EEE 598D: Analog Filter and Signal Processing Circuits
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Homework 1

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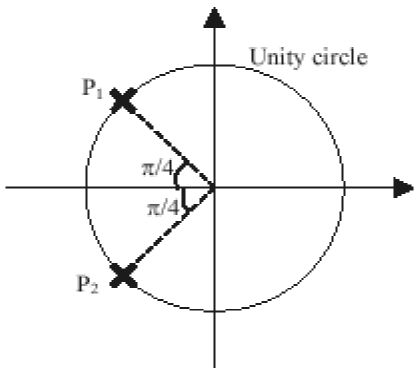
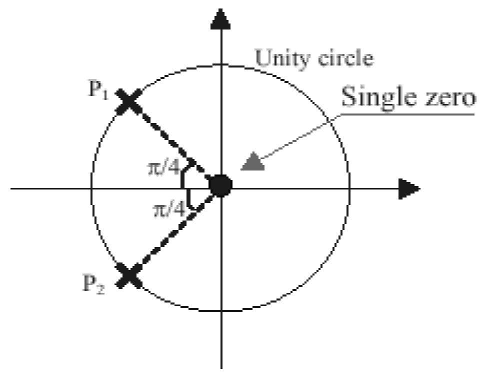
Problem 1:

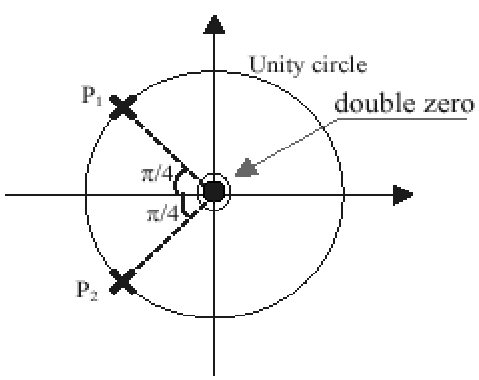
For the following filters with s-domain pole/zero location shown in the diagrams, (i) find the transfer function of the filters, (ii) sketch the gain and phase responses of the filters.



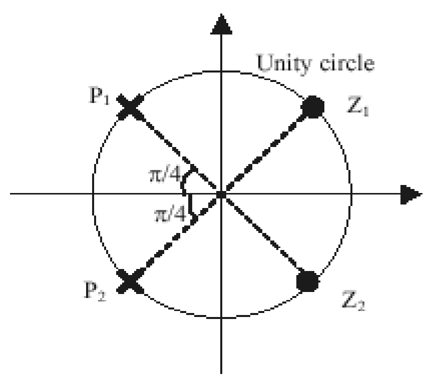
(A)
(Assuming $|H(j\omega)|_{\omega=0}=1$)



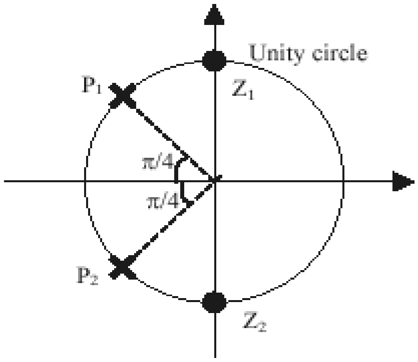
(B)
(Assuming $|H(j\omega)|_{\omega=1}=1$)



(C)
(Assuming $|H(j\omega)|_{\omega=1}=1$)



(D)
(Assuming $|H(j\omega)|_{\omega=0}=1$)



(E)
(Assuming $|H(j\omega)|_{\omega=0} = 1$)

Answer:

(A): The system has two poles at $p1,2 = -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} j$. The transfer function can be generally written as:

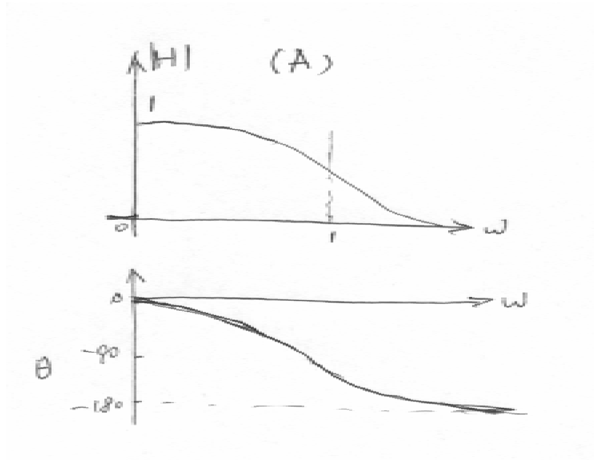
$$H(s) = \frac{K}{(s - p1)(s - p2)}$$

$$H(s) = \frac{K}{[s - (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j)][s - (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j)]} = \frac{K}{s^2 + \sqrt{2}s + 1}$$

Because $|H(j\omega)|_{\omega=0} = 1$, so K=1. Then we have:

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

It is a low pass filter with natural frequency $\omega_0 = 1$ and $Q = \frac{\sqrt{2}}{2}$. Its bode plot is shown below:



(B): The system has two poles at $p1,2 = -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} j$ and one zero at origin. The transfer function can be generally written as:

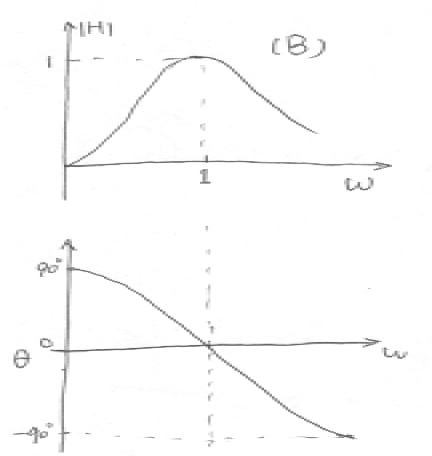
$$H(s) = \frac{Ks}{(s - p1)(s - p2)}$$

$$H(s) = \frac{Ks}{[s - (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j)][s - (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j)]} = \frac{Ks}{s^2 + \sqrt{2}s + 1}$$

Because $|H(j\omega)|_{\omega=1} = 1$, so $K = \sqrt{2}$. Then we have:

$$H(s) = \frac{\sqrt{2}s}{s^2 + \sqrt{2}s + 1}$$

It is a band pass filter with the bode plot shown below:



(C): The system has two poles at $p1,2 = -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} j$ and two zeros at the origin. The transfer function can be generally written as:

$$H(s) = \frac{Ks^2}{(s-p1)(s-p2)}$$

$$H(s) = \frac{Ks^2}{[s-(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j)][s-(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j)]} = \frac{Ks^2}{s^2+\sqrt{2}s+1}$$

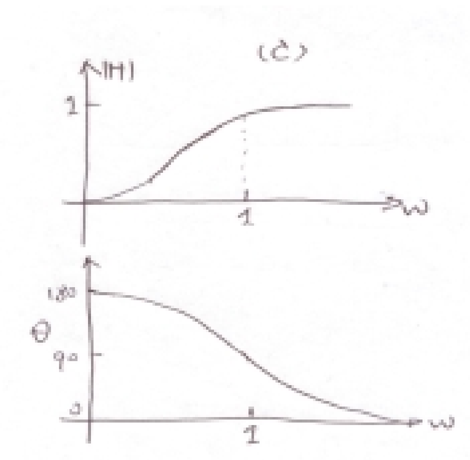
Because $\left|H(j\omega)\right|_{\omega=1} = 1$, we have:

$$\left|H(s)\right|_{\omega=1} = \left|\frac{Kj^2}{j^2+\sqrt{2}j+1}\right| = \left|\frac{-K}{\sqrt{2}j}\right| = 1$$

So we have $K = \sqrt{2}$, and:

$$H(s) = \frac{\sqrt{2}s^2}{s^2+\sqrt{2}s+1}$$

It has a high pass characteristic. Its bode plot is shown in the following figure:



(D): The system has two poles at $p1,2 = -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} j$ and two zeros at $z1,2 = \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} j$. The transfer function can be generally written as:

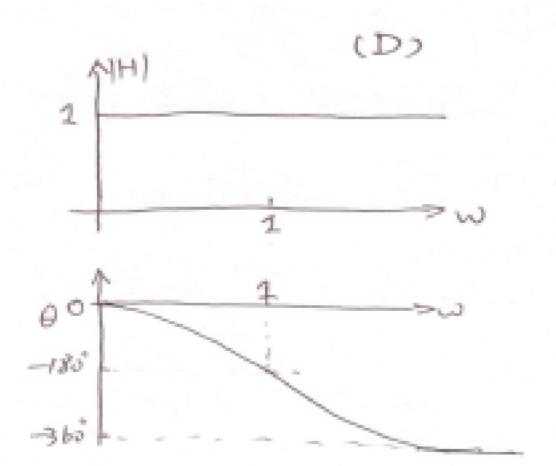
$$H(s) = \frac{K(s - z1)(s - z2)}{(s - p1)(s - p2)}$$

$$H(s) = \frac{K[s - (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j)][s - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j)]}{[s - (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j)][s - (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j)]} = \frac{K(s^2 - \sqrt{2}s + 1)}{s^2 + \sqrt{2}s + 1}$$

Because $\left|H(j\omega)\right|_{\omega=0} = 1$, so K=1. Then we have:

$$H(s) = \frac{s^2 - \sqrt{2}s + 1}{s^2 + \sqrt{2}s + 1}$$

This is a all pass filter. Its bode plot is shown below:



(E): The system has two poles at $p1,2 = -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} j$ and two zeros at $z1,2 = \pm j\omega$. The transfer function can be generally written as:

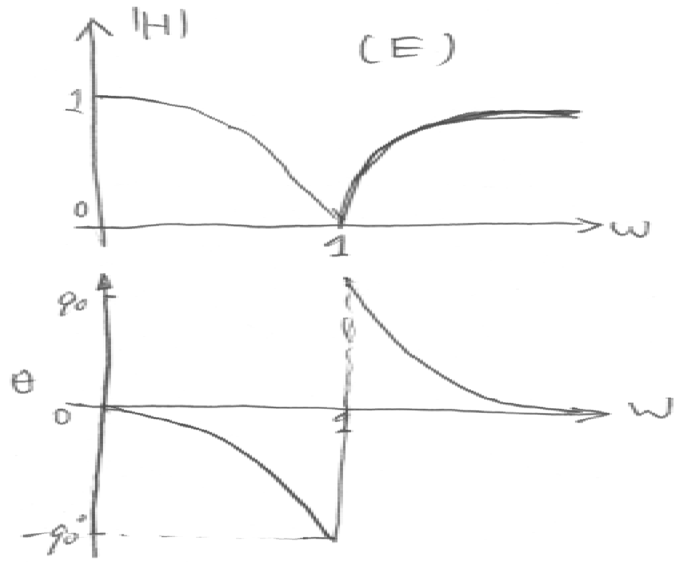
$$H(s) = \frac{K(s - z1)(s - z2)}{(s - p1)(s - p2)}$$

$$H(s) = \frac{K(s - j\omega)(s + j\omega)}{[s - (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j)][s - (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} j)]} = \frac{K(s^2 + 1)}{s^2 + \sqrt{2}s + 1}$$

Because $\left|H(j\omega)\right|_{\omega=0} = 1$, so K=1. Then we have:

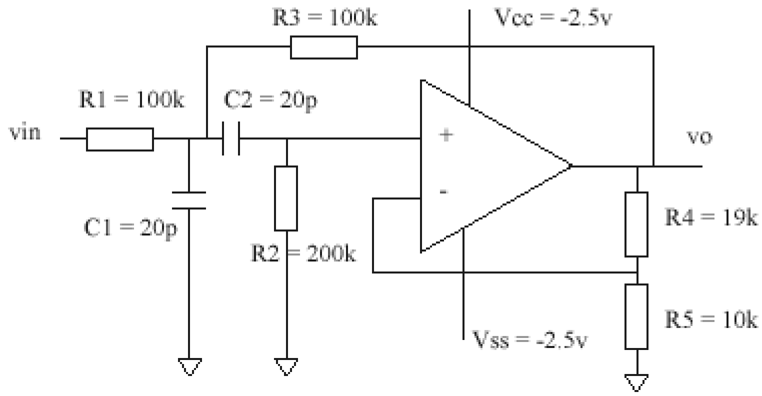
$$H(s) = \frac{s^2 + 1}{s^2 + \sqrt{2}s + 1}$$

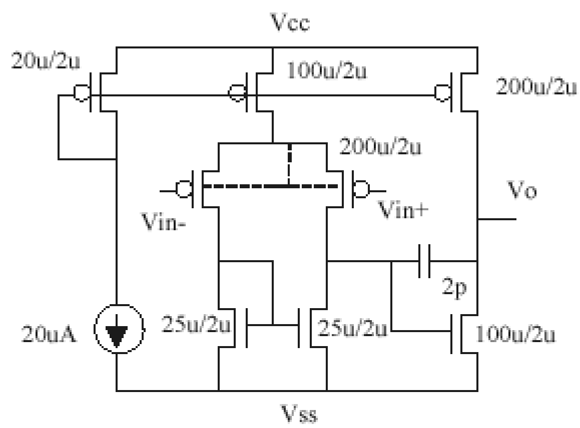
It shows a notch filter characteristic, as shown below:



Problem 2.

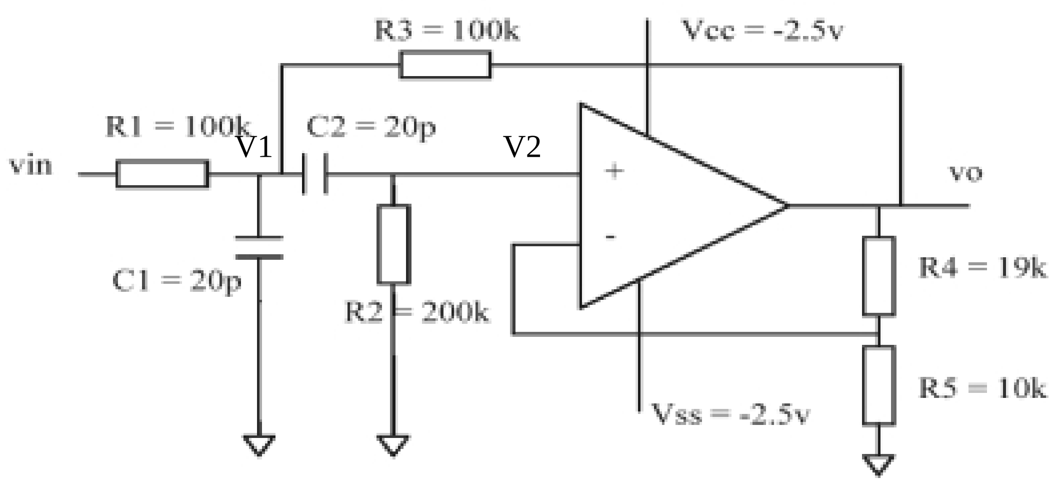
A practical VLSI active RC filter is shown in figure below. Derive the s-domain transfer function and simulate the ac responses (gain and phase) of the filter using a circuit simulate (spice/pspice/hspice or other.) with the operation amplifier provided.





Answer:

See the following figure, we define two nodes: V1 and V2.



See the above figure. The node equation at V1 is:

$$\frac{V1 - Vin}{R1} = (V2 - V1)sC2 - V1sC1 + \frac{(Vo - V1)}{R3} \quad (2.1)$$

Node equation at V2:

$$(V2 - V1)sC2 + \frac{V2}{R2} = 0 \quad (2.2)$$

Equation at node Vo:

$$\frac{(Vo - V2)}{R4} + \frac{Vo - V1}{R3} = 0 \quad (2.3)$$

We also have:

$$\frac{V2}{R5} = \frac{Vo - V2}{R4} \quad (2.4)$$

From (2.4) we get:

$$V2 = \frac{Vo \cdot R5}{R4 + R5} \quad (2.6)$$

Using (2.6) and (2.2) we get:

$$V1 = \frac{Vo \cdot R5}{R4 + R5}\left(\frac{1 + sC2R2}{sC2R5}\right) \quad (2.7)$$

Plug (2.6) and (2.7) into (2.1) we get:

$$Vo\left[\frac{R5}{(R4 + R5)R2} + \frac{R5}{(R4 + R5)}\frac{1 + sC2R2}{sC2R2}\left(\frac{1}{R1} + sC1 + \frac{1}{R3}\right) - \frac{1}{R3}\right] = \frac{Vin}{R1} \quad (2.8)$$

$$\frac{Vo}{Vin} = \frac{R4 + R5}{R1R5}\frac{1}{\left[\frac{1}{R2} + \frac{1 + sC2R2}{sC2R2}\left(\frac{1}{R1} + sC1 + \frac{1}{R3}\right)\right] - \frac{R4 + R5}{R3R5}} \quad (2.9)$$

$$\frac{Vo}{Vin}=\frac{R4+R5}{R1R5}\frac{sC2R2}{[\frac{sC2R2}{R2}+(1+sC2R2)(\frac{1}{R1}+sC1+\frac{1}{R3})]-sC2R2\frac{R4+R5}{R3R5}} \quad (2/10)$$

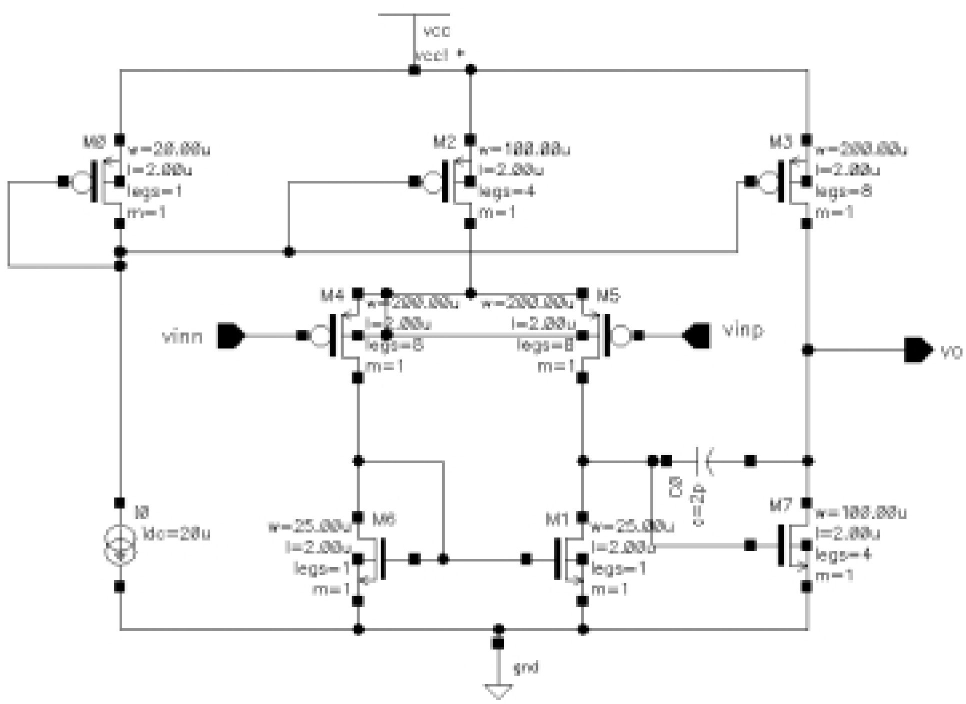
$$\frac{Vo}{Vin}=\frac{R4+R5}{R1R5}\frac{sC2R2}{s^2C1C2R2+s[\frac{C2R2}{R2}+C2R2(\frac{1}{R1}+\frac{1}{R3})-C2R2\frac{R4+R5}{R3R5}]+\frac{1}{R1}+\frac{1}{R3}} \quad (2.11)$$

Which can be written as:

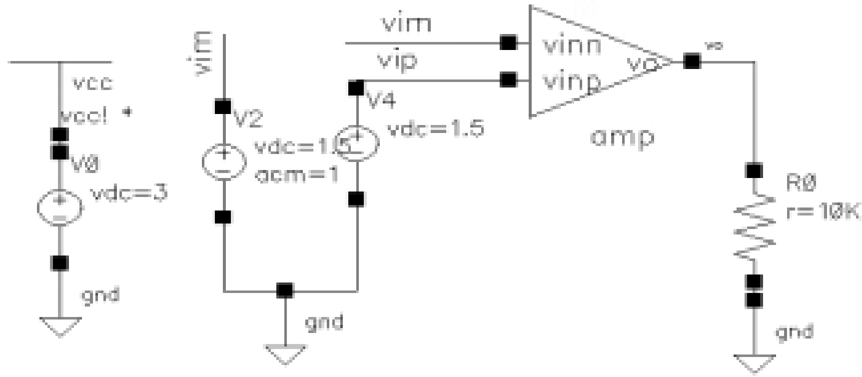
$$\frac{Vo}{Vin}=\frac{R4+R5}{R1R5C1}\frac{s}{s^2+s\frac{1}{C1}[\frac{1}{R2}+(\frac{1}{R1}+\frac{1}{R3})-\frac{R4+R5}{R3R5}]+(\frac{1}{R1}+\frac{1}{R3})\frac{1}{C1C2R2}} \quad (2.12)$$

It can be seen that it is a second order system. It has a band-pass characteristic.

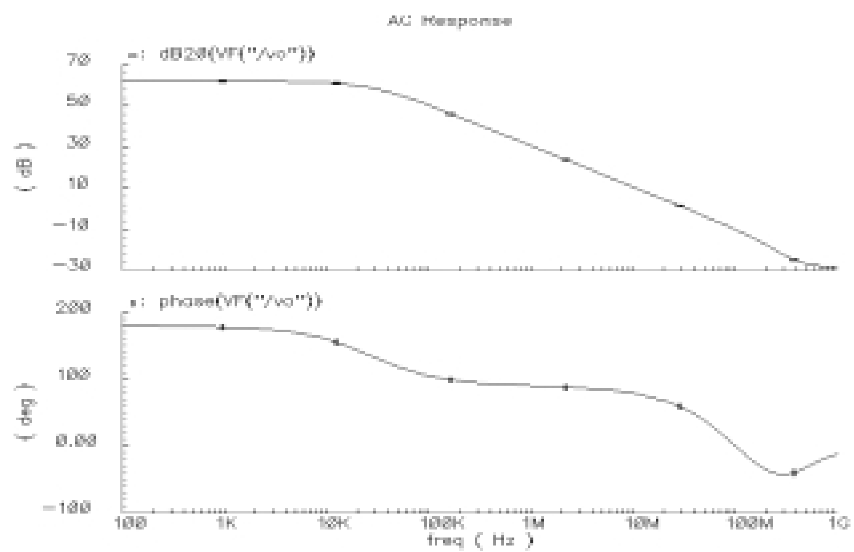
The following figure shows the actually schematic of the amplifier used for simulation:



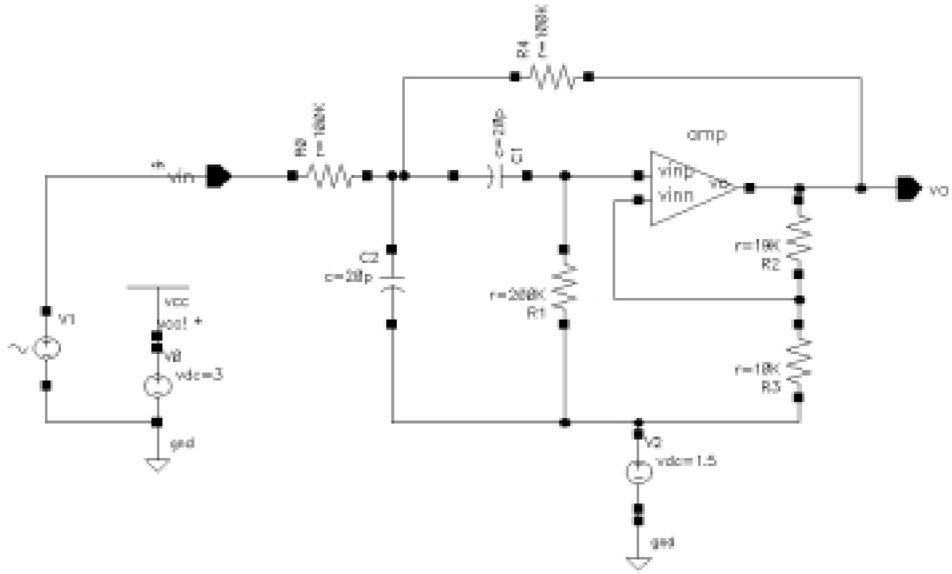
Note that there is no negative bias. The following figure shows the test circuit of the opamp. The power supply is 3V, and we use Vcc/2 as the common mode voltage.



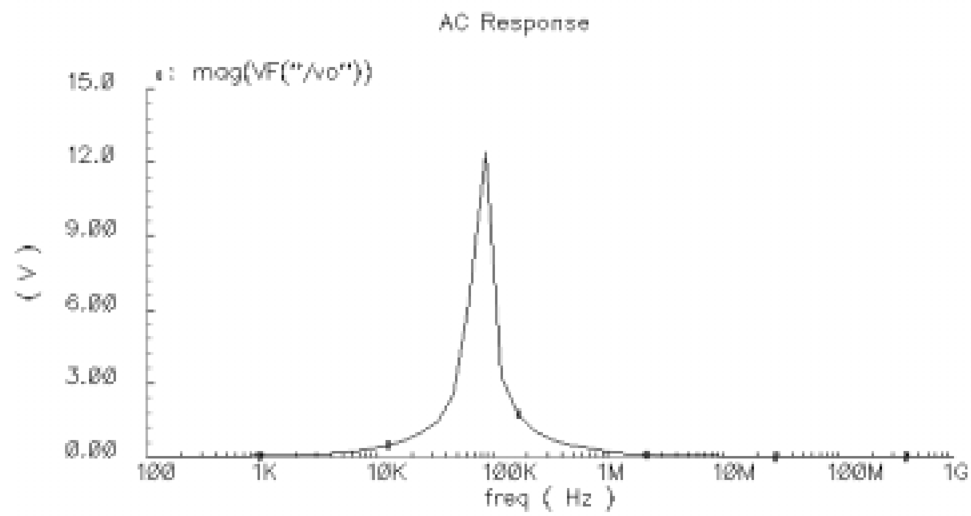
The following figure is the Bode plot of the amplifier. The load is 10K Ohms.



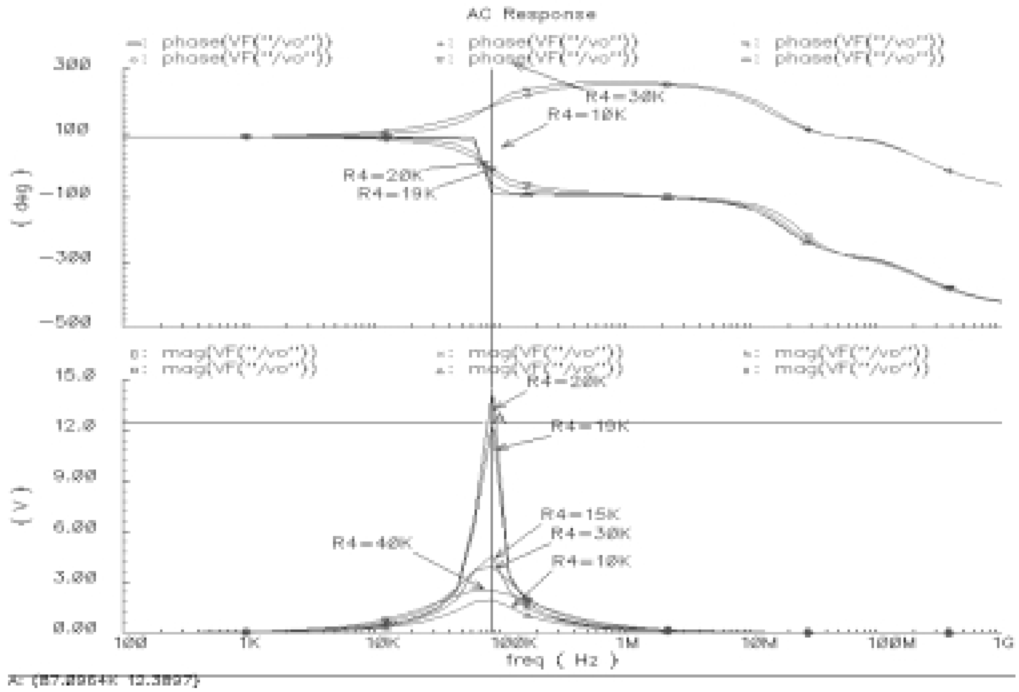
Bode plot of the Opamp with a 10K Ohms load.



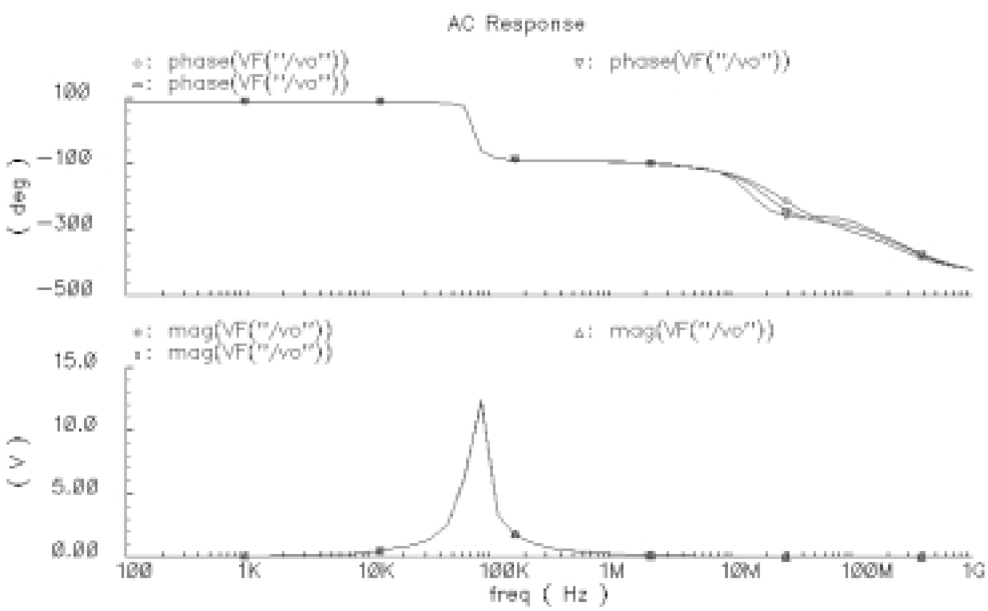
Test bench circuit of the filter. Note that the reference of the input AC signal is Vcc/2=1.5.



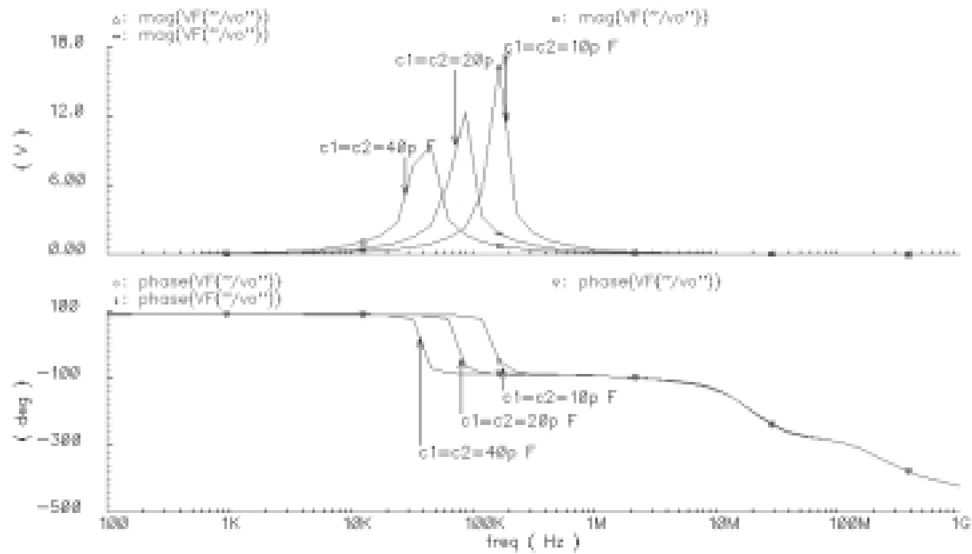
Output character of the filter. The ac magnitude of the input signal is 1V. Clearly it shows a bandpass characteristic.



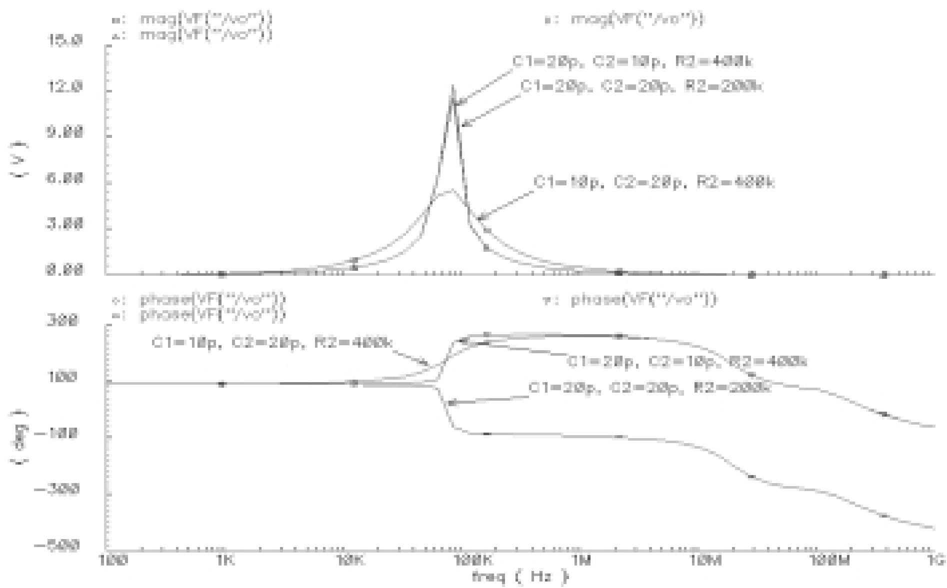
Keep the value of R5 to be constant as 10K Ohms, the value of R4 changed. The natural frequency keep constant but the Q value changes a lot.



Keep R4/R5 to be constant, the absolute values of R4 and R5 changes (R5=5K, 10K and 20K Ohms). The natural frequency and Q value do not change with the change of R4 and R5.



Change the value of C1 and C2 and keep other component constant. It can bee seen that the natural frequency changes with the change of C1 and C2.



Keep C1C2R2 to be constant. It can be see that the natural frequency also keeps to be constant but the Q value changes. This is consistent with Equation (2.12).