EEE598D Homework #3

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A fully differential CMOS transconductor circuit is shown in Fig.1.

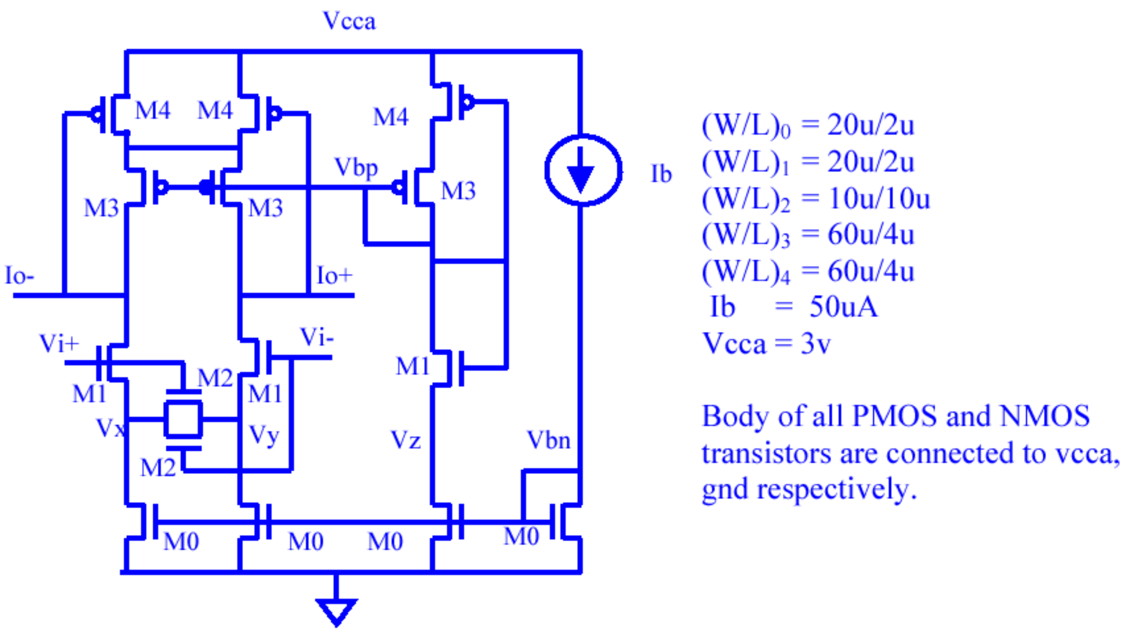


Fig.1 Fully differential CMOS transconductor circuit

Problem 1. Derive the expression of differential transconductance gm of this circuit and calculate gm for the 0.5um CMOS process (given in Homework#1). (Assuming 1.5v input common-mode voltage and neglecting body effects)

Problem 2. Simulate the differential gm for the 1V peak-to-peak input voltage and a 1k resistive load across I+ and I-. Plot I (across R) and gm vs. (vi_+ -vi_-). Extract a_2 and a_3 from your simulation data using the curve fitting technique. Where a_2 and a_3 are non-linearity parameters defined as:

$$I^+ - I^- = g_m(V_i^+ - V_i^-) + a_2(V_i^+ - V_i^-)^2 + a_3(V_i^+ - V_i^-)^3$$

Problem 1:

This transconductor is first proposed by *Krummenacher and Joehl (A 4-MHz CMOS continuous–time filter with on-chip automatic tuning, IEEE JSSC, Vol.23(3), 1988).*

The following figure shows their original schematics.

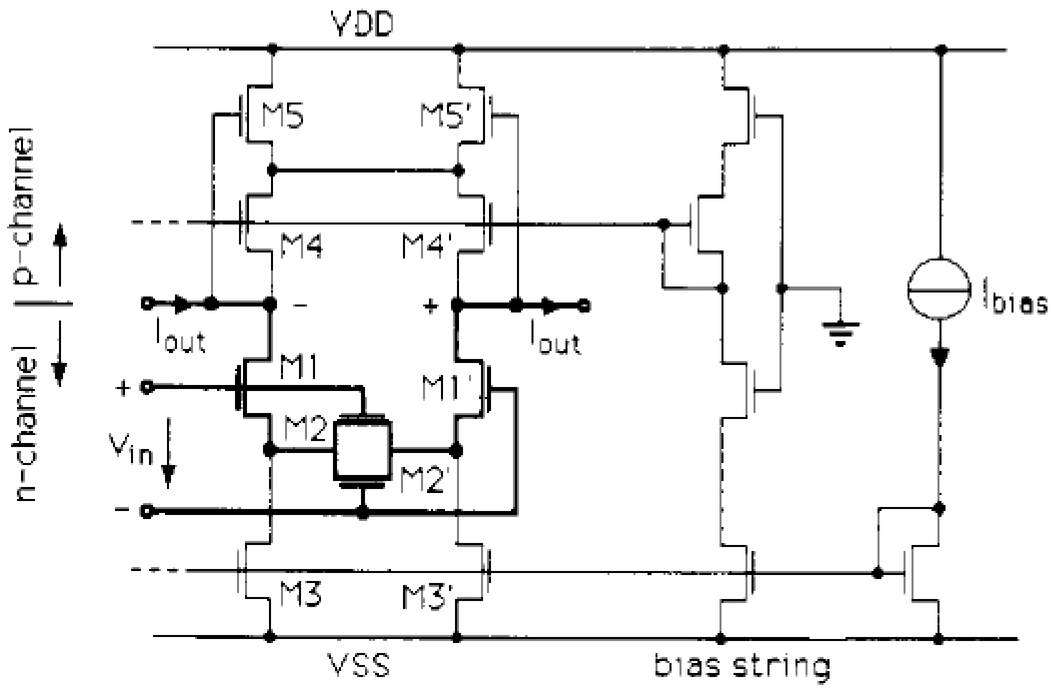


Figure.2 Original schematics proposed by *Krummenacher and Joehl.*

Its can be simplified as the following figure:

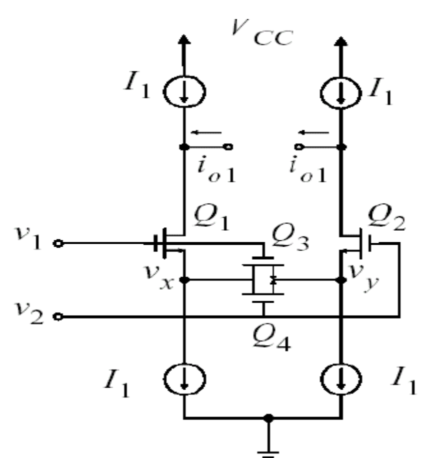


Figure.3 Simplified schematic shows the operation principle

The current pass through transistor Q1 is:

$$I_1 + I_0 = \frac{\beta_1}{2}(v_i^+ - v_x - v_T)^2 \tag{1}$$

The current pass through transistor Q2 is (assuming $\beta_1 = \beta_2$):

$$I_1 - I_0 = \frac{\beta_1}{2}(v_i^- - v_y - v_T)^2 \tag{2}$$

Then we have:

$$2I_0 = \frac{\beta_1}{2}[v_i^- - v_i^- - (v_x - v_y)][v_i^- + v_i^- - 2v_T - (v_x + v_y)] \tag{3}$$

The current pass through transistors Q3 and Q4 is equal to I_0, so we have:

$$\begin{aligned} I_0 &= \beta_3\{[v_i^+ - v_T - \frac{(v_x + v_y)}{2}](v_x - v_y) + [v_i^- - v_T - \frac{(v_x + v_y)}{2}](v_x - v_y)\} \\ &= \beta_3(v_x - v_y)[v_i^+ + v_i^- - 2v_T - (v_x + v_y)] \end{aligned} \tag{4}$$

Using (3) and (4) we have:

$$\frac{\beta_1}{4}[v_i^+ - v_i^- - (v_x - v_y)] = \beta_3(v_x - v_y) \tag{5}$$

We have:

$$v_x - v_y = \frac{\beta_1}{\beta_1 + 4\beta_3}(v_i^+ - v_i^-) \tag{6}$$

Now if we assume I_0 is very small compared to I_1, then we can write the equations of Q1 and Q2 as:

$$\begin{aligned} \sqrt{\frac{2I_1}{\beta_1}} &= v_i^+ - v_x - v_T \\ \sqrt{\frac{2I_1}{\beta_1}} &= v_i^- - v_y - v_T \end{aligned} \tag{7}$$

From (7) we have:

$$2\sqrt{\frac{2I_1}{\beta_1}} = v_i^+ + v_i^- - (v_x + v_y) - 2v_T \tag{8}$$

Plug (6) and (8) into (4) we have:

$$I_0 = \beta_3 \frac{\beta_1}{\beta_1 + 4\beta_3} 2\sqrt{\frac{2I_1}{\beta_1}}(v_i^+ - v_i^-) \qquad (9)$$

So the transconductance can be written as:

$$\boxed{G_m = \frac{I_0}{(v_i^+ - v_i^-)} = \frac{2\beta_1\beta_3}{\beta_1 + 4\beta_3}\sqrt{\frac{2I_1}{\beta_1}}} \qquad (10)$$

$$G_m = \frac{I_0}{(v_i^+ - v_i^-)} = \frac{2\sqrt{\beta}_1\beta_3}{\beta_1 + 4\beta_3}\sqrt{2I_1} \qquad (11)$$

We introduce the following parameter:

$$\alpha = 1 + \beta_1 / 4\beta_3 \qquad (12)$$

Equation (11) can be written as:

$$G_m = \frac{I_0}{(v_i^+ - v_i^-)} = \frac{1}{1 + \frac{\beta_1}{4\beta_3}}\sqrt{\frac{I_1\beta_1}{2}} = \frac{1}{\alpha}\sqrt{\frac{I_1\beta_1}{2}} \qquad (13)$$

From equation (7) we know:

$$\sqrt{\frac{I_1\beta_1}{2}} = \frac{I_b}{(V_{gs} - v_T)_{Q1}} \qquad (14)$$

So we have:

$$\boxed{G_m = \frac{I_b}{\alpha(V_{gs} - v_T)_{Q1}}} \qquad (15)$$

$\beta = u_n C_{ox}(W/L)$. In this example, we have $(W/L)_1$=20u/2u; $(W/L)_2$=10u/2u;

For the 0,5 um process, form the MOSIS web we know:

T_{ox}=1.4E-8 m=1.4E-2 um

μ_n=452.9910821 cm2/V S

ε_{ox}=3.97 X 8.85 X 10^{-14}=34.5aF/um
$C_{ox}=\varepsilon_{ox}/T_{ox}$=2.46 fF/um^2.
$\mu_n C_{ox}$=111 uA/V^2.

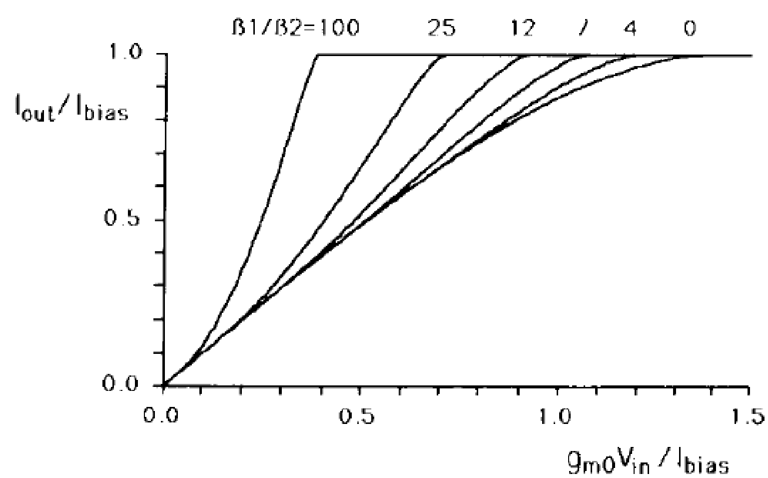
We have: $\beta_1 = u_n C_{ox}(W/L)_1 = 1110uA/V^2$ and $\beta_3 = u_n C_{ox}(W/L)_3 = 111uA/V^2$
I_1=50uA.
Plug these parameters into equation (12) we get α=3.5.
Using equation (13) we get

$$\mathbf{gm=47.5\ uS} \qquad \textbf{(16)}$$

See the simulation results in the next problem. The simulated results is about 48uS. The calculated result is in good agreement with the simulated results.

More about this circuit:

There is a optimum value of the β_1/β_3 for the best linearity performance. It appears to be about 7 (see the following figure). More detailed results please see *Krummenacher and Joehl (A 4-MHz CMOS continuous–time filter with on-chip automatic tuning, IEEE JSSC, Vol.23(3), 1988).*



Problem 2

The following schematics is the simulation circuit.

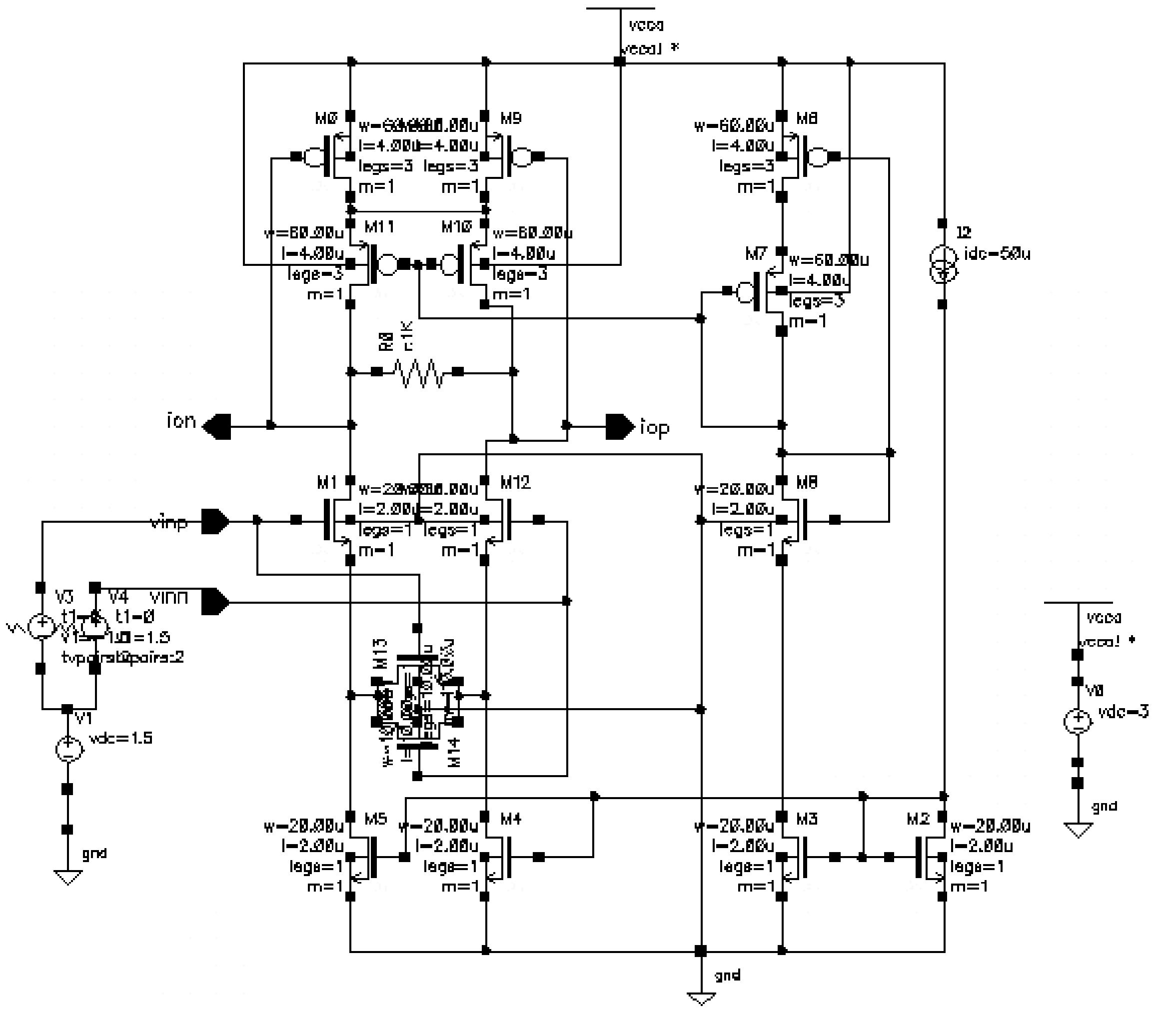


Figure.4 Simulation schematics used in this homework

The transistors M4 in figure.1 (or M5 and M5’ in figure.2) operate in the triode region. See the following figure.

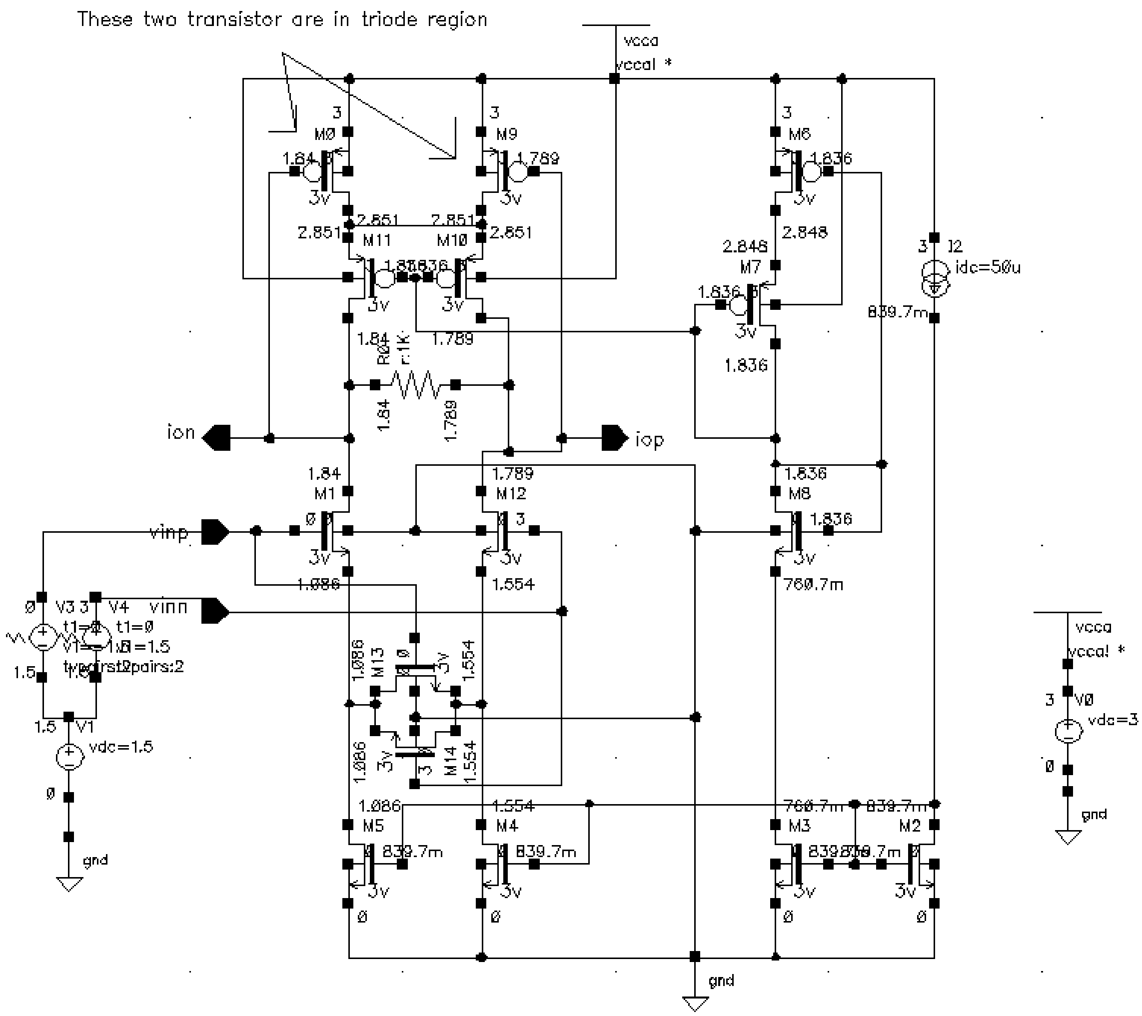


Figure.5 The DC node voltage of the circuit. It can be seen that the transistors of M4 in figure is in triode region.

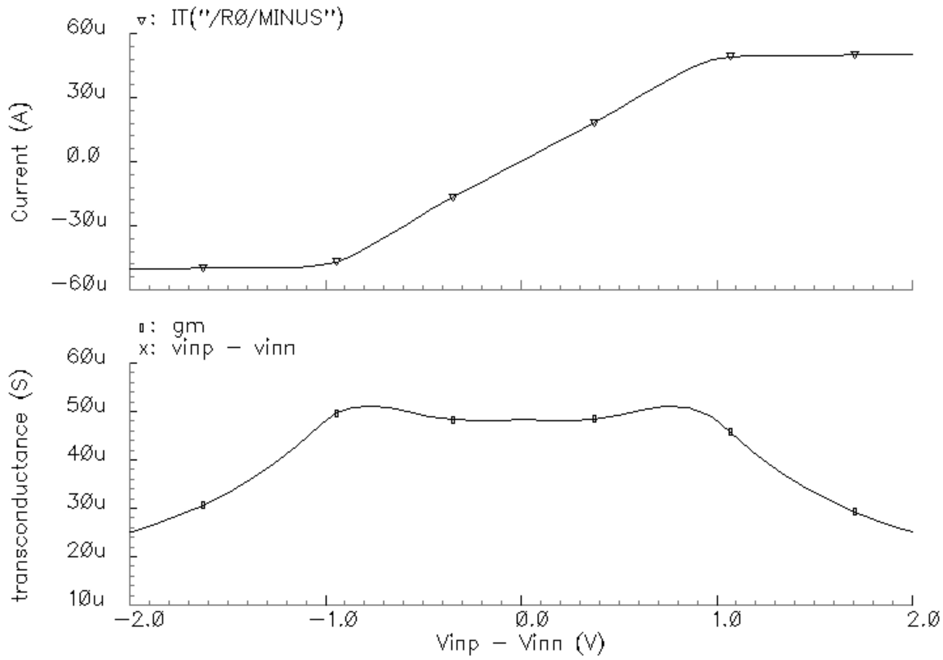


Figure.6 The simulation results: I (across R) and gm vs. the differential iunput.

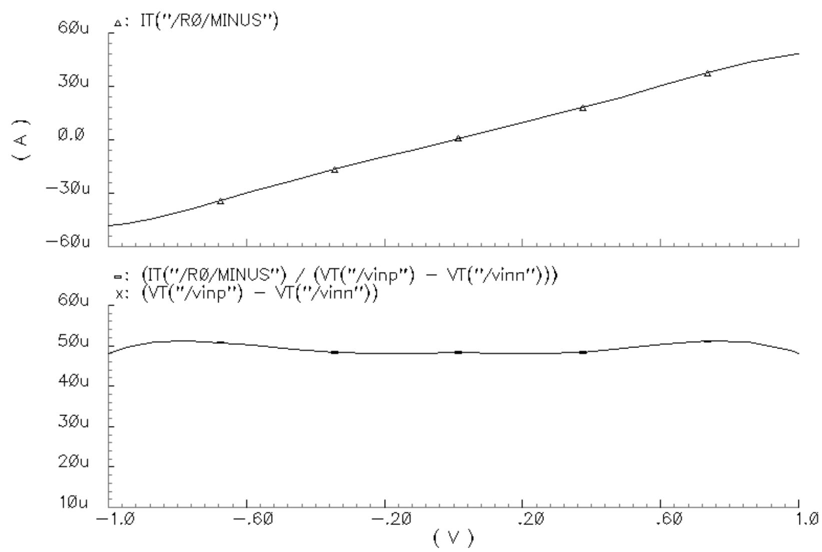


Figure.7 Detailed I and gm vs differential input in the region of –1V to 1V.

Note that the maximum output current is I_b=50uA.

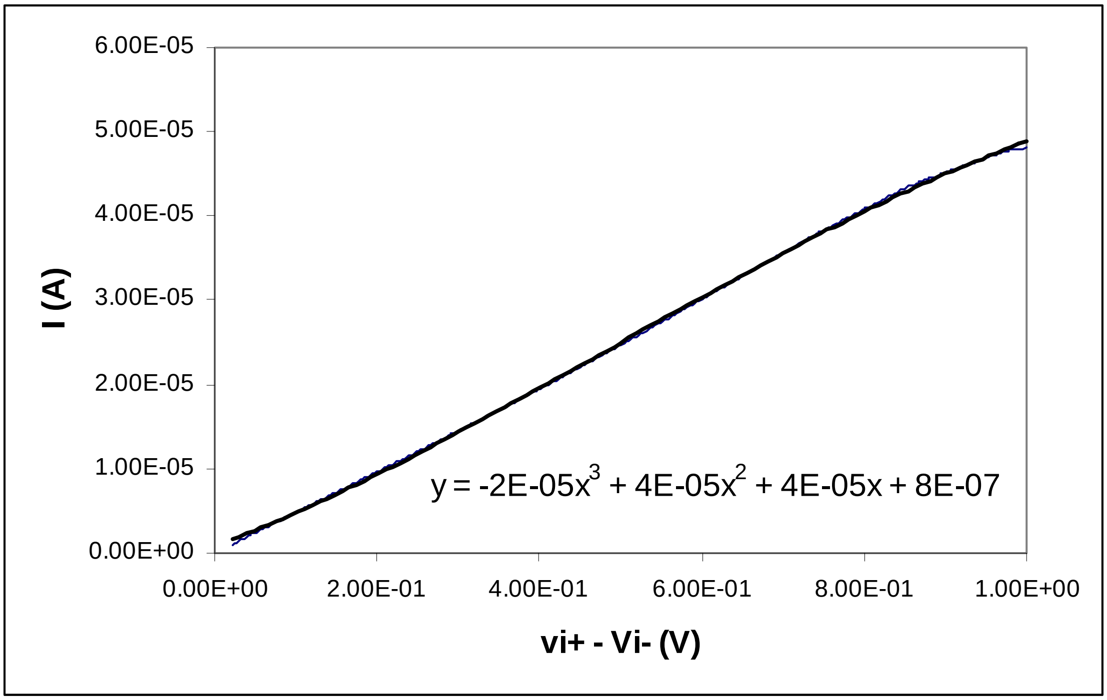


Figure.7 Curve fitting. This figure shows the value of gm is about 40uS. α_1=4E-5, α_2=4E-5 and α_3=2E-5.

Now we characterize its nonlinearity distortion.

Harmonic distortion:

For a memoryless, time-variant system, assume:

$$y(t) = \alpha_1 x(t) + \alpha_2 x(t)^2 + \alpha_3 x(t)^3 \tag{17}$$

If the input *x(t)* is a sinusoidal signal given by:

$$x(t) = A\cos(\omega t) \tag{18}$$

The output is:

$$\begin{aligned} y(t) &= \alpha_1 A\cos(\omega t) + \alpha_2 A^2 \cos^2(\omega t) + \alpha_3 A^3 \cos^3(\omega t) \\ &= \alpha_1 A\cos(\omega t) + \alpha_2 A^2 \frac{1+\cos(2\omega t)}{2} + \alpha_3 A^3 \frac{3\cos(\omega t) + \cos(3\omega t)}{4} \\ &= \frac{\alpha_2 A^2}{2} + \left[\alpha_1 A\cos(\omega t) + \frac{3\alpha_3 A^3}{4}\right]\cos(\omega t) + \frac{\alpha_2 A^2}{2}\cos(2\omega t) + \frac{\alpha_3 A^3}{4}\cos(3\omega t) \end{aligned} \tag{19}$$

It can be seen that the amplitude of the *n*th harmonic grows approximately in proportional A^n. For a fully differential system, all even terms are small, so typically α_3 dominates and the output signal can be approximately expressed as:

$$y(t) = \alpha_1 A\cos(\omega t) + \frac{\alpha_3 A^3}{4}\cos(3\omega t) \tag{20}$$

The fundamental is:

$$H_{D1} = \alpha_1 A \tag{21}$$

The 3rd order harmonic is:

$$H_{D3} = \frac{\alpha_3 A^3}{4} \tag{22}$$

The 3rd order harmonic distortion is defined as:

$$HD_3 = \frac{H_{D3}}{H_{D1}} = \frac{A^2}{4}\frac{\alpha_3}{\alpha_1} \tag{23}$$

Note that this distortion lies at $3\omega t$ for a single sinusoidal input at ωt.
For the circuit configuration we just simulated, we A=1V, α_3=2E-5 and α_3=4E-5. Use this data we find that:

$$HD_3 = -18dB \tag{24}$$

Intermodulation:

If the input signal consists of two equally sized sinusoidal signals:

$$x(t) = A\cos(\omega_1 t) + A\cos(\omega_2 t) \tag{25}$$

The output signal is:

$$\begin{aligned} y(t) &= \alpha_1 x(t) + \alpha_2 x(t)^2 + \alpha_3 x(t)^3 \\ &= (\alpha_1 A + \frac{9\alpha_3}{4}A^3)[\cos(\omega_1 t) + \cos(\omega_1 t)] \\ &+ \frac{\alpha_3}{4}A^3[\cos(3\omega_1 t) + \cos(3\omega_2 t)] \\ &+ \frac{\alpha_3}{4}A^3[\cos(2\omega_1 t + \omega_2 t) + \cos(2\omega_2 t + \omega_1 t)] \\ &+ \frac{\alpha_3}{4}A^3[\cos(2\omega_1 t - \omega_2 t) + \cos(2\omega_2 t - \omega_1 t)] \end{aligned} \tag{26}$$

Define:

$$I_{D1} = \alpha_1 A \tag{27}$$

$$I_{D3} = \frac{3\alpha_3}{4} A^3 \tag{28}$$

The 3rd intermodulation is defined as:

$$ID_3 = \frac{I_{D3}}{I_{D1}} = \frac{3A^2}{4}\frac{\alpha_3}{\alpha_1} \tag{29}$$

Use the simulated data we can get:

$$ID_3 = \frac{I_{D3}}{I_{D1}} = \frac{3A^2}{4}\frac{\alpha_3}{\alpha_1} = -8.5dB \tag{30}$$

More simulation results:

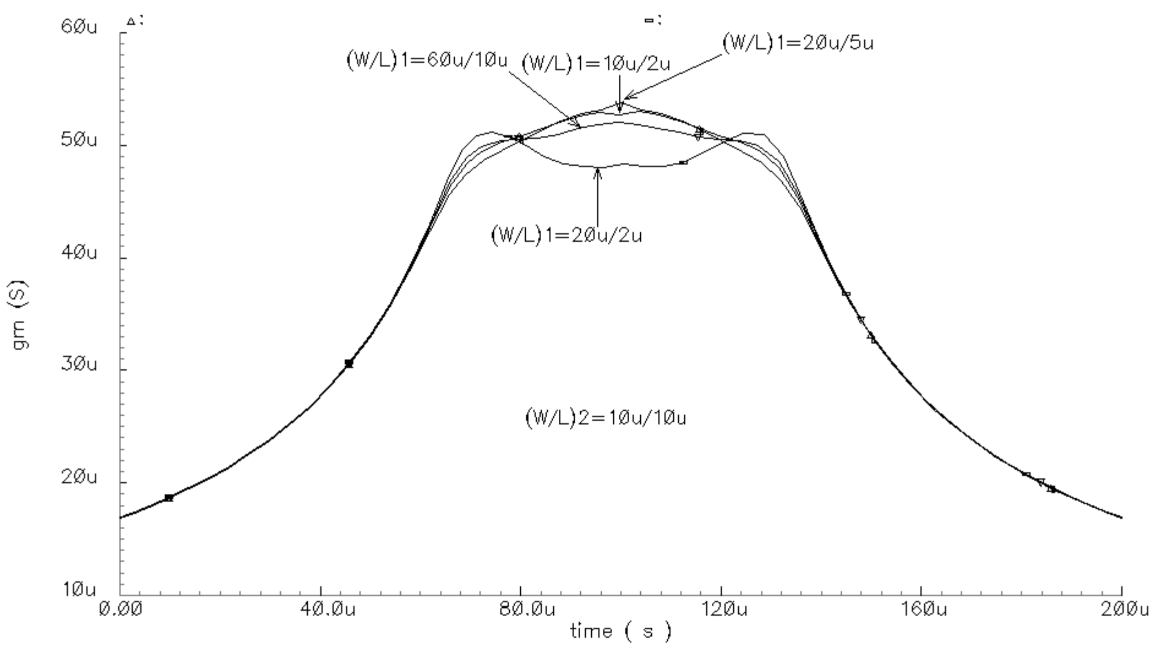


Figure 8: gm vs β_1/β_2. In this figure, β_2 is kept to be constant and β_1 changes.