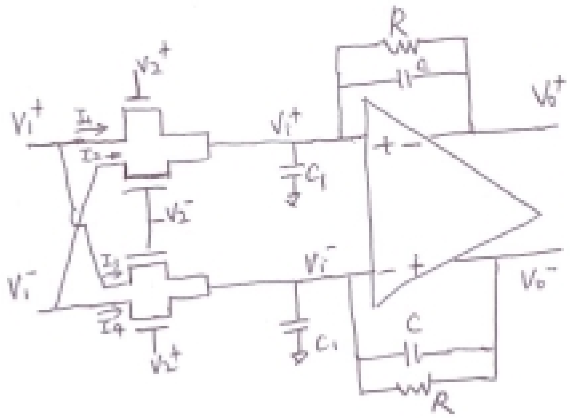
HW#2 Fuding Ge
Registered at ASU EAST

EEE 598O
Analog Filter & signal processing Circuits.
Instructor: Hongjiang Song



$$I = \beta\left(V_{gs} - V_T - \frac{V_{ds}}{2}\right)(V_d - V_s)$$

1. Ideal Opamp

a) Ignore C_1 and assume a 0V common-mode voltage.

$$I_1 = \beta\left(V_2^+ - V_1^+ - V_T - \frac{V_1^+ - V_i^+}{2}\right)(V_1^+ - V_i^+) \quad ①$$

$$I_2 = \beta\left(V_2^- - V_1^- - V_T - \frac{V_1^- - V_i^+}{2}\right)(V_1^- - V_i^+) \quad ②$$

$$I_3 = \beta\left(V_2^- - V_1^+ - V_T - \frac{V_1^+ - V_i^-}{2}\right)(V_1^+ - V_i^-) \quad ③$$

$$I_4 = \beta\left(V_2^+ - V_1^- - V_T - \frac{V_1^- - V_i^-}{2}\right)(V_1^- - V_i^-) \quad ④$$

$$V_o^+ = R /\!/ \frac{1}{SC} \cdot (I_1 + I_2) = \frac{R}{1+SRC}(I_1 + I_2) \quad ⑤$$

$$V_o^- = R /\!/ \frac{1}{SC}(I_3 + I_4) = \frac{R}{1+SRC}(I_3 + I_4) \quad ⑥$$

$$V_o^+ - V_o^- = \frac{R}{1+SRC}(I_1 + I_2 - I_3 - I_4) = \frac{R\beta}{1+SRC}\left[(V_1^+ - V_i^-)(V_2^+ - V_2^-) + (V_1^- - V_i^+)(V_2^- - V_2^+)\right]$$

$$= \frac{R\beta}{1+SRC}(V_2^+ - V_2^-)(V_1^+ - V_i^- - V_1^- + V_i^+) = \frac{R\beta}{1+SRC}(V_2^+ - V_2^-)(V_1^+ - V_1^- + V_i^+ - V_i^-) \quad ⑦$$

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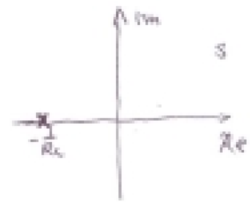
$$\left.\frac{V_o^+ - V_o^-}{V_2^+ - V_2^-}\right|_{(V_1^+ - V_1^-)=\text{Const}} = \frac{R\beta\ (V_1^+ - V_1^- + V_1^+ - V_1^-)}{1+SRC} \quad ⑧$$

V_1^+ & V_1^- are common-mode voltage, so $V_i^+ = V_i^- = 0$

we have:

$$\boxed{\begin{aligned}\left.\frac{V_o^+ - V_o^-}{V_2^+ - V_2^-}\right|_{(V_1^+ - V_1^-)=\text{Const}} &= \frac{R}{1+SRC}\beta(V_1^+ - V_1^-) \\ \left.\frac{V_o^+ - V_o^-}{V_1^+ - V_1^-}\right|_{(V_2^+ - V_2^-)=\text{Const}} &= \frac{R}{1+SRC}\beta(V_2^+ - V_2^-)\end{aligned}} \quad ⑨$$

(b) From equation ⑨ we know there is one pole at $\omega_p = -\frac{1}{RC}$



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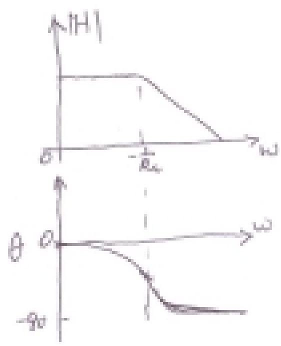

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SFG Flow:

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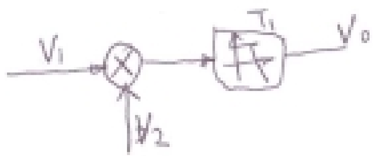

2. Non-ideal opamp.

$$\frac{V_o^+ - V_o^-}{V_i^+ - V_i^-} = \frac{A_o}{1 + \frac{S}{\omega_o}} \qquad (10)$$

we have

$$V_i^+ - V_i^- = \frac{V_o^+ - V_o^-}{A_o}\left(1 + \frac{S}{\omega_o}\right) \qquad (11)$$

plug (11) into (8) we have

$$\left.\frac{V_o^+ - V_o^-}{V_2^+ - V_2^-}\right|_{(V_1^+ - V_1^-) = Const} = \frac{R}{1+SRC}\beta\left[V_1^+ - V_1^- + \frac{V_o^+ - V_o^-}{A_o}\left(1 + \frac{S}{\omega_o}\right)\right]$$

$$= \frac{R}{1+SRC}\beta\left(V_1^+ - V_1^-\right) + \frac{R}{1+SRC}\beta\frac{V_o^+ - V_o^-}{A_o}\left(1 + \frac{S}{\omega_o}\right) \qquad (12)$$

To simplify the expression we use single-ended signal

$$\left.\frac{V_o}{V_2}\right|_{V_1 = Const} = \frac{R}{1+SRC}\beta V_1 + \frac{R}{1+SRC}\beta\frac{1 + \frac{S}{\omega_o}}{A_o} V_o \qquad (13)$$

$$V_o = \frac{R}{1+SRC}\beta V_1 V_2 + \frac{R}{1+SRC}\beta\frac{1 + \frac{S}{\omega_o}}{A_o} V_o V_2 \qquad (14)$$

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Denote $T_1(s) = \dfrac{R\beta}{1+sRC}$ $\qquad T_2(s) = \dfrac{A_0}{1+\frac{s}{\omega_0}}$

we have

$$V_0 = T_1(s)\,V_1 V_2 + \frac{T_1(s)}{T_2(s)}(s)\,V_0 V_2 \qquad (15)$$

It is can been see that the nonideality of the opamp introduce nonlinearity.

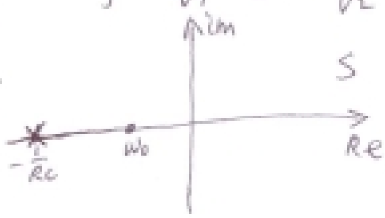
$$\boxed{\begin{aligned} \frac{V_0}{V_1} &= \frac{T_1(s)}{1-\frac{T_1}{T_2}V_2}\,V_2 \\ \frac{V_0}{V_2} &= T_1(s)\,V_1 + \frac{T_1(s)}{T_2(s)}\,V_0 \end{aligned}} \qquad (16)$$

(b) $\dfrac{T_1}{T_2} = \dfrac{R\beta}{1+sRC}\,\dfrac{1+\frac{s}{\omega_0}}{A_0} = \dfrac{R\beta}{1+\frac{s}{\omega_1}}\,\dfrac{1+\frac{s}{\omega_0}}{A_0}$

we know $\omega_0 << \frac{1}{RC} = \omega_1$

$\frac{T_1(s)}{T_2(s)}$ shows an all-pass characteristic

so the pole of $\frac{V_0}{V_1}$ & $\frac{V_0}{V_2}$ is still dominated by $T_1(s)$.



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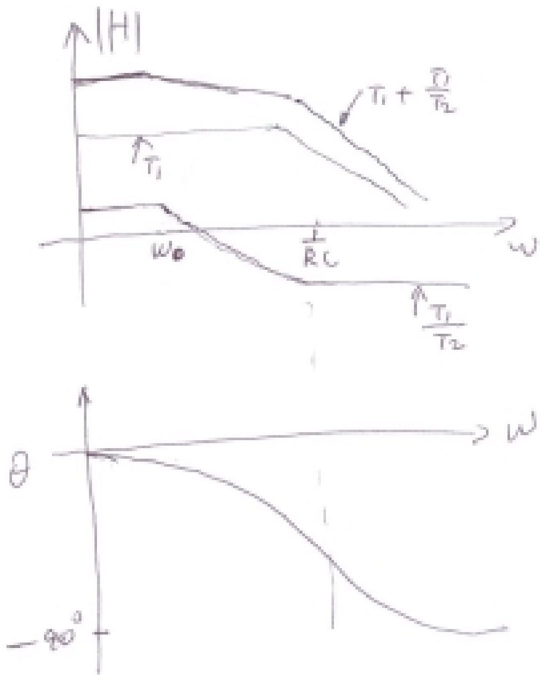

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