

Analysis of Bursty Crosstalk in DSL

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Abstract—In this paper we propose a simple model for the bursty operation of digital subscriber line (DSL) modems. Based on this model, we derive a power spectrum density (PSD) for the bursty crosstalk. This analysis suggests that both the duty cycle and the bursty activity rate must be optimized when bursty crosstalk is used to increase the performance of crosstalk-limited systems.

I. INTRODUCTION

DIGITAL subscriber line allows high speed transmission using the twisted cooper lines originally used for telephone services. Twisted pairs are deployed using telephone cables in which twisted pairs are packed closely together. Crosstalk electromagnetically coupled from neighboring twisted pairs also carrying DSL signals can severely limit the performance of DSL systems [1].

In general, DSL modems are designed to operate continuously, even in those periods of time in which there is no information to transmit. In order to reduce the crosstalk introduced in the telephone cable, it is desirable that DSL modems transmit in bursty mode, i.e., they stop transmitting when there is no information to send.

In this paper we propose a simple model for the bursty operation of DSL modems. Based on this model, we derive a PSD for the bursty crosstalk. The PSD is used to analyze the effects of both the duty cycle and bursty activity rate.

II. CROSSTALK REPRESENTATION

A. Continuous Crosstalk

Continuous crosstalk is represented by

$$c(t) = \sum_i c_i(t),$$

where $c_i(t)$ are the individual components of the crosstalk, i.e., neighboring high speed data signals coupled electromagnetically. All $c_i(t)$ are assumed to be mean-zero, independent of each other, and wide sense stationary.

The correlation function of $c(t)$ is

$$R_c(\tau) = \mathcal{E}\{c(t)c(t+\tau)\} = \sum_i R_i(\tau),$$

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where $R_i(\tau)$ is the autocorrelation of $c_i(t)$, and $\mathcal{E}\{\cdot\}$ is the expected value operator. The PSD of $c(t)$ is

$$S_c(f) = \mathcal{F}\{R_c(\tau)\} = \sum_i S_i(f),$$

where $S_i(f)$ is the PSD of $c_i(t)$, and $\mathcal{F}\{\cdot\}$ denotes the Fourier transform.

B. Bursty Function

Bursty operation is achieved multiplying each $c_i(t)$ by a bursty function $b_i(t)$ that takes the values 1 and 0 depending if there is information to transmit or not, respectively. Hence, the 1 and 0 values appears in a random fashion. We assume that the functions $b_i(t)$ are all independent of each other, identically distributed, and wide sense stationary. We further assume that $c_i(t)$ and $b_k(t)$ are independent for all values of i, k .

The expected value of $b_i(t)$ is \bar{b} and the correlation function of $b_i(t)$ is

$$R_b(\tau) = \mathcal{E}\{b_i(t)b_i(t+\tau)\},$$

for all values of i . The PSD of $b_i(t)$ is

$$S_b(f) = \mathcal{F}\{R_b(\tau)\}.$$

C. Bursty Crosstalk

Bursty crosstalk is represented by

$$\tilde{c}(t) = \sum_i b_i(t)c_i(t).$$

The correlation function of $\tilde{c}(t)$ is

$$\tilde{R}_c(\tau) = \mathcal{E}\{\tilde{c}(t)\tilde{c}(t+\tau)\} = \sum_i \tilde{R}_i(\tau),$$

where $\tilde{R}_i(\tau)$ is the autocorrelation of $b_i(t) \cdot c_i(t)$. The PSD of $\tilde{c}(t)$ is

$$\tilde{S}_c(f) = \mathcal{F}\{\tilde{R}_c(\tau)\} = \sum_i \tilde{S}_i(f),$$

where $\tilde{S}_i(f)$ is the PSD of $b_i(t) \cdot c_i(t)$.

III. BURSTY CROSSTALK ANALYSIS

Using the previous definitions and assumptions, we can write

$$\tilde{R}_c(\tau) = R_b(\tau) \cdot R_c(\tau),$$

and therefore

$$\tilde{S}_c(f) = S_b(f) * S_c(f),$$

where $*$ denotes convolution operation.

The function $b_i(t)$ under consideration is known as *random telegraph waveform* [2]. Fig. 1 depicts one possible realization of $b_i(t)$.

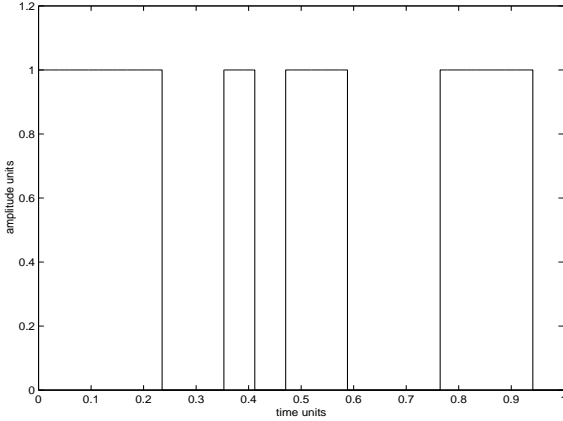


Fig. 1. The bursty function $b_i(t)$

The $b_i(t)$ may assume at any instant of time either the value $b_i(t) = 0$ with probability p_0 , or the value $b_i(t) = 1$ with probability p_1 , and it makes independent random traversals from one value to the other. The expected value is

$$\bar{b} = p_1.$$

When $p_1 = \frac{1}{2}$ the probability that k traversals occur in a time interval of length T is given by the Poisson distribution

$$P(kT) = \frac{(aT)^k}{k!} \exp(-aT),$$

where a is the average number of traversals per unit time. The autocorrelation of $b_i(t)$ is

$$\begin{aligned} R_b(\tau) &= \text{Prob}(b_i(t) = 1, b_i(t + \tau) = 1) \\ &= p_1 \text{Prob}(k \text{ even}, T = |\tau|) \\ &= \frac{p_1}{2} [1 + \exp(-2a|\tau|)], \end{aligned}$$

and the PSD of $b_i(t)$ is

$$S_b(f) = \frac{p_1}{2} \left[\delta(f) + \frac{1}{\pi} \frac{(2a/2\pi)}{(2a/2\pi)^2 + f^2} \right].$$

Therefore, the autocorrelation of $\tilde{c}(t)$ is

$$\tilde{R}_c(\tau) = \left[\frac{p_1}{2} R_c(\tau) + \frac{p_1}{2} R_c(\tau) \exp(-2a|\tau|) \right],$$

and the PSD of $\tilde{c}(t)$ is

$$\tilde{S}_c(f) = \tilde{S}_1(f) + \tilde{S}_2(f),$$

where

$$\tilde{S}_1(f) = \frac{p_1}{2} S_c(f),$$

and

$$\tilde{S}_2(f) = p_1 H_a(f) * S_c(f),$$

and

$$H_a(f) = \frac{1}{2\pi} \frac{(2a/2\pi)}{(2a/2\pi)^2 + f^2}.$$

The PSD $\tilde{S}_1(f)$ is a scaled version of the original crosstalk PSD $S_c(f)$. The PSD $\tilde{S}_2(f)$ is a “filtered” version of the PSD $S_c(f)$. Clearly, the total power in $\tilde{S}_c(f)$ is proportional to p_1

$$\tilde{R}_c(0) = p_1 R_c(0),$$

for any value of a . The effect of the value of a on $\tilde{S}_c(f)$ is illustrated with an example showing the effect of a on the disturber’s PSD causing crosstalk.

A. Effect of a on the disturber’s PSD

In this example we assume that the crosstalk $c(t)$ is caused by one type of disturber 2B1Q HDSL with PSD [1]

$$\text{PSD}_{\text{HDSL}}(f) = K_{\text{HDSL}} \frac{2}{f_0} \frac{[\sin(\pi f/f_0)]^2}{(\pi f/f_0)^2} \frac{1}{1 + (f/f_{3\text{dB}})^8},$$

where $f_0 = 392$ KHz, $f_{3\text{dB}} = 196$ KHz, $K_{\text{HDSL}} = \frac{5}{9} \cdot \frac{V_p^2}{R}$, $V_p = 2.7$ Volts, and $R = 135 \ \Omega$. Fig. 2(a) depicts the disturber’s PSD for $p_1 = 0.5$ and $a = 0, 1, 10, 100$. The original PSD correspond to $a = 0$.

B. Effect of a on the modem training

The use of $b(t)$ impacts the training of a modem in the presence of bursty crosstalk. To analyze this, consider the time-average estimate of the power of $b(t)$

$$\hat{\mathcal{R}}_b(0) \simeq \frac{1}{2T} \int_{-T}^T b^2(t) dt,$$

where $2T$ is the observation interval used for training the modem. Using the results in [3], it can be shown that $\hat{\mathcal{R}}_b(0)$ has variance

$$\sigma_b^2 = \frac{1}{2T} \int_{-2T}^{2T} C(\alpha, 0) \left(1 + \frac{|\alpha|}{2T}\right) d\alpha,$$

where

$$C(\alpha, 0) = \frac{p_1}{2} [1 + \exp(-2a|\alpha|)] - (p_1)^2,$$

and $C(\alpha, \tau)$ is the autocovariance of the product $x(t)x(t+\tau)$. Fig. 2(d) depicts (σ_b^2/p_1) for $p_1 = 0.5$ and $a = 0.1, 1, 10, 100$.

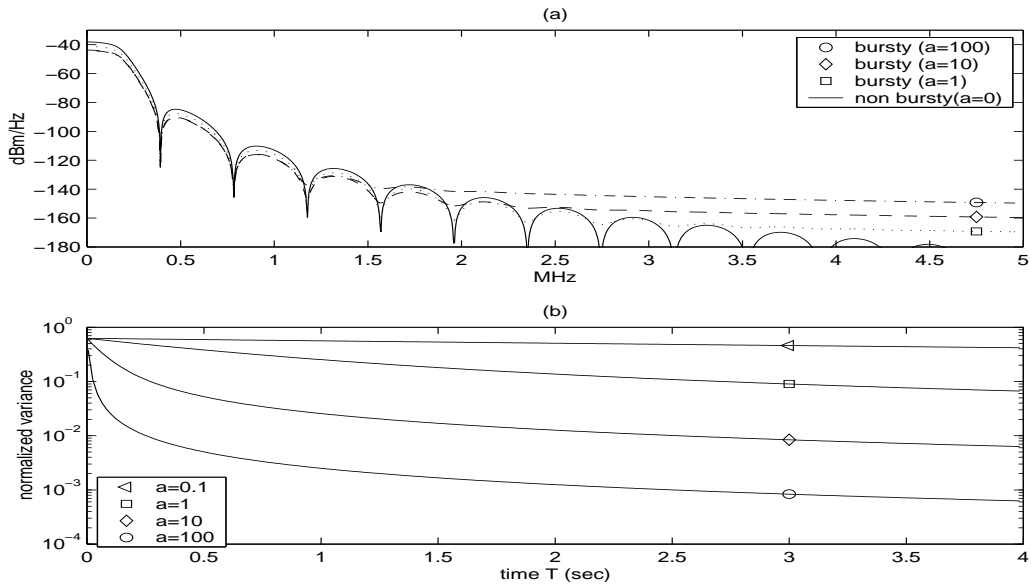


Fig. 2. Figure (a) compares the original 2B1Q HDSL disturber's PSD with the modified disturber's PSD for $p_1 = 0.5$ and $a = 0.1, 1, 10, 100$. Figure (b) depicts (σ_b^2/p_1) as a function of T for different a .

IV. DISCUSSION AND CONCLUSION

From the example and the expressions for $\tilde{R}_c(\tau)$, $\tilde{S}_c(f)$ and σ_b^2 we can make three conclusions.

First, it is clear that the total power of the bursty crosstalk is proportional to the duty cycle (i.e., to p_1). In general, the more the duty cycle is reduced, the more the power in the bursty crosstalk will be reduced. This result is intuitive.

Second, observation of Figs. 2(a), 2(b), and 2(c) shows that higher values of a increases the sidelobes of the modified disturber's PSD. This in turn could translate in increase of sidelobes in $\tilde{S}_c(f)$. In these cases, a lower value of a is desirable.

Third, Fig. 2(d) shows that for a fixed observation interval T ,

a higher value of a results in a smaller variance of the power estimate $\hat{R}_b(0)$. In this case, a higher value of a is desirable. For example, for $T = 2$ we need $a \geq 100$ to get an estimate variance of less than 0.001.

The results in this analysis suggest that both the duty cycle and the bursty activity rate must be optimized when bursty crosstalk is used to increase the performance of crosstalk-limited systems.

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