Mobile Radio Channel – modeling of time-variant multi-path fading channel

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Abstract— **The time-variant description of the mobile channel is mainly determined by the surrounding environment between Base and Mobile stations. Different channels models can be derived according to the nature of the surrounding. The present document follows the models described by the COST 207 which include rural area, urban area and hilly terrain. The description of the mobile channel is given according to the Bello's systems functions which include the Power delay profile, Doppler Power Spectrum and the Scattering function. In order to get the mentioned functions is necessary the simulation of the channel which is implemented as a tap-delay line or time varying FIR filter. For such objective, the filter coefficients were derived following the Rice method to produce the appropriate statistical response according to the COST model.**

Index Terms— **COST 207, Time-varying channel, fading, complex Gaussian colored random process, Doppler shift.**

I. INTRODUCTION

Two important features of the mobile radio channel are its time varying response and its random behavior. The first is due mainly to the movement of the mobile station and the second is given by the unknown and unpredictable environment that surrounds both Base Station (BS) and the Mobile Station (MS). The creation of a model to study the channel response includes the effect of reflected, scattered, diffracted and absorbed electromagnetic waves. In the study of the mobile channel the distinction between them is not useful due that there is no knowledge of the location and physical dimension of the different obstacles. For this reason normally all of these effects are group and called scattering. Besides, a very important effect produced by the several trajectories and the movement is the Doppler shift. The conjunction of the multi-path signals which represent several versions of the transmitted signal with different time delays, amplitudes, phase and Doppler shifts produces what is called fast fading as a result of the superposition. This superposition might be constructive as also destructive causing rapid variations in the range of 40 dB for small intervals of time or distances in the order of few tens of the wave-length.

Section II of the present document starts by presenting shortly the Tapped-delay line representation of frequency-selective and time variant channel and the Bello's System functions and their corresponding interpretation. Afterwards, the Rice method for the generation of the filter coefficients is presented in Section III, special consideration to achieve a good approximation to the desired model are also exposed. Section IV is dedicated to present the COST 207 model. Finally, an explanation of the deterministic channel simulation and the corresponding consideration of a discrete model are illustrated in section V.

II. FREQUENCY-SELECTIVE CHANNEL MODEL

Typically a channel model can be classified either as frequency-nonselective or frequency selective channel. The first correspond to channels where the differences between the propagation delays of the received electromagnetic waves can be ignored compared to the symbol intervals. However as soon as the symbol interval became shorter (higher data rates), this difference cannot be ignored anymore and therefore must be used a frequency selective model. To describe the non-selective channel usually are employed the Rice, Rayleigh or Susuki models which can be realized by the combination of real-valued colored Gaussian random process. The same can be said for the frequency selective case and the section III is dedicated to explain how to get that kind of random processes.

A complete set of systems function were derive by Bello to describe the frequency selective stochastic channel or also so called WSSUS – Wide Sense stationary Uncorrelated Scatering. The main assumption is a stationary behavior during the observation time interval and the un-correlation between the different trajectories. Experimentally, it has been shown that this assumption is valid when the mobile unit covers a distance in the order of less than a few tens of wavelengths.

The equation (1) shows the impulse response of the mobile radio channel, where $a_n(t)$ correspond to the delay coefficients,

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$\tau_n(t)$ to the propagation delays and $\alpha_n(t)$ to the variant phases (merge the complex part of the delay coefficients and the phase shift).

$$\underline{h}(\tau,t) = \sum_{n=0}^{N-1} a_n(t) e^{-j\alpha_n(t)} \delta(\tau - \tau_n(t)). \qquad (1)$$

With the inclusion of the receptor system which incorporate a sampling over the channel response no all the trajectories are resolvable and the channel can be seen as show in equation (2)

$$\underline{h}_{RX}(\tau,t) = \sum_{m=1}^{M} \left[\delta(\tau - m\tau_m) \cdot \sum_{n=0}^{N-1} \left[a_n(t) e^{-j\alpha_n(t)} \delta(\tau - \tau_n(t)) \right]_{\tau = m\tau_m} \right] = \sum_{m=1}^{M} \delta(\tau - m\tau_m) \cdot \underline{z}_m(t) \qquad (2)$$

The inner summation can be treated as one term $\underline{z}_m(t)$ which correspond to a complex Gaussian random stochastic process due to the fact that many paths of the physical impulse response have an effect to each sample of the reconstructed impulse response and taking into account the *Central Limit Theorem*. It can be found that the amplitude of $\underline{z}_m(t)$ follows the Rice distribution given place to the Rice fading and in the case of no line-of-sight (LOS) the Rayleigh distribution. Similarly, the study of the Doppler frequency statistics gives us as a result the well-known Jakes Spectrum.

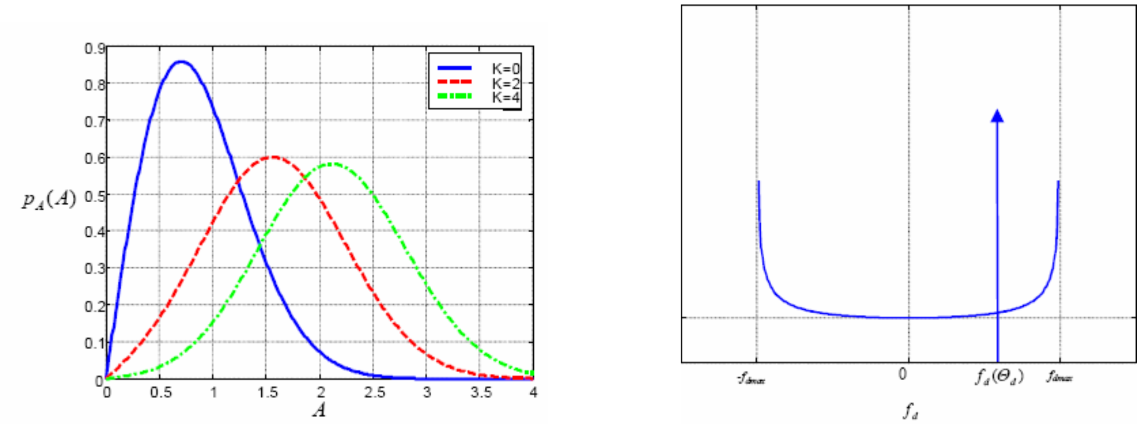

Fig1. Rice Probability density function of the amplitude (left) and Jakes probability density function of the Doppler frequency (right).

According with the equation (2) a mobile radio channel can be simulated as a FIR filter with time-varying coefficients as is shown in the figure 2.

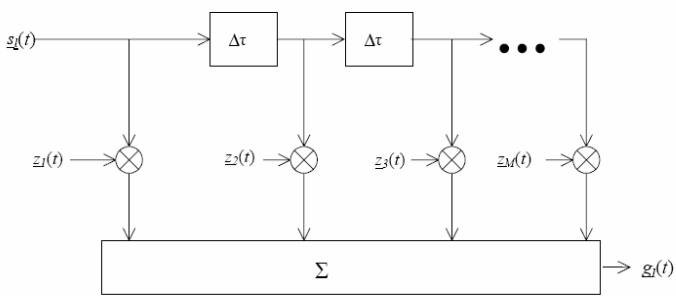

Fig2. FIR-structure for mobile radio channel simulations.

A useful way to describe a random process is by the use of correlations functions and spectral density functions. For the case of WSSUS models, a set of function is set up and called Bello's system functions. Initially the autocorrelation of the impulse response is calculated as

$$\underline{r}_{hh}(\tau_1,\tau_2,t_1,t_2) = \frac{1}{2} E\left\{ \underline{h}^*(\tau_1,t_1)\underline{h}(\tau_2,t_2) \right\}. \qquad (3)$$

Taking into account the assumption of wide sense stationary and uncorrelated scattering it is obtained the expression given in the equation (4) which is called *multipath time-covariance* and represent the average power output as function of the time delay τ and the difference Δt in the observation time.

$$\underline{r}_{hh}(\tau_1,\tau_2,\Delta t) = \begin{cases} \frac{1}{2} E\{h^*(\tau_1,t)h(\tau_1,t+\Delta t)\} = \underline{r}_{hh}(\tau_1,\Delta t) & \text{for} \quad \tau_1 = \tau_2 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

If we set Δt =0, it is obtained the *power delay profile* $\underline{r}_{hh}(\tau) \equiv \underline{r}_{hh}(\tau,0)$ which describes the spread of the power in the τ - domain due to the multipath propagation.

To see the behavior in frequency domain is natural to think to take the Fourier transform of the impulse response and find the transfer function $\underline{H}(f,t) = \int_{-\infty}^{\infty} \underline{h}(\tau,t)e^{-2\pi f\tau}d\tau$. In this point is appropriate notice that the Fourier transform is done in the τ - domain which results in a description of the Doppler shift frequency for the different delay paths.

Likewise to the multipath time-covariance a new correlation function of the transfer function can be found and simplified after use the fact of wide sense stationary. Additionally it's easy to show that this new function correspond to the Fourier transform of the multipath time-covariance.

$$\underline{r}_{HH}(f_1,f_2,\Delta t) = \frac{1}{2} E\{\underline{H}^*(f_1,t)\underline{H}(f_2,t+\Delta t)\} = \int_{-\infty}^{\infty} \underline{r}_{hh}(\tau_1,\Delta t)e^{-2\pi(f_2-f_1)\tau_1}d\tau_1 . \quad (5)$$

With the substitution $\Delta f = f_1 - f_2$, it yields to $\underline{r}_{HH}(\Delta f,\Delta t) = \int_{-\infty}^{\infty} \underline{r}_{hh}(\tau_1,\Delta t)e^{-2\pi\Delta f\tau_1}d\tau_1$. This last function is called *spaced-frequency, spaced-time correlation function* and due to the term Δf it can be said that the model is also WSS in the frequency domain. From this function can be determined the *Doppler Power Spectrum* setting $\Delta f = 0$ and taking its Fourier transform.

$$\underline{S}_{HH}(f_d) = \int_{-\infty}^{\infty} \underline{r}_{HH}(\Delta t)e^{-2\pi f_d \Delta t}d\Delta t . \quad (6)$$

To conclude this brief survey over the correlation functions and the power spectrum a important function that describes the channel in both τ -domain and the f_d -domain is included. This one is the *Scattering function* that corresponds to the Fourier transform of the multipath time-covariance.

$$\underline{S}_{hh}(\tau,f_d) = \int_{-\infty}^{\infty} \underline{r}_{hh}(\tau,\Delta t)e^{-2\pi f_d \Delta t}d\Delta t . \quad (7)$$

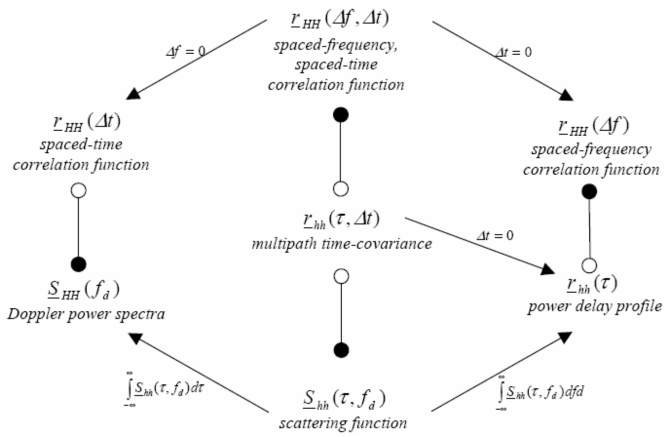

Fig3. Relationships among the channel correlation functions and power spectra

III. RICE METHOD FOR GENERATION OF COMPLEX GAUSSIAN RANDOM PROCESS

In the previous section was showed that for the simulation of the mobile channel as a FIR filter are needed L time-variant complex coefficients for the L different trajectories. Mainly these coefficients must hold the Jakes Spectrum. In section III will be showed that for long delays path the distribution has a Gaussian shape. In any case, the generation for these kinds of processes can be achieved basically either by the filter method or the Rice method. Here a short description of the filter method is presented whereas the Rice method is showed in more detail since it was the method employed for the project. Despite the employed method, the generation of 2 $\mathcal{L}$ real valued colored Gaussian random process is required and play a main role in order to get the desired spectrum.

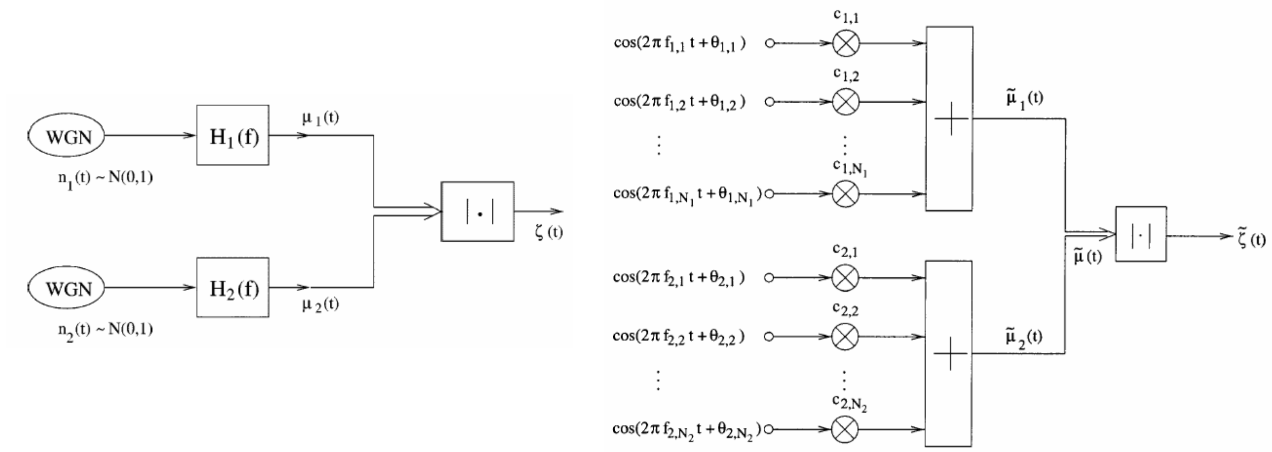

Fig4. Stochastic reference models for colored Gaussian random process $\mu_i(t)$. Left, Filter method. Right, Rice method.

When using the filter method, as shown in the left of figure 1, white Gaussian noise (WGN) $v_i(t)$ is given to the input of a line time-invariant filter, whose transfer function is denoted by $H_i(f)$. In the following, we assume that the filter is ideal, so the transfer function $H_i(f)$ can be adjusted to any given frequency response with arbitrary precision. If $n_i(t) \sim N(0,1)$, then we have a zero-mean stochastic Gaussian random process $\mu_i(t)$ at the filter output, obtaining that the power spectral density $S_{\mu\mu}(f)$ of $\mu_i(t)$ matches the square of the absolute value of the transfer function, $S_{\mu\mu}(f) = |H_i(f)|^2$. Hence, by filtering of white Gaussian noise $v_i(t)$, we obtain a colored Gaussian random process $\mu_i(t)$.

The Rice Method consists in the superposition of an infinite number of weighted harmonic functions with equidistant frequencies and harmonic random phases. Mathematically the process $\mu_i(t)$ can be described as: $\mu_i(t) = \lim_{N_i \to \infty} \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n})$, where $c_{i,n} = 2\sqrt{\Delta f_i \cdot S_{\mu_i \mu_i}(f_{i,n})}$; $f_{i,n} = n\Delta f_i$. Here Δf_i is chosen in such a way $f_{i,n}$ covers the whole relevant frequency range $[-f_{d\max}, f_{d\max}]$ due to the Doppler shift. The phases $\theta_{i,n}$ with n = 1,2, … , N_i are uniform distributed random variables in the interval (0, 2π].

Both methods ideally result in identical stochastic process however, they are not perfectly realizable. In the case of the Filter method it is needed an ideal filter and also the white Gaussian Noise is not realizable exactly. The situation is not better for the Rice method where only the desired Gaussian random process can be achieved with the summation of an infinite number of harmonic functions N_i, as shown in the right side of figure 2. Using a finite number of harmonic functions the $\mu_i(t)$ random process is rewritten as:

$$\hat{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \text{ , where } \hat{\mu}_i(t) \to \mu_i(t) \text{ as } N_i \to \infty$$

The model shown above can be considered deterministic as soon the coefficients, frequencies and phases are established. It can be said that the resultant deterministic process or function is a sample function of the stochastic process $\mu_i(t)$. Then rewriting again the process is obtained as $\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n})$.

Until now we have obtained the real deterministic Gaussian process $\tilde{\mu}_i(t)$ to obtain the complex deterministic Gaussian process is sufficient compute $\tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t)$ and for the Rayleigh process $\zeta(t) = |\tilde{\mu}(t)|$. To include the existence of a line of sight (LOS) is included a $m(t)$ process which has a specific frequency and phase. Hence, $\xi(t) = |\tilde{\mu}(t) + m(t)|$ results in a Rice process.

IV. COST 207 MODELS

As a part of the preparation of the pan-European mobile communication system GSM, the Conference of European Posts and Telecommunications Administrations (CEPT) created the European working group COST 207 (European cooperation in the Field of Scientific and Technical Research). The main objective of this group was developed appropriate channel models for typical propagation environments. The typical environments are classifiable into areas with rural character (RA, Rural Area), areas typical for cities and suburbs (TU, Typical Urban), densely built urban areas with bad propagation conditions (BU, Bad Urban), and hilly terrain (HT, Hilly Terrain). Basing on the WSSUS assumption, the working group COST 207 developed specifications for the delay power spectral density and the Doppler power spectral density for these four classes of propagation environments [COS86, COS89].

The delay power spectral density functions $S_{\tau'\tau'}(\tau')$ of the channel models according to the COST 207 are shown in Table 1 and in the figure 5. The real valued constant quantities C_{RA}, C_{TU}, C_{BU} and C_{HT} introduced there can in principle be chosen arbitrarily. Hence, they can be determined in such a way that the average delay power is equal to one for example, $\int_0^\infty S_{\tau',\tau'}(\tau')d\tau' = 1$. In this case, it holds:

$$C_{RA} = \frac{9.2}{1 - e^{-6.44}}, C_{TU} = \frac{1}{1 - e^{-7}}, C_{BU} = \frac{2}{3(1 - e^{-5})} \quad \text{and} \quad C_{HT} = \frac{1}{(1 - e^{-7})/3.5 + (1 - e^{-5})/10}.$$

Table 1. Specification of typical delay power spectral densities $S_{\tau'\tau'}(\tau')$ according to the COST 207 [COS89].

Propagation Area	**Delay power spectral density** $S_{\tau'\tau'}(\tau')$	**Delay spread** $B^{(2)}_{\tau'\tau'}$
Rural Area (RA)	$C_{RA} e^{-9.2\tau'/\mu s}, \quad 0 \le \tau' < 0.7\mu s$ $0, \quad$ else	0.1µs
Typical Urban(TU)	$C_{TU} e^{-\tau'/\mu s}, \quad 0 \le \tau' < 0.7\mu s$ $0, \quad$ else	0.98µs
Bad Urban(BU)	$C_{BU} e^{-\tau'/\mu s}, \quad 0 \le \tau' < 0.5\mu s$ $C_{BU} \frac{1}{2} e^{(5-\tau'/\mu s)}, \quad 0.5\mu s \le \tau' < 10\mu s$ $0, \quad$ else	2.53µs
Hilly Terrain (HT)	$C_{HT} e^{-3.5\tau'/\mu s}, \quad 0 \le \tau' < 2\mu s$ $C_{HT} 0.1 e^{(15-\tau'/\mu s)}, \quad 15\mu s \le \tau' < 20\mu s$ $0, \quad$ else	6.88µs

Table 2 shows the four types of Doppler power spectral densities $S_{\mu\mu}(f)$ specified by the COST 207. They are also presented graphically in figure 6. In order to assure that $\int_{-\infty}^{\infty} S_{\mu\mu}(f)df = 1$, the real constants A_1 and A_2 are chosen as $A_1 = 50/(\sqrt{2\pi}3f_{\max})$ and $A_2 = 10^{1.5}/[\sqrt{2\pi}(\sqrt{10} + 0.15)f_{\max}]$. The classical Jakes power spectral density only occurs in the case of very short propagation delays $\tau' < 0.5\mu s$ [see figures …]. Only in this case, the assumptions that the amplitudes of the scattering components are homogenous and the angles of the arrival are uniformly distributed between (0, 2π] are justified. For scattering components with medium and long propagation delays τ' (far echoes), however, it is assumed that the corresponding Doppler frequencies are normally distributed, resulting in a Doppler spectral density with a Gaussian shape [see figure 7]. This Gaussian shape is generally shifted from the origin of the frequency plane, suggesting that the existence of a certain direction of preference for far echoes.

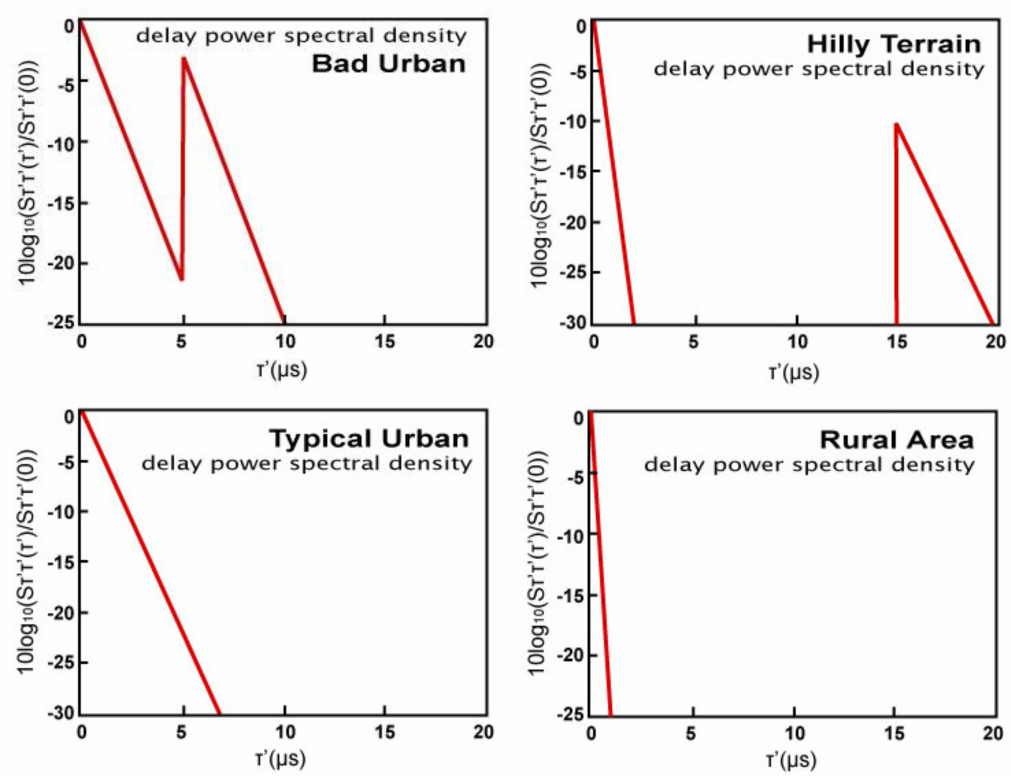

Fig5. Delay Power Spectral Density for the different Typical Areas in COST 207.

Table 2. Specification of typical Doppler power spectral densities $S_{\mu\mu}(f)$ according to the COST 207 [COS89], where

$$G(A_i, f_i, s_i) = A_i \exp\left\{\frac{(f - f_i)^2}{2s_i^2}\right\}$$

Type	Doppler power spectral density $S_{\mu\mu}(f)$	Propagation delay, τ'	Doppler spread $B_{\mu\mu}^{(2)}$
Jakes	$\frac{1}{\pi f_{\max}\sqrt{1-(f/f_{\max})^2}}$	$0 \le \tau' < 0.5\mu s$	$f_{\max}/\sqrt{2}$
Gauss I	$G(A_1, -0.8 f_{\max}, 0.05 f_{\max}) + G(A_1/10, 0.4 f_{\max}, 0.1 f_{\max})$	$0.5 \le \tau' < 2\mu s$	$0.45 f_{\max}$
Gauss II	$G(A_2, 0.7 f_{\max}, 0.1 f_{\max}) + G(A_1/10^{1.5}, -0.4 f_{\max}, 0.15 f_{\max})$	$\tau' \ge 2\mu s$	$0.25 f_{\max}$
Rice	$\frac{0.41^2}{\pi f_{\max}\sqrt{1-(f/f_{\max})^2}} + 0.91^2 \delta(f - 0.7 f_{\max})$	$\tau' = 0\mu s$	$0.39 f_{\max}$

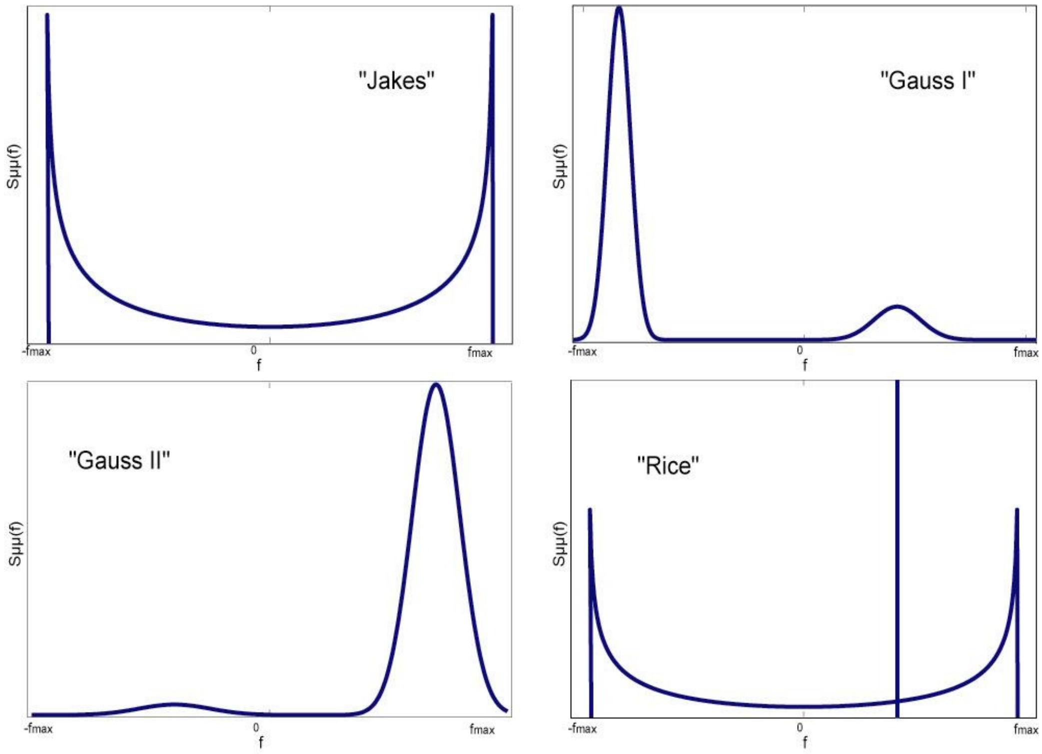

Fig6. Different categories of Power Spectrum Density defined in COST 207.

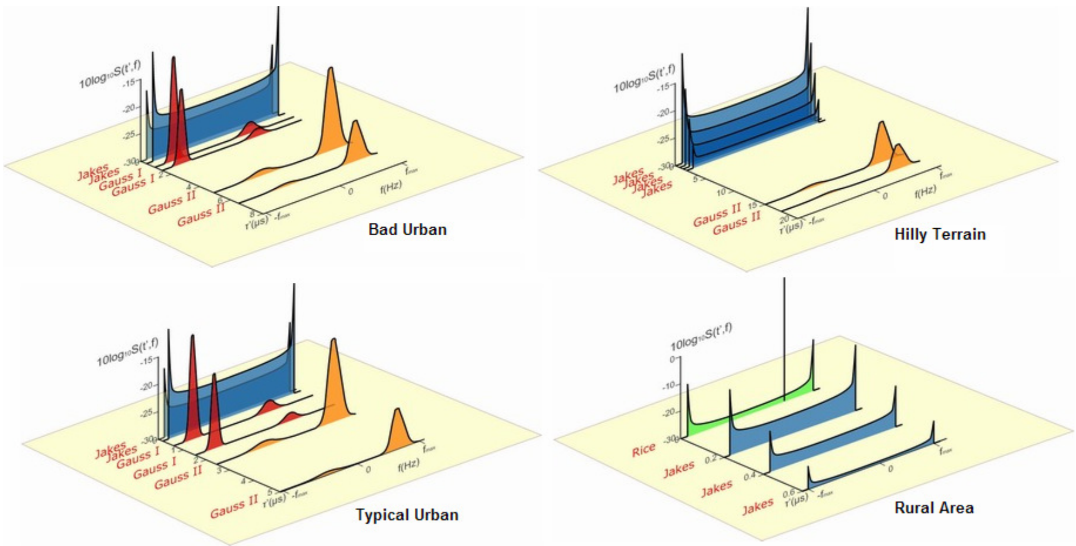

Fig7. Doppler Power Spectrum for the typical area defined in COST 207.

Another important default assumption into the COST 207 is given by the default mobile speeds for the different channels shown in the Table 3. which with a carrier frequency of 900 MHz result in the corresponding maximum Doppler shift frequencies.

Table 3. Default mobile speeds for the channels models

Type	Mobile Speed	Maximum Doppler shift
TUx	3 Km/h	2.5 Hz
RA	120 km/h	100 Hz
HT	120 km/h	100 Hz

V. Deterministic Discrete Channel Simulation

In order to implement a channel simulator the main idea is implement a FIR filter with variant coefficients which follow a specific probability distribution and with some specific delays between the filter taps according with the tables given by the COST model. First of all, the impulse response as shown in the equation (2) is discretized substituting t by kT_s, where T_s is the sampling interval and $k \in Z$. The next natural step is to determine the needed parameters to generate the filter coefficients. These parameters are those showed in the equation (8) and usually are referred as $c_{i,n}$: Doppler coefficients, $f_{i,n}$: Doppler frequencies, and $\theta_{i,n}$: Doppler phases. Where the sub index i take the value of 1 or 2 to refers respectively to the real part and the imaginary part of the process μ. The sub index n is defined from 1 to N_i and indicates the number of harmonics functions summed up to generate the process μ.

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \quad (8)$$

In the section III was stated that $c_{i,n} = 2\sqrt{\Delta f_i \cdot S_{\mu_i\mu_i}(f_{i,n})}$; $f_{i,n} = n\Delta f_i$. Here Δf_i is chosen in such a way $f_{i,n}$ covers the whole relevant frequency range $[-f_{d\max}, f_{d\max}]$ due to the Doppler shift. Where $f_d(\theta) = (v/\lambda)\cos\theta$ and $f_{d\max}$ is reached for $\cos\theta = 1$. The easiest way to cover the whole range with N_i harmonics is divide the given range in a number of segments with the same length. This method is know as Method of Equal distances and can be described as follow:

$$f_{i,n} = \frac{\Delta f_i}{2}(2n-1); \quad n = 1,2,..., N_i \text{ and } \Delta f = f_{i,n} - f_{i,n-1}; \quad n = 2,3,..., N_i$$

In order to compute the Doppler coefficients, it is defined the interval $I_{i,n}$ and the following integral must be that the hold for every i=1,2 and every subinterval (for every n=1,2,…,N_i).

$$I_{i,n} = \left[f_{i,n} - \frac{\Delta f_i}{2}, f_{i,n} + \frac{\Delta f_i}{2} \right]; \quad n = 1,2,..., N_i \qquad \int_{f \in I_{i,n}} S_{\mu_i\mu_i}(f)df = \int_{f \in I_{i,n}} \tilde{S}_{\mu_i\mu_i}(f)df$$

Then, the Doppler coefficients are computed as $c_{i,n} = 2\sqrt{\int_{f \in I_{i,n}} S_{\mu_i\mu_i}(f)df}$.

The procedure mentioned above is the general regardless the desired power spectrum density $S_{\mu_i\mu_i}(f)$. However a specific formula can be found for the Jakes and the Gauss PSD following the method of equal distances.

A. *Parameter of Jakes PSD.*

Again a range of Doppler frequencies is specified by $|f| \le f_{d\max}$. With the use of N_i harmonics functions we obtain $\Delta f_i = f_{d\max}/N_i$ and $f_{i,n} = \frac{f_{d\max}}{2}(2n-1); \quad n = 1,2,..., N_i$ and $i = 1,2$. Then the Doppler coefficients are computed as $c_{i,n} = \frac{2\sigma_0}{\sqrt{\pi}}\left[arcSin\left(\frac{n}{N_i}\right) - arcSin\left(\frac{n-1}{N_i}\right)\right]^{1/2}$. An important point becomes visible here. To produce a uncorrelated imaginary and real part into the $\mu(t)$ process is necessary to assure the summation of harmonics at different frequencies. For that purpose normally is followed that $N_2 = N_1 + 1$.

B. *Parameters of Gaussian PSD.*

For the Jakes PSD the relevant range is $|f| \le f_{d\max}$, however for the Gaussian the relevant range is smaller and normally is chosen such that include at least the 99.99% of the total power. For that purpose it is defined as $|f| \le \kappa_c f_c$, where f_c corresponds to the 3dB cut-off frequency and $\Delta f_i = \kappa_c f_c / N_i$, $f_{i,n} = \frac{\kappa_c f}{2N_i}(2n-1); \quad n = 1,2,..., N_i$ and $i = 1,2$.

Then the Doppler coefficients are computed as $c_{i,n} = \sigma_0\sqrt{2}\left[erf\left(\frac{n\kappa_c\sqrt{\ln 2}}{N_i}\right) - erf\left(\frac{(n-1)\kappa_c\sqrt{\ln 2}}{N_i}\right)\right]^{1/2}$.

Likewise to the Jakes PSD to produce real and imaginary part uncorrelated N_2 is set such that $N_2 = N_1 + 1$.

C. Fast Channel Simulators

So far, the procedure to discretize the model and calculate the model parameters has been shown. At this point two approaches can be chosen. One option is a direct realization, that means that the simulation calculate all parameters at each instant *k* during the simulation. This choice is heavy in the number of multiplications, additions and harmonics calculations. A second approach is an indirect implementation that takes advantage of the periodicity of the harmonic functions. The procedure for indirect implementation can be divided in three phases as follows:

1. Set-up Phase, each of the N_i harmonic function is sampled only once within its basic period.
2. The samples are stored in N_i tables.
3. During the simulation, the entries of each table are read out cyclically and summed up.

Basically the saving is done on the time consumption for trigonometric operations

CONCLUSIONS

The mobile channel is known as a very complicated channel due to the different effects that takes part during the wave propagation. In order to include all the effects present in the mobile channel into the simulation of a UMTS transmission is needed to create a good impulse channel response that make possible simulate a situation close to a real scenario. The heart of the channel simulator is the creation of a complex Gaussian random process responsible to produce the time-varying coefficients of the FIR filter. The simulation of typical environment is achieved taking the parameters given by the different tables created for the COST 207 group which defines the delay, power and type of Doppler PSD for the different propagation trajectories.

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