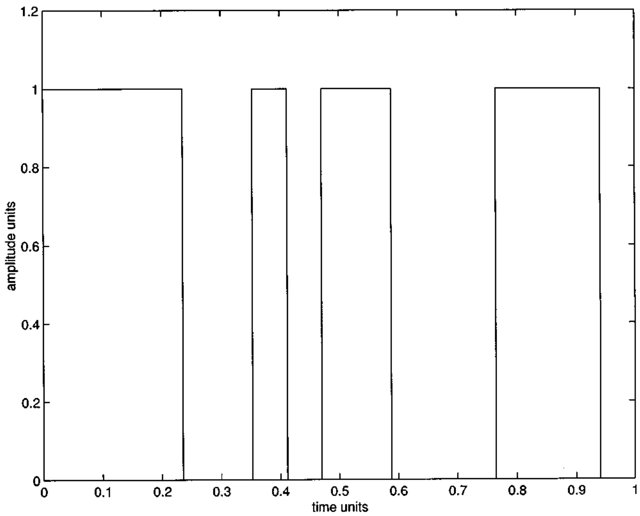


Fig. 1. The bursty function $b_i(t)$.

III. Analysis of Bursty Crosstalk

Using the previous definitions and assumptions, we can write

$$\tilde{R}_c(\tau) = R_b(\tau) \cdot R_c(\tau)$$

and therefore

$$\tilde{S}_c(f) = S_b(f) * S_c(f)$$

where $*$ denotes convolution operation.

The function $b_i(t)$ under consideration is known as *random telegraph waveform* [3]. Fig. 1 depicts one possible realization of $b_i(t)$.

The $b_i(t)$ may assume at any instant of time either the value $b_i(t) = 0$ with probability p_0, or the value $b_i(t) = 1$ with probability p_1, and it makes independent random transitions from one value to the other. The expected value is

$$\bar{b} = p_1.$$

When $p_1 = 1/2$ the probability that k transitions occur in a time interval of length T is given by the Poisson distribution

$$P(kT) = \frac{(aT)^k}{k!} \exp(-aT)$$

where a is the average number of transitions per second. Hence, the autocorrelation of $b_i(t)$ is

$$\begin{aligned} R_b(\tau) &= \text{Prob}(b_i(t) = 1,\, b_i(t+\tau) = 1) \\ &= p_1 \text{Prob}(k \text{ even},\, T = |\tau|) \\ &= \frac{p_1}{2} \left[1 + \exp(-2a|\tau|)\right] \end{aligned}$$

and the PSD of $b_i(t)$ is

$$S_b(f) = \frac{p_1}{2} \left(\delta(f) + \frac{1}{\pi} \frac{(2a/2\pi)}{(2a/2\pi)^2 + f^2} \right).$$

Therefore, the autocorrelation of $\tilde{c}(t)$ is

$$\tilde{R}_c(\tau) = \left[\frac{p_1}{2} R_c(\tau) + \frac{p_1}{2} R_c(\tau) \exp(-2a|\tau|) \right]$$

and the PSD of $\tilde{c}(t)$ is

$$\tilde{S}_c(f) = \tilde{S}_1(f) + \tilde{S}_2(f)$$

where

$$\begin{aligned} \tilde{S}_1(f) &= \frac{p_1}{2} S_c(f) \\ \tilde{S}_2(f) &= p_1 H_a(f) * S_c(f) \\ H_a(f) &= \frac{1}{2\pi} \frac{(2a/2\pi)}{(2a/2\pi)^2 + f^2}. \end{aligned}$$

The PSD $\tilde{S}_1(f)$ is a scaled version of the original crosstalk PSD $S_c(f)$. The PSD $\tilde{S}_2(f)$ is a "filtered" version of the PSD $S_c(f)$. Clearly, the total power in $\tilde{S}_c(f)$ is proportional to p_1

$$\tilde{R}_c(0) = p_1 R_c(0)$$

for any value of a. The effect of the value of a on $\tilde{S}_c(f)$ is illustrated with an example showing how a affects the disturber's PSD causing crosstalk.

A. Effect of a on the Disturber's PSD

In this example we assume that the crosstalk $c(t)$ is caused by one type of disturber 2B1Q HDSL with PSD [1]

$$\text{PSD}_{\text{HDSL}}(f) = K_{\text{HDSL}} \frac{2}{f_0} \frac{[\sin(\pi f/f_0)]^2}{(\pi f/f_0)^2} \frac{1}{1 + (f/f_{3\text{dB}})^8}$$

where $f_0 = 392$ kHz, $f_{3\text{dB}} = 196$ kHz, $K_{\text{HDSL}} = 5/9 \cdot V_p^2/R$, $V_p = 2.7$ V, and $R = 135\ \Omega$. Fig. 2(a) depicts the disturber's PSD for $p_1 = 0.5$ and $a = 0,\ 1,\ 10,\ 100$. The value $a = 0$ corresponds to the original PSD.

B. Effect of a on Modem Training

The use of $b(t)$ impacts the training of a modem in the presence of bursty crosstalk. To analyze this, consider the time-average estimate of the power of $b(t)$

$$\hat{\mathcal{R}}_b(0) \simeq \frac{1}{2T} \int_{-T}^{T} b^2(t) dt$$

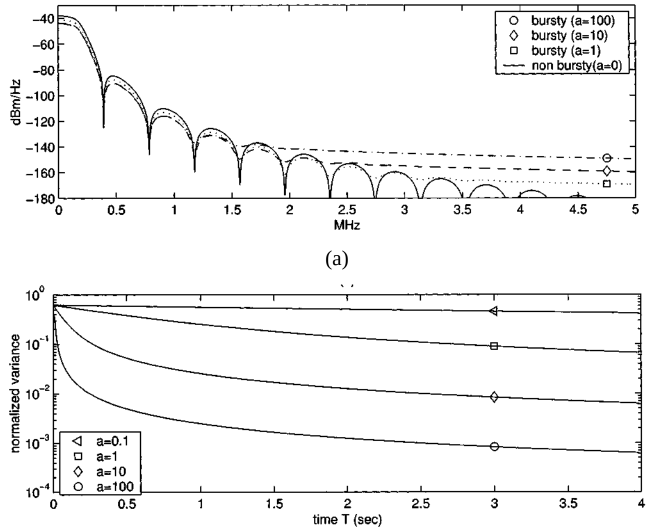
where $2T$ is the observation interval used for training the modem. Using the results in [4], it can be shown that $\hat{\mathcal{R}}_b(0)$ has variance

$$\sigma_b^2 = \frac{1}{2T} \int_{-2T}^{2T} C(\alpha,\, 0) \cdot \left(1 - \frac{|\alpha|}{2T}\right) \cdot d\alpha$$

where

$$C(\alpha,\, 0) = \frac{p_1}{2} \left[1 + \exp(-2a|\alpha|)\right] - (p_1)^2$$

and $C(\alpha,\, \tau)$ is the autocovariance of the product $b(t)b(t+\tau)$. Fig. 2(b) depicts (σ_b^2/p_1) for $p_1 = 0.5$ and $a = 0.1,\ 1,\ 10,\ 100$.



(b)

Fig. 2. (a) Compares the original 2B1Q HDSL disturber's PSD with the modified disturber's PSD for $p_1 = 0.5$ and $a = 0, 1, 10, 100$. (b) Depicts (σ_b^2/p_1) as a function of T for $a = 0.1, 1, 10, 100$.

IV. Discussion and Conclusion

From the example and the expressions for $\tilde{R}_c(\tau)$, $\tilde{S}_c(f)$ and σ_b^2 we can make three conclusions.

First, it is clear that the total power of the bursty crosstalk is proportional to the duty cycle (i.e., to p_1). In general, the more the duty cycle is reduced, the more the power in the bursty crosstalk will be reduced. This result is intuitive.

Second, observation of Fig. 2(a) shows that higher values of a increases the sidelobes of the modified disturber's PSD. This in turn could translate in increase of sidelobes in $\tilde{S}_c(f)$. In these cases, a lower value of a is desirable. For example, to keep the sidelobes of the modified disturber's PSD below -140 dBm/Hz for $f > 2$ MHz we need $a < 100$.

Third, Fig. 2(b) shows that for a fixed observation interval T, a higher value of a results in a smaller variance of the power estimate $\hat{\mathcal{R}}_b(0)$. In this case, a higher value of a is desirable. For example, for $T = 2$ seconds we need $a > 100$ to get an estimate variance of less than 0.001.

The results in this analysis suggest that both the duty cycle and the bursty activity rate must be optimized when bursty crosstalk is used to increase the performance of crosstalk-limited systems.

Appendix

The previous analysis was done for $p_1 = p_0 = 0.5$. In this particular case only, the discontinuity points of $b(t)$ have Poisson distribution. For other values of p_1 and p_0, the process $b(t)$ is modeled as a Markov process [5]. In this general case $b(t)$ is asymptotically stationary with mean $\overline{b} = p_1$, and autocorrelation

$$R_b(\tau) = \left(p_1 - p_0 + \frac{1}{2}\right)^2 + p_1 p_0 \exp\left(-\mu_1 \left(1 + \frac{p_1}{p_0}\right) |\tau|\right).$$

Using the results in [4], it can be shown that

$$C(\alpha, 0) = p_1 + p_1 p_0 \exp\left(-\mu_1 \left(1 + \frac{p_1}{p_0}\right)|\tau|\right) - \left[\left(p_1 - p_0 + \frac{1}{2}\right)^2 + p_1 p_0\right]^2$$

where μ_1 is the transition probability rate for transitions from $b(t) = 1$ to $b(t) = 0$.

Acknowledgment

The author thanks the anonymous reviewers for their valuable comments.

References

[1] T. Starr, J. M. Cioffi, and P. J. Silverman, *Understanding Digital Subscriber Line Technology*: Prentice Hall, 1999.
[2] F. Ramírez-Mireles, "Analysis of bursty crosstalk in DSL," in *IEEE Global Communications Conf. (Globecom'2001)*, Nov. 2001.
[3] W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*. New York: IEEE Press, 1987, pp. 61–62.
[4] A. Papolulis, *Signal Analysis*. Singapore: McGraw Hill, 1984, pp. 359–360.
[5] ——, *Probability, Random Variables, and Stochastic Process*. Singapore: McGraw Hill, 1984, pp. 392–393.