Spring 2002

EEE598D: Analog Filter & Signal Processing Circuits

Instructor:

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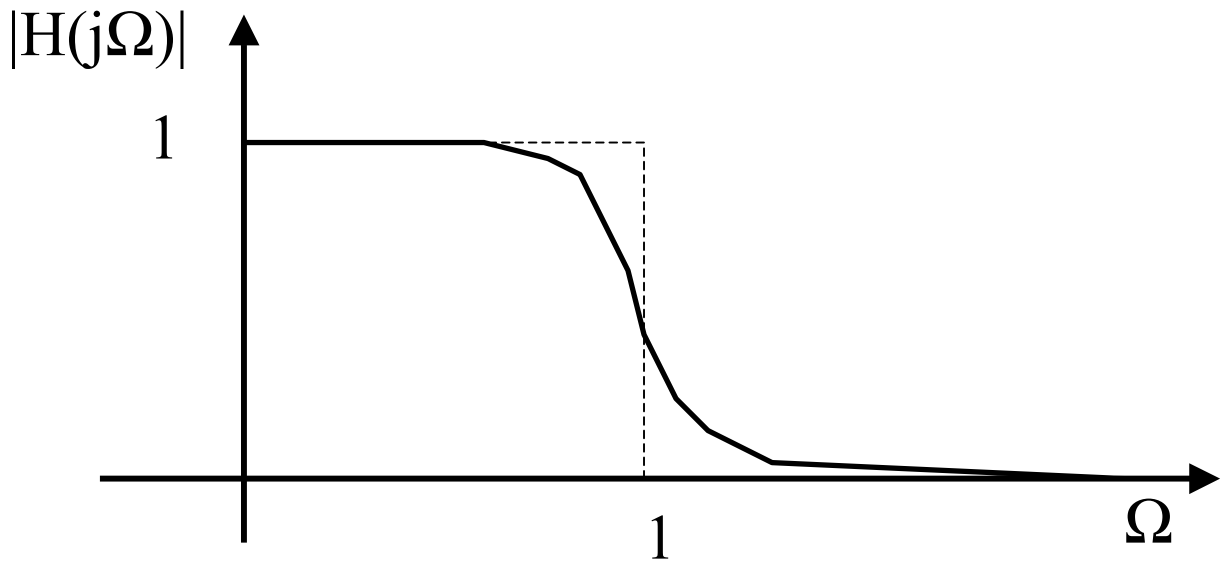
Today:

Frequency Scaling and Transformation

- Lowpass Prototype Filter
- Frequency Scaling
- Frequency Transformation

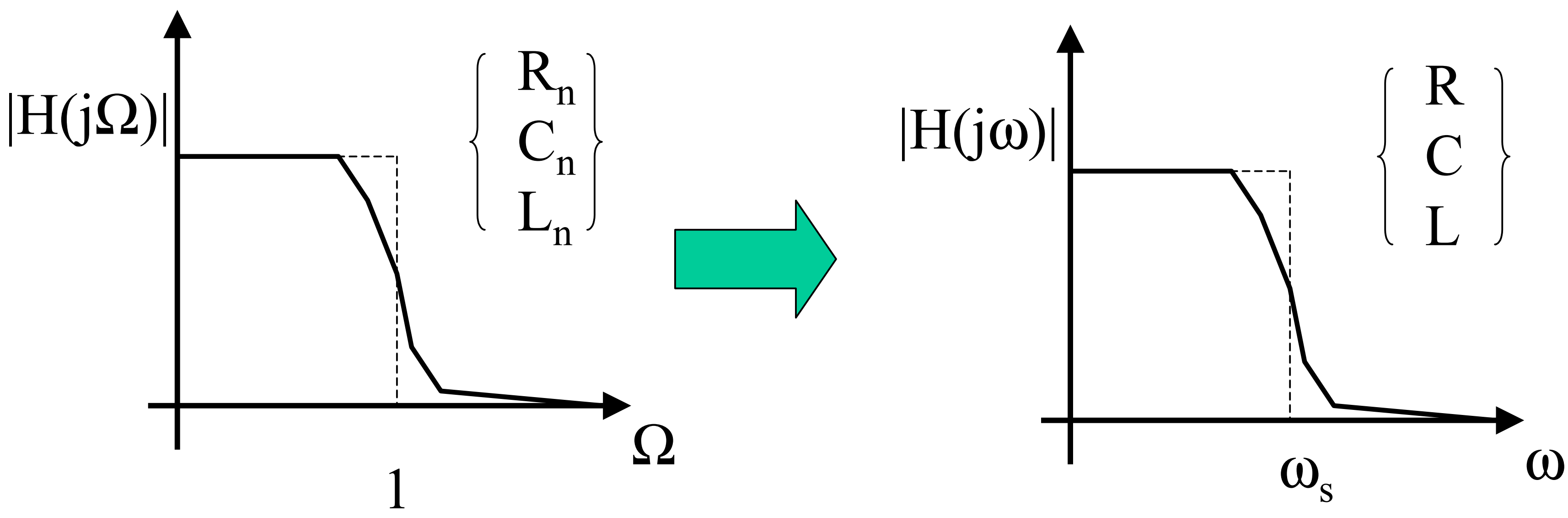
Lowpass Prototype Filter

- It is a normalized filter.
- Passband $0 < \Omega < 1$
- Stopband $1 < \Omega$



Frequency Scaling

- A lowpass filter with cutoff frequency ω_s can be built from the prototype lowpass filter



Frequency Scaling

by scaling the the values of RLC with respect to the RLC of normalize (prototype) filter in the following way:

$$R = R_s R_n$$

$$C = \frac{C_n}{\omega_s R_s}$$

$$L = \frac{R_s}{\omega_s} L_n$$

Arbitrary resistor scaling factor (R_s)

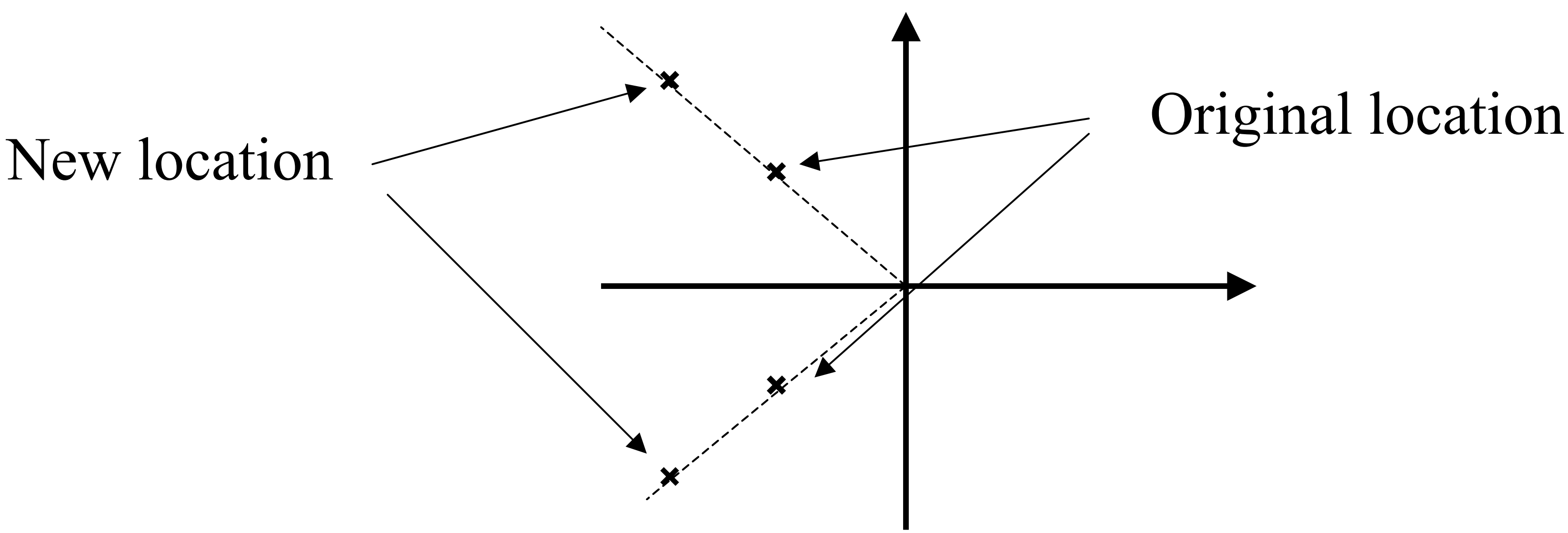
Frequency scaling factor (ω_s)

Frequency Scaling

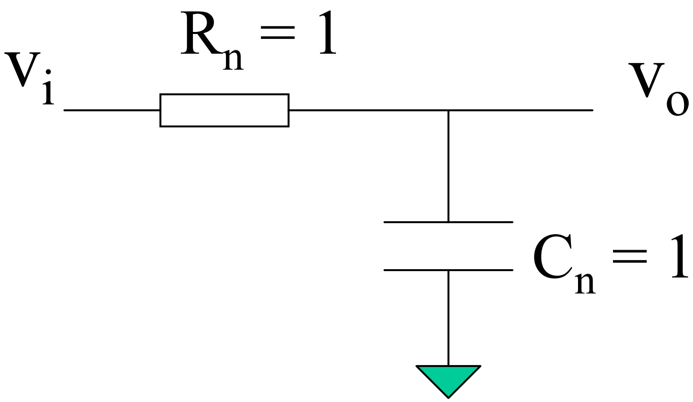
Location of pole and zero of the transfer function:

$$p \Rightarrow \omega_s p$$

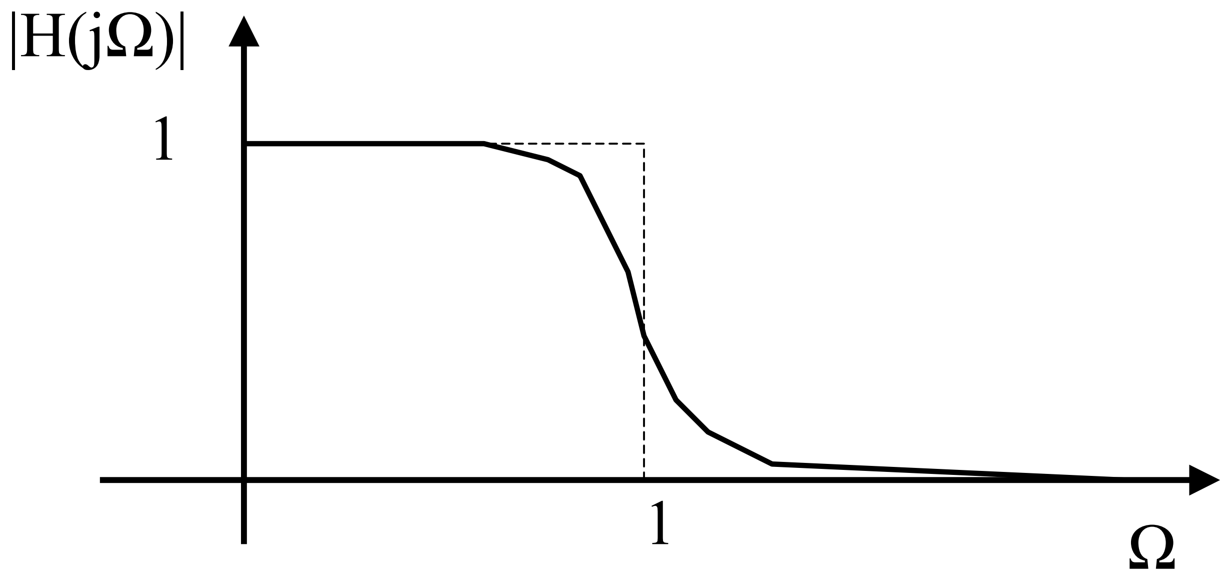
$$z \Rightarrow \omega_s z$$



Example: Frequency Scaling



$$H_n(S) = \frac{1}{1+S}$$

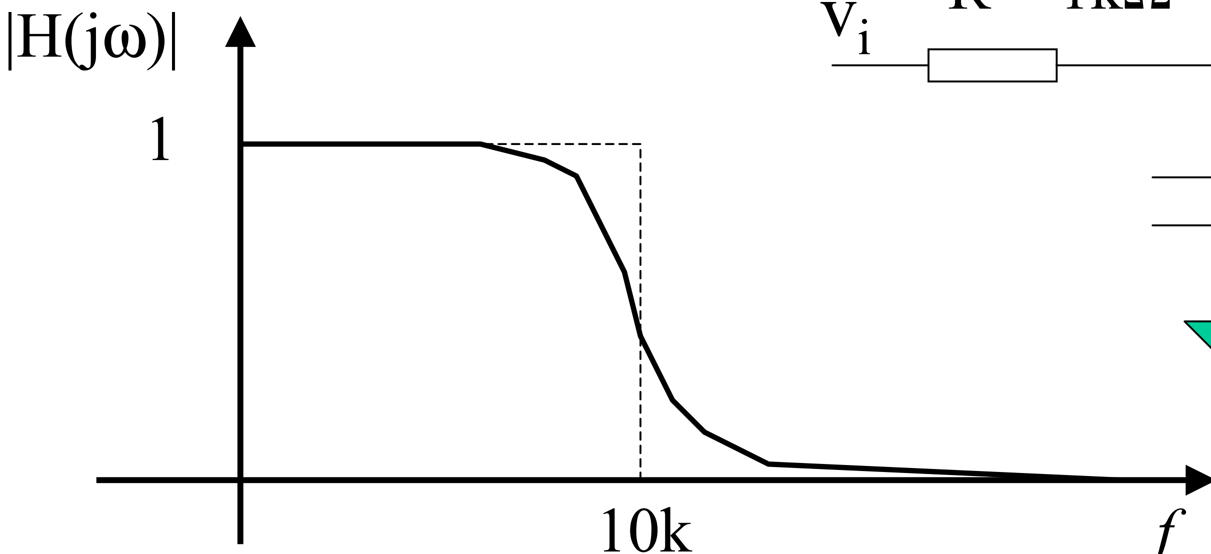


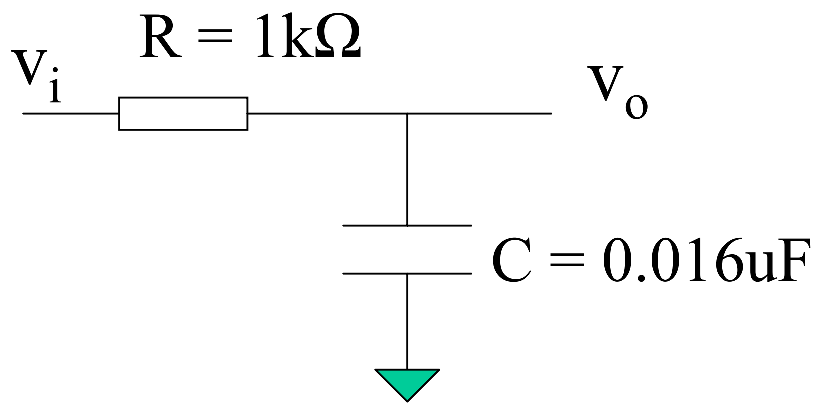
Example: Frequency Scaling (I)

Let R_s = 1k, $\omega_s/2\pi$ = 10kHz, we have

$$R = R_s R_n = 1k\Omega$$

$$C = \frac{C_n}{\omega_s R_{s\ n}} = 0.016uF$$



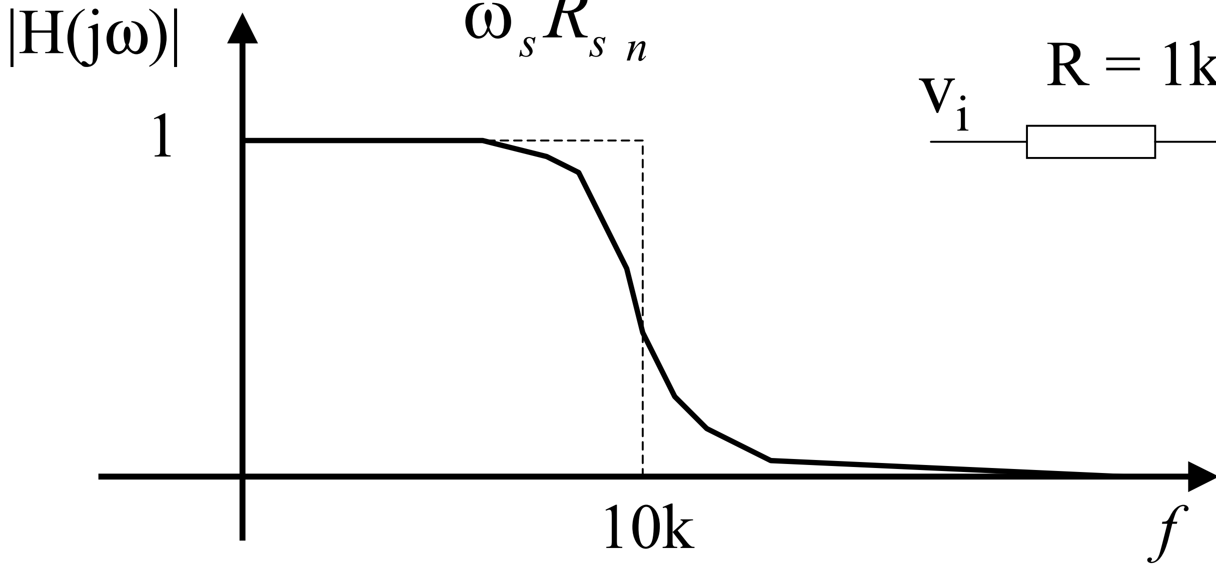


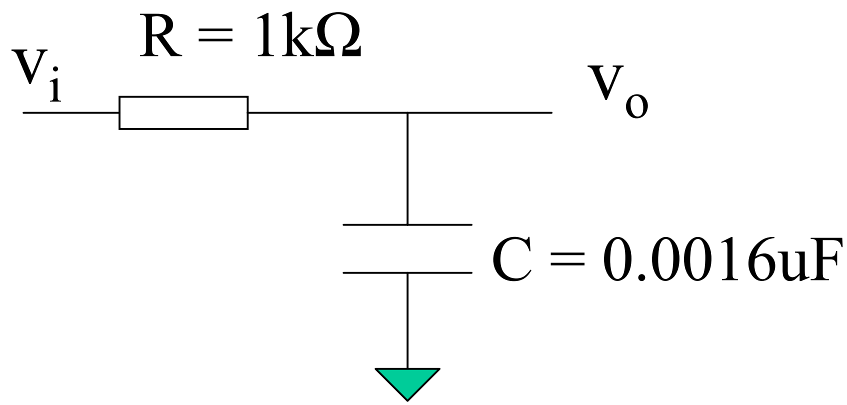
Example: Frequency Scaling (II)

If let $R_s = 10k$, $\omega_s/2\pi = 10kHz$, we have

$$R = R_s R_n = 10k\Omega$$

$$C = \frac{C_n}{\omega_s R_s \, {}_n} = 0.0016uF$$





The solution is not unique!!

Typical Component Values in IC

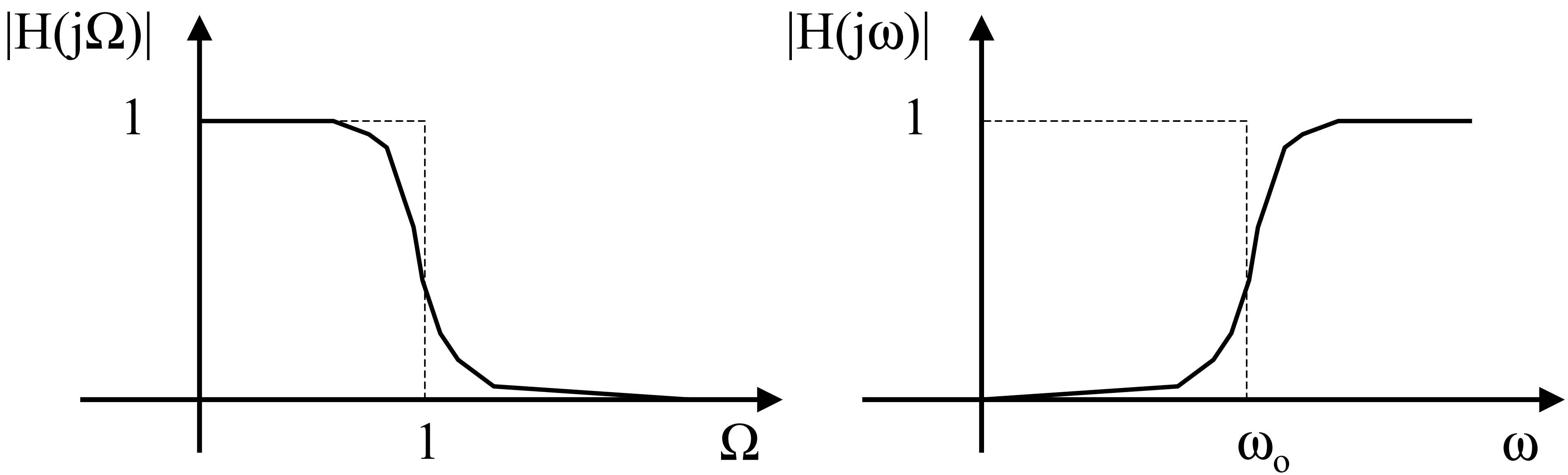
Tolerances:	10 ~ 40% absolute
	0.1 ~ 1% for ratio
Resistor:	50 ~ 100kΩ
Capacitor:	0.5 ~ 50pf
Inductor:	<10nH (lossy)

Frequency Transformation

- Lowpass-to-highpass transformation
- Lowpass-to-bandpass transformation
- Lowpass-to-bandstop transformation
- Lowpass-to-multi-bandpass transformation

Lowpass-To-Highpass Transformation

$$\Omega = -\frac{\omega_o}{\omega} \quad or \quad S = \frac{\omega_o}{s}$$



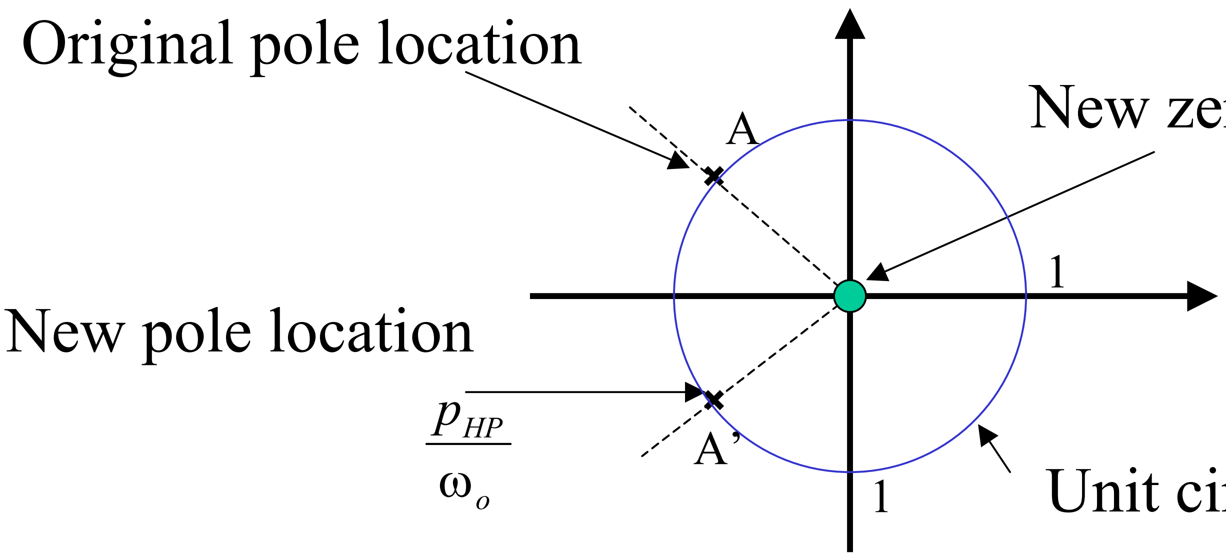
A) Lowpass prototype

B) Highpass normalized

Lowpass-To-Highpass Transformation

- Location of poles of the transfer function

$$p_{LP} \Rightarrow \frac{p_{HP}}{\omega_o} = \frac{1}{p_{LP}} = \left|\frac{1}{p_{LP}}\right| e^{-\theta_{PL}}$$



Example: LP-to-HP Transformation

- LP filter:

$$H_L(s) = \frac{1}{1+s}$$

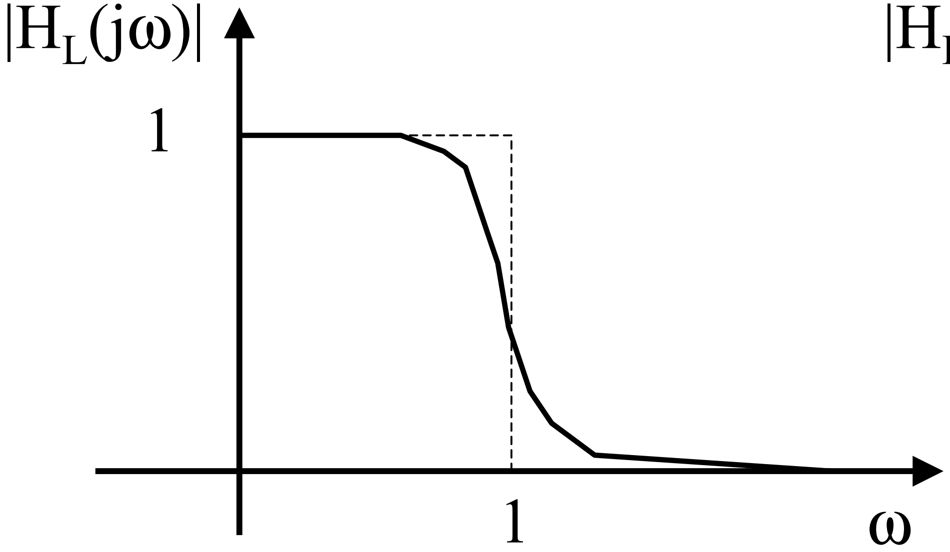
- Transformation

$$s \Rightarrow \frac{1}{s}$$

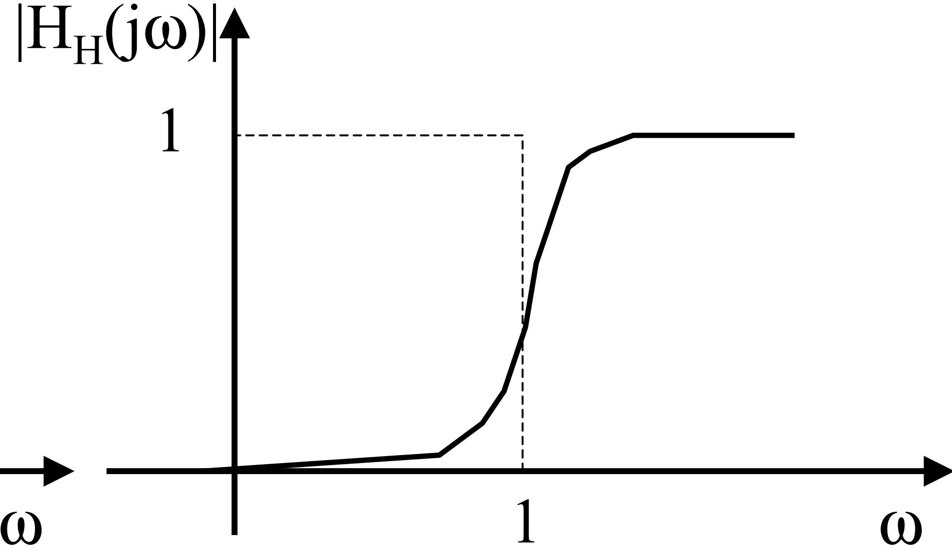
- LP filter

$$H_H(s) = H_L(\Omega)|_{\Omega=1/s} = \frac{1}{1+1/s} = \frac{s}{1+s}$$

Example: LP-to-HP Transformation



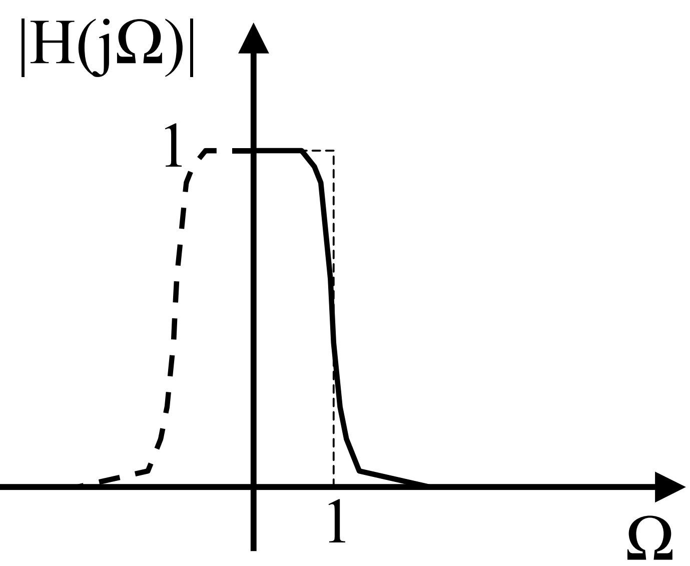
A) Lowpass filter



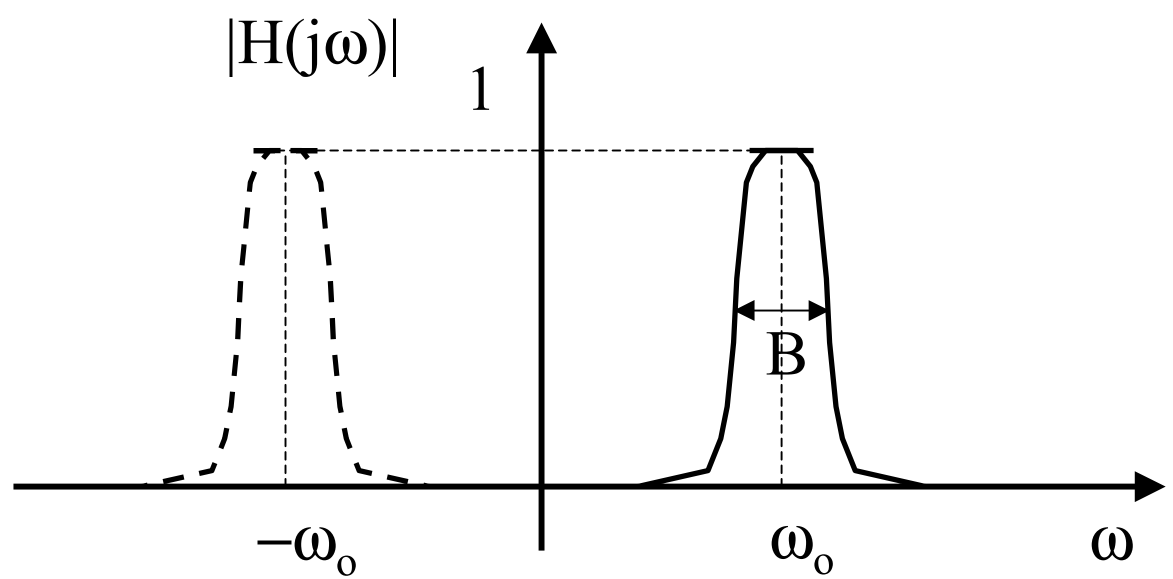
B) Highpass filter

Lowpass-To-Bandpass Transformation

$$\Omega = \frac{1}{B}\frac{\omega^2 - \omega_o^2}{\omega} \qquad or \qquad S = Q\left(\frac{s}{\omega_o} + \frac{\omega_o}{s}\right)$$



A) Lowpass prototype



B) bandpass

$$Q = \frac{\omega_o}{B}$$

Pole Locations

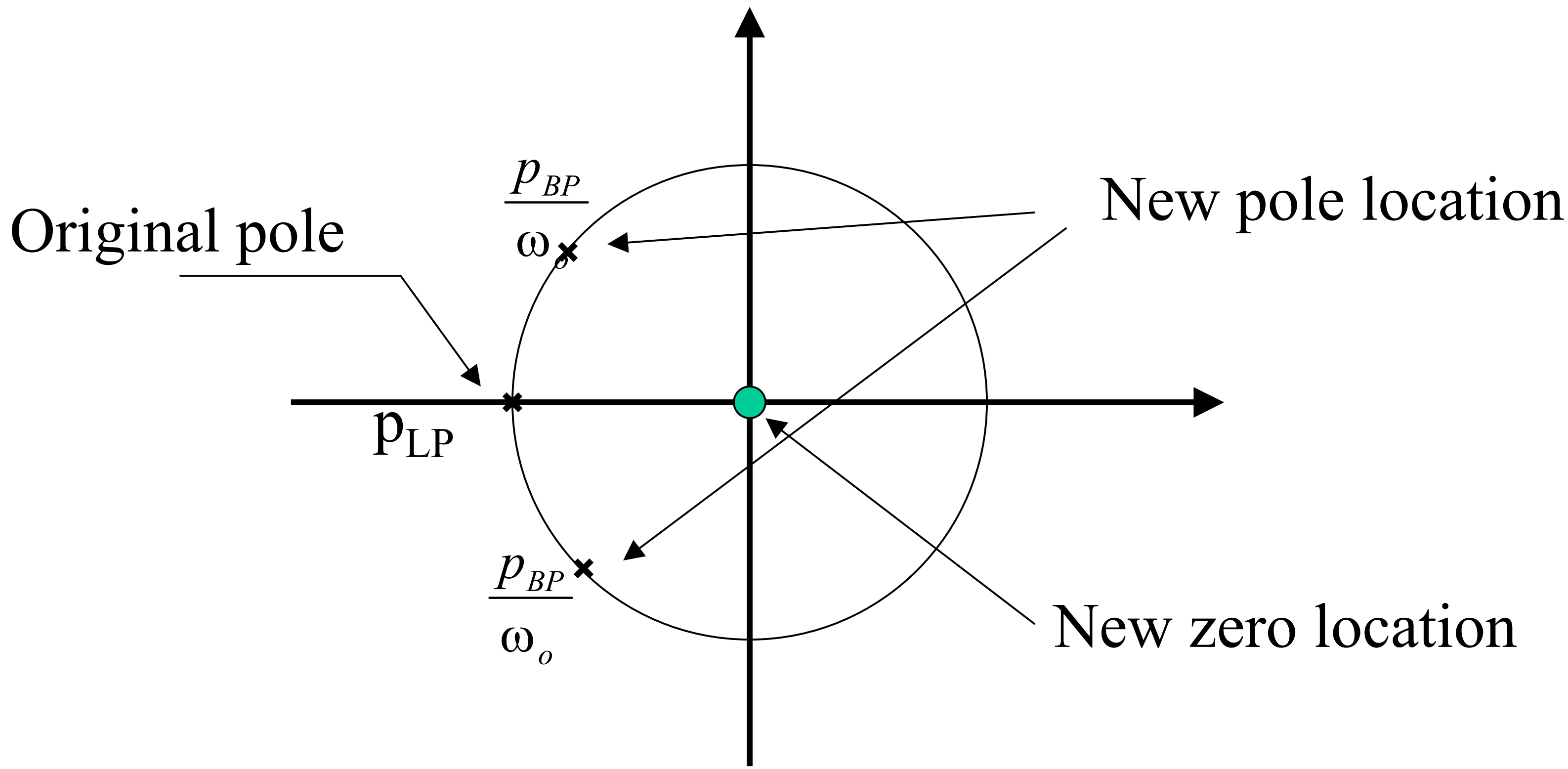
- For the first-order lowpass prototype:

$$s - p_{Lp} = 0 \Rightarrow Q(\frac{s}{\omega_o} + \frac{\omega_o}{s}) - p_{Lp} = 0$$

$$\Rightarrow \left(\frac{s}{\omega_o}\right)^2 - \frac{p_{Lp}}{Q}\left(\frac{s}{\omega_o}\right) + 1 = 0$$

$$\frac{p_{BP}}{\omega_o} = (\frac{p_{LP}}{2Q} \pm j\sqrt{1 - (\frac{p_{LP}}{2Q})^2})$$

Pole Locations



Pole Locations

- For the second-order lowpass prototype:

$$s^2 + \frac{s}{Q_{LP}} + 1 = (s - p_{LP1})(s - p_{LP2}) = 0$$

$$\Rightarrow (Q(\frac{s}{\omega_o} + \frac{\omega_o}{s}) - p_{Lp1})(Q(\frac{s}{\omega_o} + \frac{\omega_o}{s}) - p_{Lp2}) = 0$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{p_{BP1}}{\omega_o} = \frac{p_{LP1}}{2Q} \pm \sqrt{(\frac{p_{LP1}}{2Q})^2 - 1} \\ \frac{p_{BP2}}{\omega_o} = \frac{p_{LP2}}{2Q} \pm \sqrt{(\frac{p_{LP2}}{2Q})^2 - 1} \end{array} \right.$$

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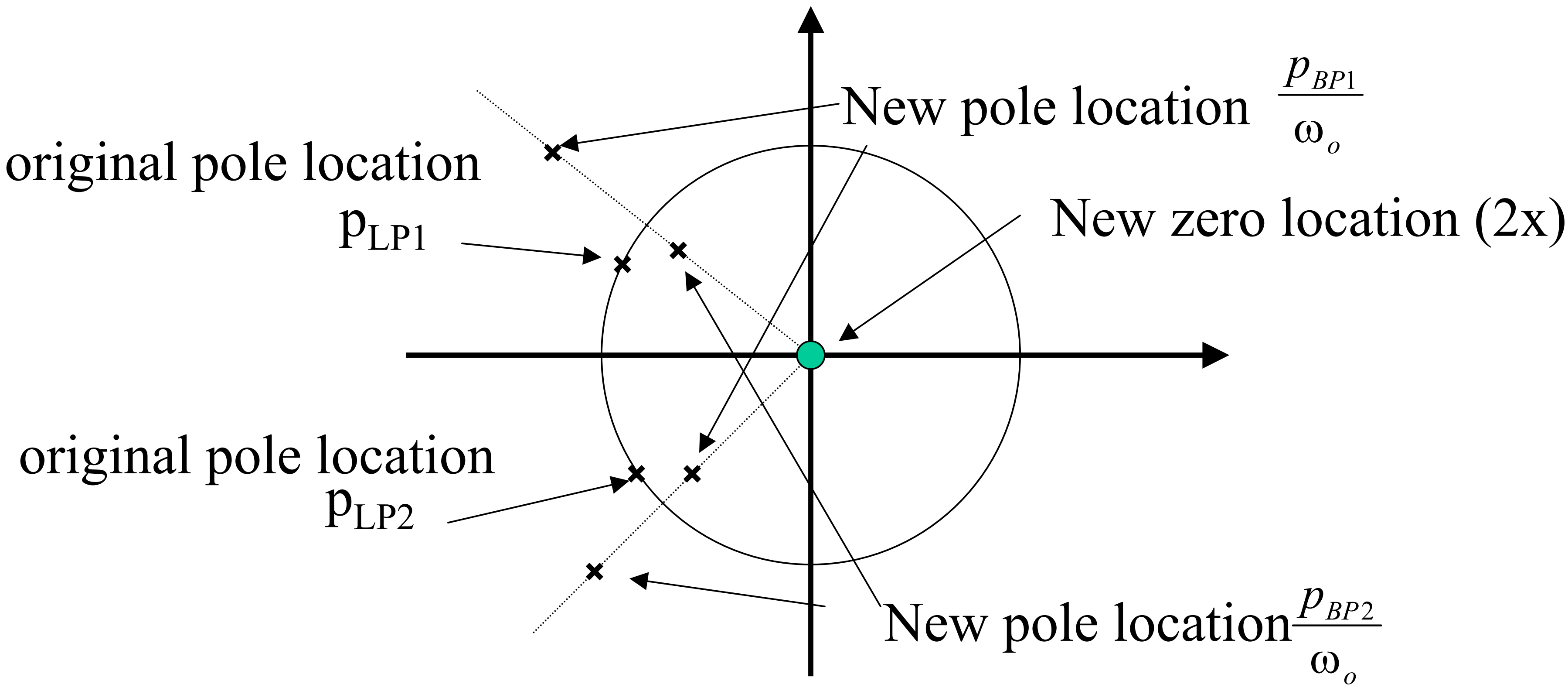
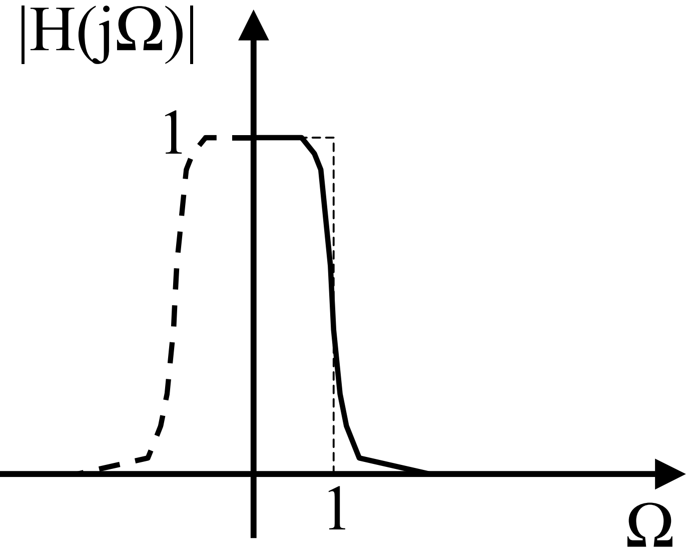
Pole Locations:

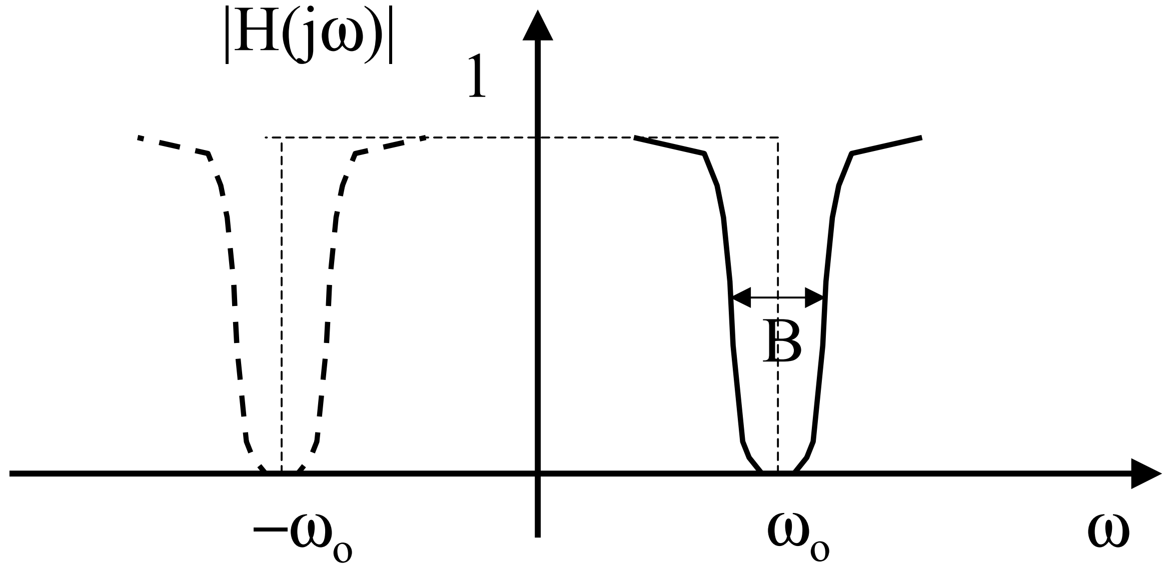


Lowpass-To-Bandstop Transformation

$$\Omega = -B\frac{\omega}{\omega^2 - \omega_o^2} \qquad or \qquad S = \frac{1}{Q}\frac{1}{(\frac{s}{\omega_o} + \frac{\omega_o}{s})}$$



A) Lowpass prototype



B) bandpass

$$Q = \frac{\omega_o}{B}$$

Pole Location

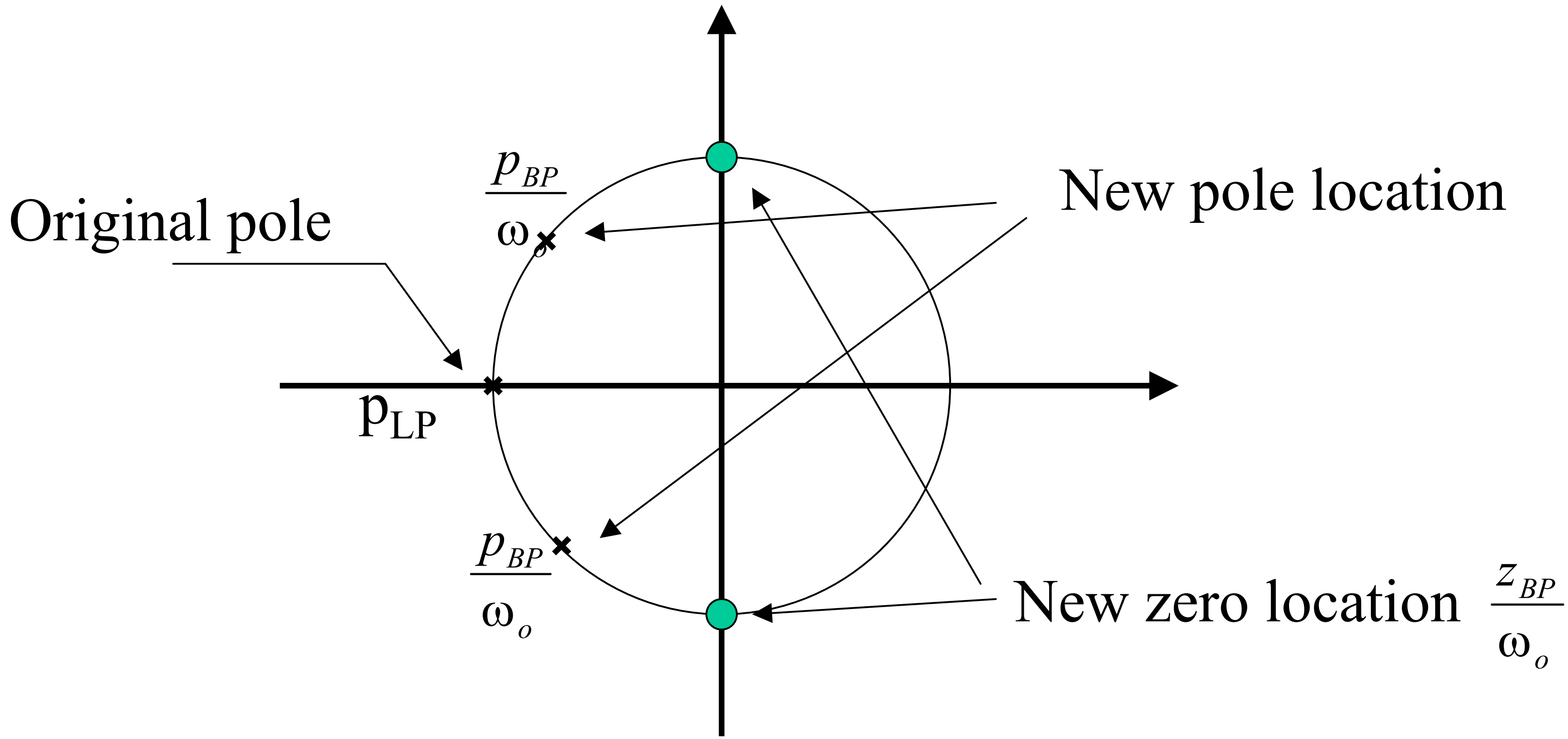
- For first-order lowpass prototype

$$s - p_{Lp} = 0 \Rightarrow \frac{1}{Q(\frac{s}{\omega_o} + \frac{\omega_o}{s})} - p_{Lp} = 0$$

$$\Rightarrow \left(\frac{s}{\omega_o}\right)^2 - \frac{1}{Qp_{Lp}}\left(\frac{s}{\omega_o}\right) + 1 = 0$$

$$\frac{p_{BP}}{\omega_o} = (\frac{1}{2Qp_{LP}} \pm j\sqrt{1 - (\frac{1}{2Qp_{LP}})^2})$$

Pole Location



Pole Locations

- For the second-order lowpass prototype:

$$s^2 + \frac{s}{Q_{LP}} + 1 = (s - p_{LP1})(s - p_{LP2}) = 0$$

$$\Rightarrow \left(\frac{1}{Q(\frac{s}{\omega_o} + \frac{\omega_o}{s})} - p_{Lp1}\right)\left(\frac{1}{Q(\frac{s}{\omega_o} + \frac{\omega_o}{s})} - p_{Lp2}\right) = 0$$

$$\Rightarrow \begin{cases} \dfrac{p_{BP1}}{\omega_o} = \dfrac{1}{2Qp_{LP1}} \pm \sqrt{(\dfrac{1}{2Qp_{LP1}})^2 - 1} \\ \dfrac{p_{BP2}}{\omega_o} = \dfrac{1}{2Qp_{LP2}} \pm \sqrt{(\dfrac{1}{2Qp_{LP2}})^2 - 1} \end{cases}$$

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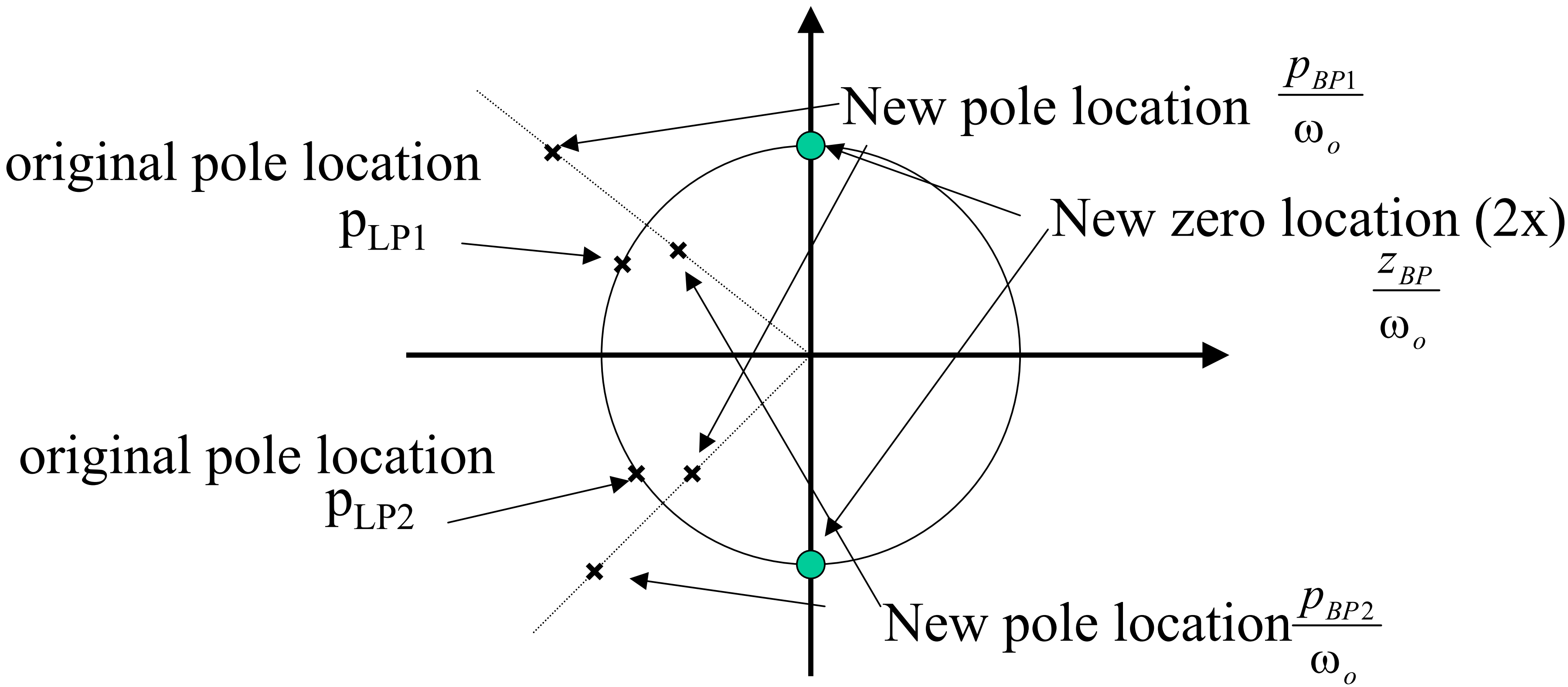
Pole Locations:



Lowpass-To-Multi-Passband Transformation

$$\Omega = \frac{K}{\omega}\coprod_{k=1}^{n}\frac{(\omega^2 - \omega^2{}_{0(2k-1)})}{(\omega^2 - \omega^2{}_{0(2k)})}$$

$$S = \frac{K}{s}\coprod_{k=1}^{n}\frac{(s^2 + \omega^2{}_{0(2k-1)})}{(s^2 + \omega^2{}_{0(2k)})}$$