Spring 2002

EEE598D: Analog Filter & Signal Processing Circuits

Instructor:

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Tuesday January 22, 2002

Today: Continuous-Time Filter Fundamental

- Pole & Zero of Transfer Function
- Bode Plots
- SFG & Block Diagram
- Classification of Filter Response

Pole & Zero of the TF

- Transfer function of a general CT filter can be expressed as

$$H(s) = \frac{a_m}{b_n} \frac{\prod_{k=1}^{m}(s - s_k)}{\prod_{k=1}^{n}(s - s_k)} = \frac{a_m}{b_n} \frac{(s - z_m)(s - z_{m-1})\ldots(s - z_1)}{(s - p_n)(s - p_{n-1})\ldots(s - p_1)} \qquad (2.1)$$

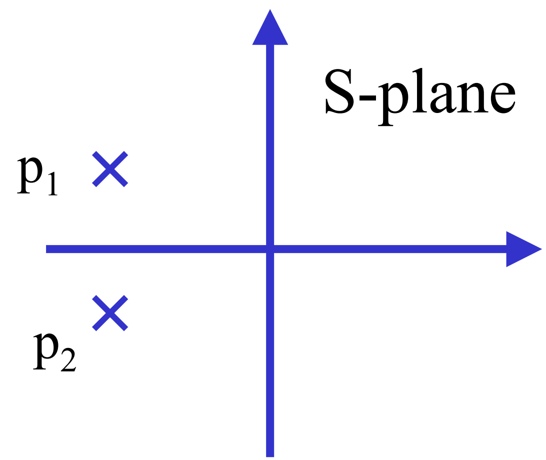
where $\{z_k\}$ and $\{p_k\}$ are s-domain zeros and poles of the transfer function

Example: Pole and Zero of the TF

- For the given example:

$$
\begin{cases}
H(s) = \dfrac{1}{s^2LC + sRC + 1} = \dfrac{1}{LC}\dfrac{1}{(s-p_1)(s-p_2)} \\
p_1 = -\dfrac{R}{2L} + j\sqrt{\dfrac{1}{LC} - (\dfrac{R}{2L})^2} \\
p_2 = -\dfrac{R}{2L} - j\sqrt{\dfrac{1}{LC} - (\dfrac{R}{2L})^2}
\end{cases}
\quad (2.\ 2)
$$

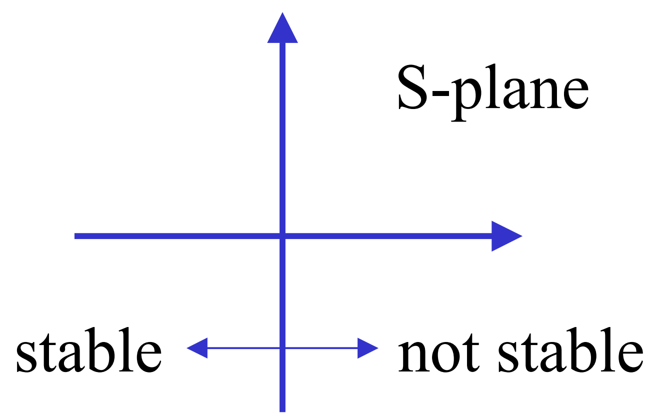
- It has two poles.



Stability of the Filter

- For stable filters, ALL poles of the transfer function have to be on the left side of the s-plane

$$H(s) = \frac{a_m}{b_n} \frac{(s - z_m)(s - z_{m-1})\ldots(s - z_1)}{(s - p_n)(s - p_{n-1})\ldots(s - p_1)} \qquad (2.3)$$



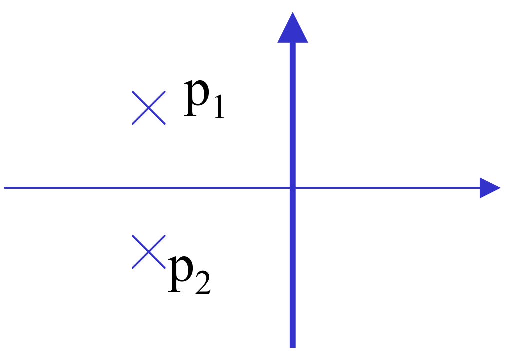
Example: Stability of the Filter System

- For the given example:

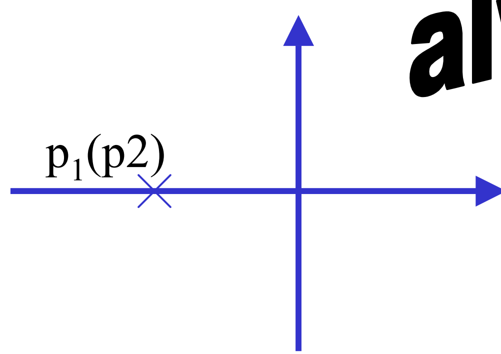
$$p_1 = -\frac{R}{2L} + j\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$$

$$p_2 = -\frac{R}{2L} - j\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2} \qquad (2.3)$$

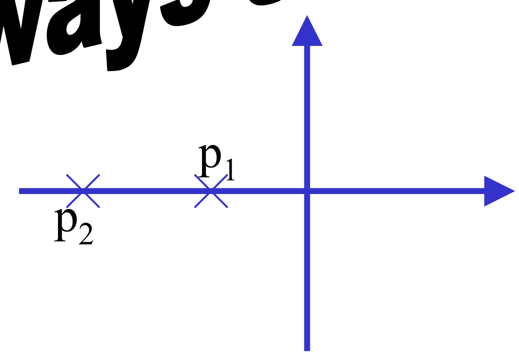
always stable!



$$\frac{1}{LC} - (\frac{R}{2L})^2 > 0$$



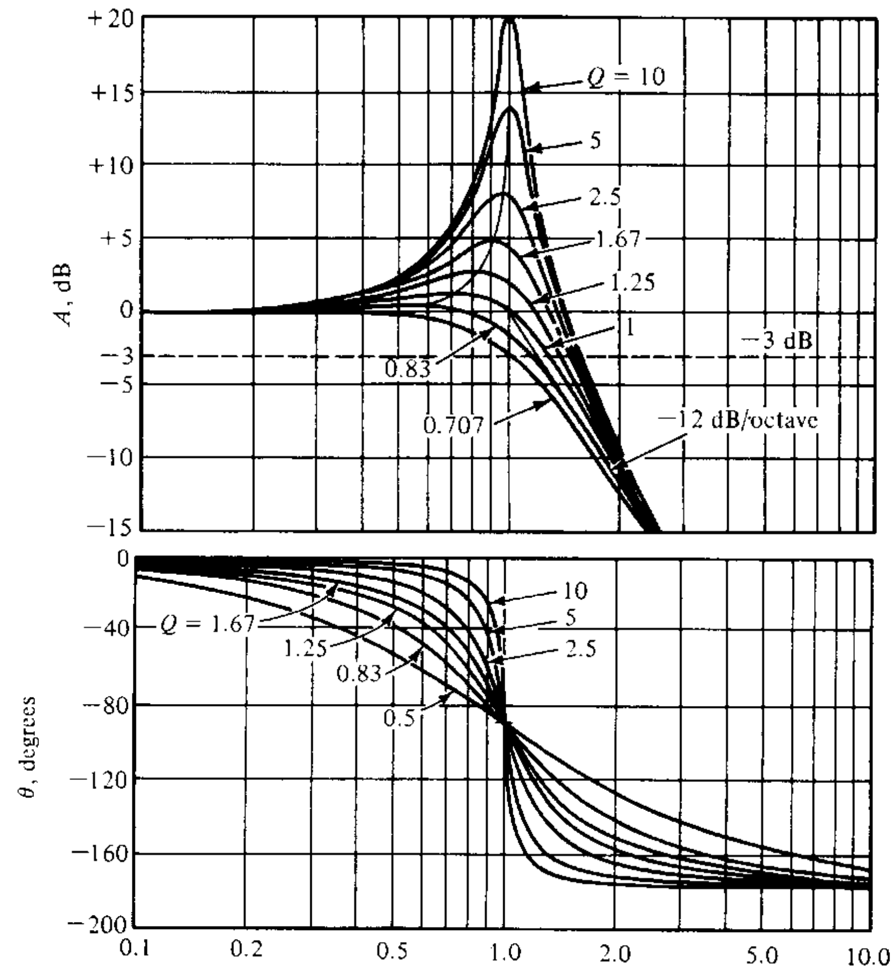
$$\frac{1}{LC} - (\frac{R}{2L})^2 = 0$$



$$\frac{1}{LC} - (\frac{R}{2L})^2 < 0$$

Bode Plots

- The plots of the gain and phase responses of the system vs. frequency

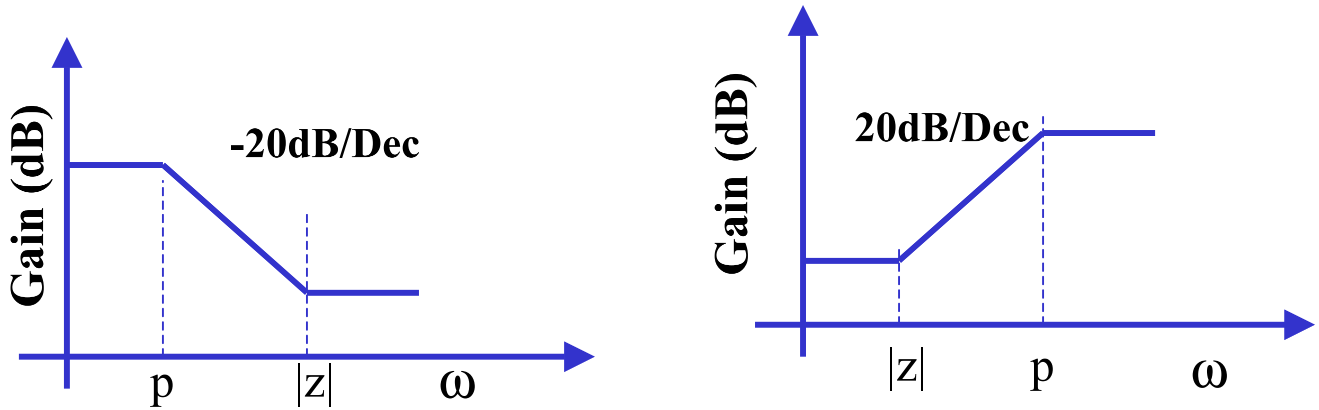


$\omega\sqrt{LC}$

Bode Plots - the First-Order Systems

- For the first order system, we have

$$H(s) = K\frac{(s-z)}{(s-p)} \qquad (2.4)$$



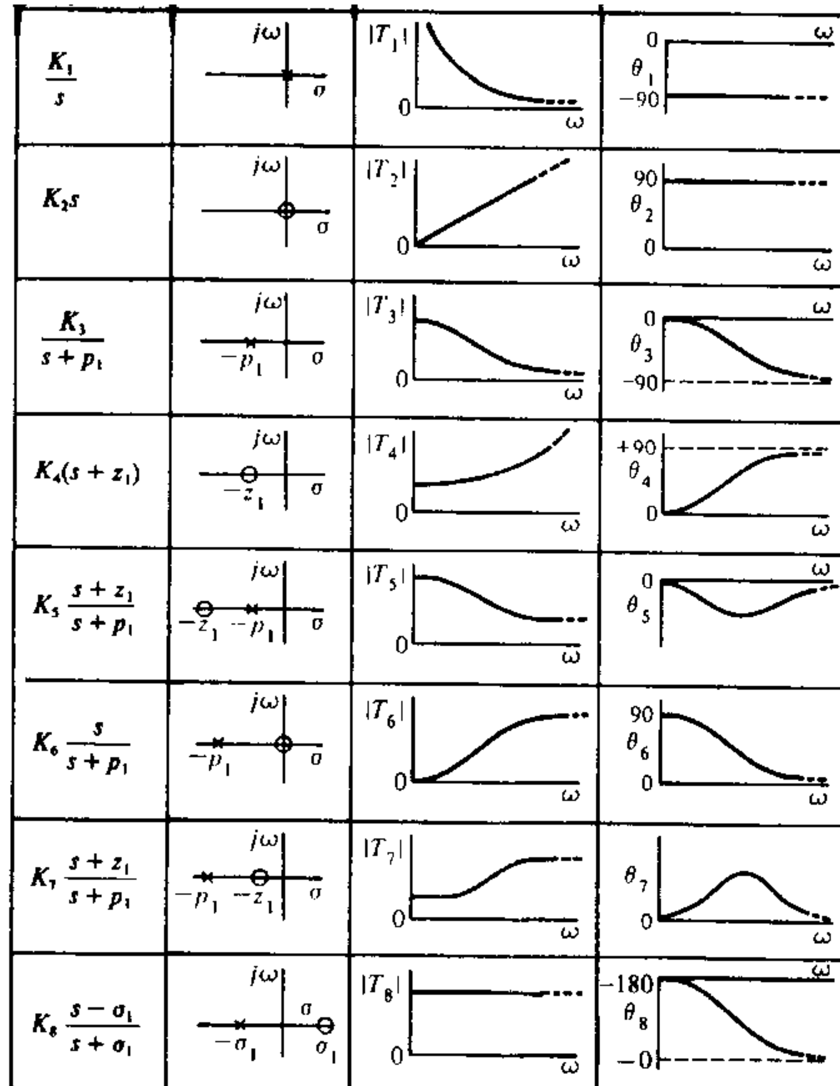
A) |z| > p

B) |z| < p

Bode Plots - the First-Order Systems

- For the first order system, we have (assume K >0)

$$H(s)(=T(s)) = K\frac{(s-z)}{(s-p)}$$



* All K_j are assumed positive.

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Bode Plots - Important Features

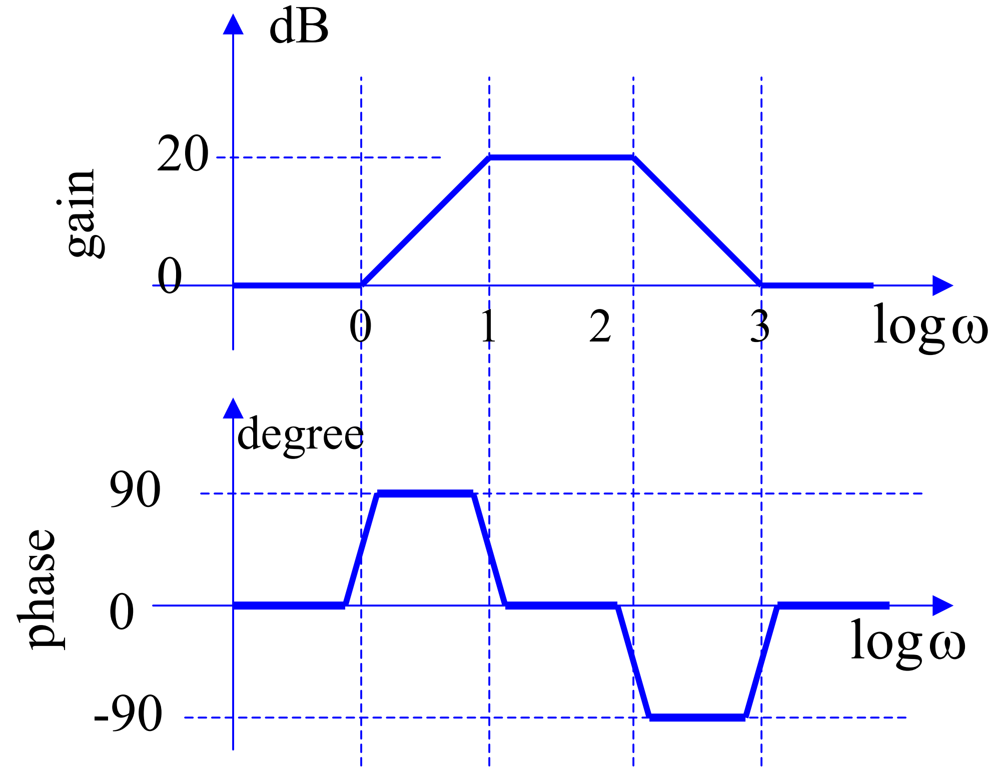
- Both frequency and magnitude are plotted on logarithm scales.
- Logarithm measure of the magnitude make it possible to add and subtract rather than multiply or divide.
- The slope of all lines for bilinear function is +/-6dB/Oct or +/-20dB/dec. In general case all asymptotic lines are integer multiples these two numbers.

Bode Plots - Important Features(Con't)

- In many applications we deal only with asymptotic plots without "filling the corner" to obtain the actual plots.
- The phase angle plots are added and subtracted to obtain total angle response.

Example: Bode Plots

$$H(s) = \frac{(s+1)(s+1000)}{(s+10)(s+100)}$$



SFG & Block Diagram

- We have seen that a CT filter can be represented by its
  - Schematic
  - Differential Equations
  - Transfer Function
- It can be also expressed as
  - Signal-Flow Graph (SFG), or
  - Block Diagram

SFG & Block Diagram

- Basic elements of CT filter

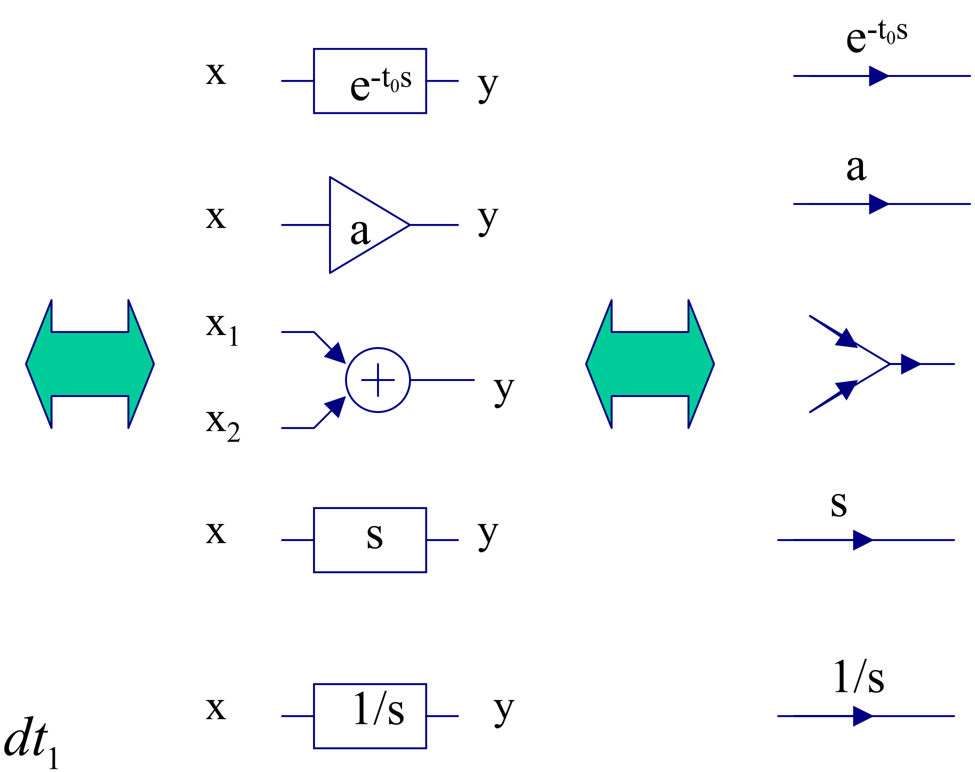
$$y(t) = x(t - t_0)$$

$$y(t) = ax(t)$$

$$y(t) = x_1(t) + x_2(t)$$

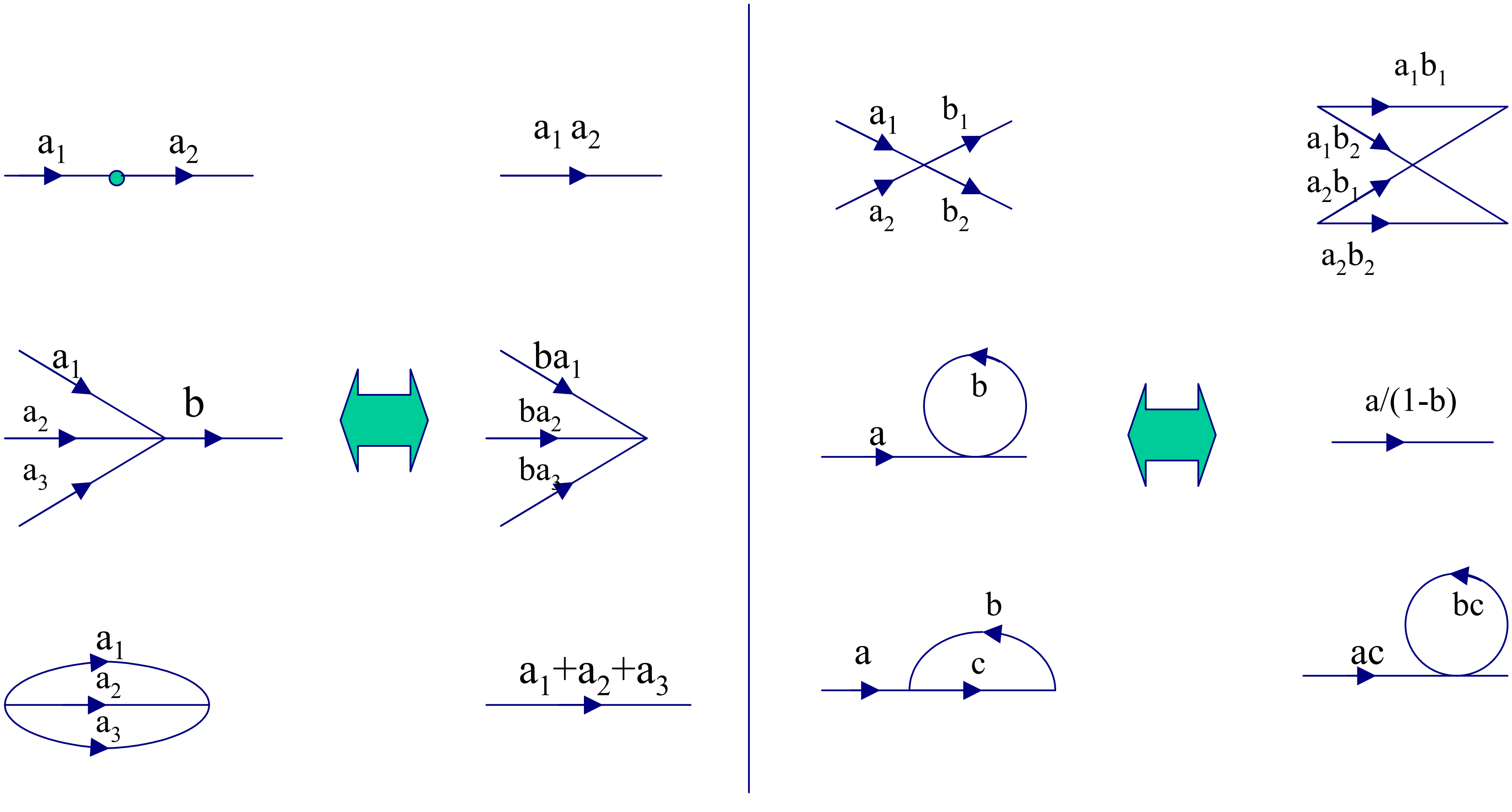
$$y(t) = \frac{dx(t)}{dt}$$

$$y(t) = y(0) + \int_0^t x(t_1)dt_1$$



(2.5)

Rules for SFG Reduction



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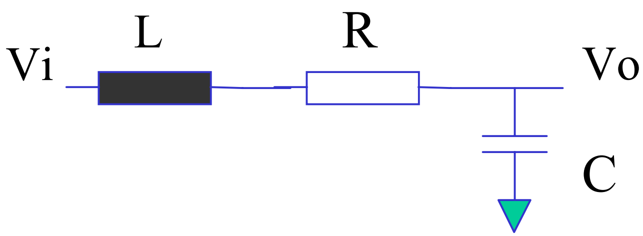
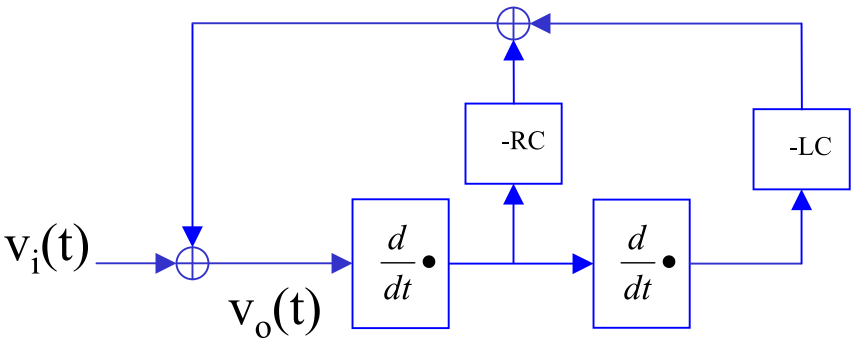
Example: SFG and Block Diagram

- For the filter described below



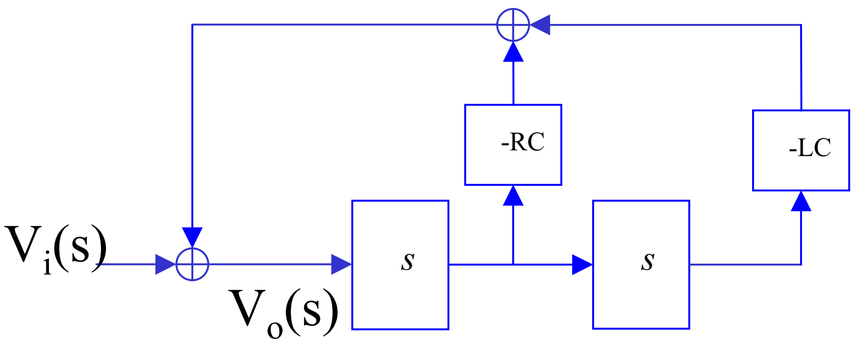
$$v_o(t) = v_i(t) - LC\frac{d^2 v_o(t)}{dt^2} - RC\frac{dv_o(t)}{dt} \quad (2.6)$$

- We have real-space block diagram:

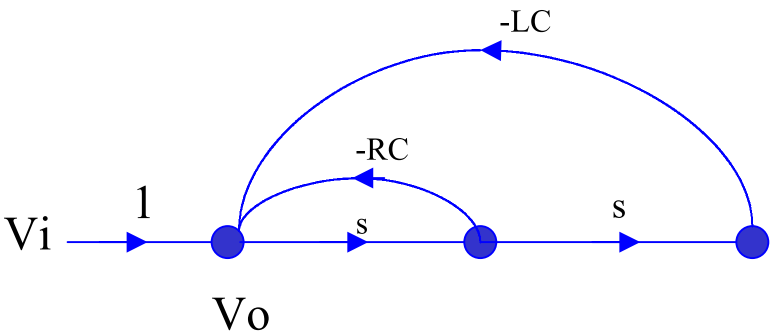


Example: SFG and Block Diagram

- S-domain block diagram:

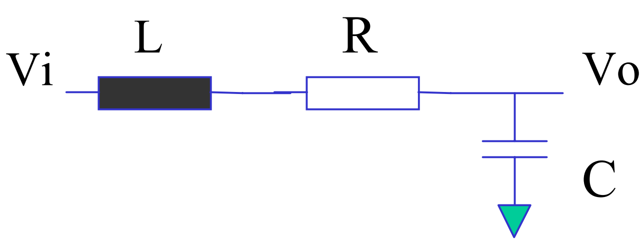


- SFG:



Alternative SFG and Block Diagram

- In practical cases, differentiator tends to amplify the noise. Therefore integrator is preferred in the filter design.
- For the filter discussed in the example, we may have alternative equation



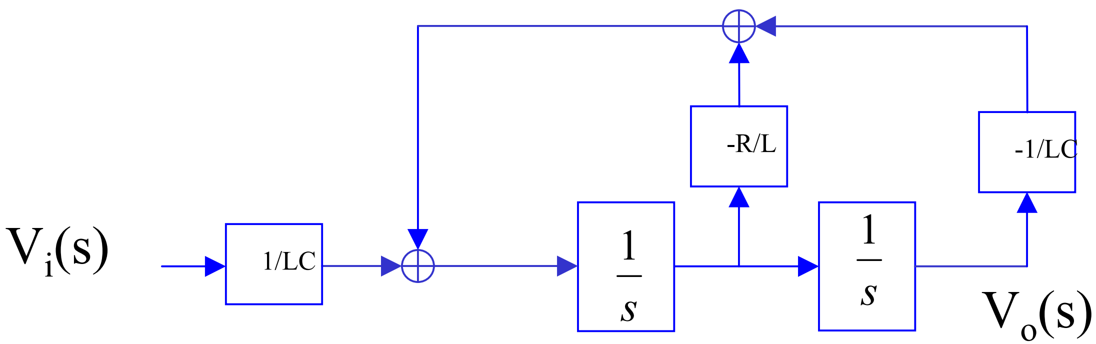
$$LC\frac{d^2 v_o(t)}{dt^2} + RC\frac{dv_o(t)}{dt} + V_o(t) = v_i(t)$$

$$\Rightarrow (LCs^2 + RCs + 1)V_o(s) = V_i(s) \qquad (2.7)$$

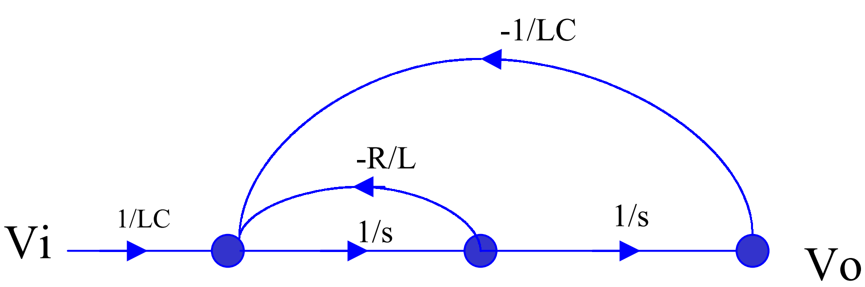
$$\Rightarrow V_o(s) = \frac{1}{s^2}\frac{V_i(s)}{LC} - \frac{1}{s^2}\frac{V_o(s)}{LC} - \frac{1}{s}\frac{R}{L}V_o(s)$$

Alternative Block Diagram and SFG

- System built using integrators:



- With the alternative SFG:



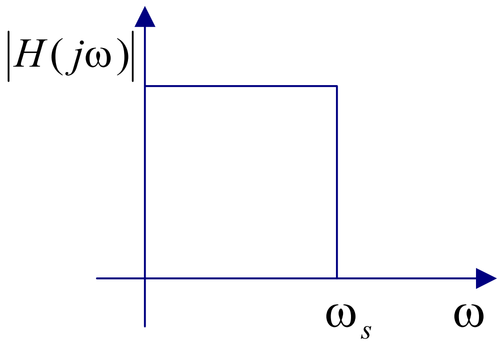
Classification of Filters Response

- Filters are classified according to the function they are to perform.
- In ideal case, a passband is the range of frequency of the filter where $|H(j\omega)|=1$.
- In a idea stopband $|H(j\omega)|=0$.

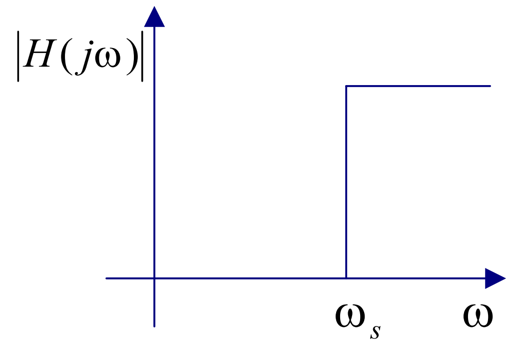
Classification of Filters Response

- The pattern of passbands and stopbands that give rise to the most common filters are defined as:
  - Lowpass (LP) filters
  - Highpass (HP) filters
  - Bandpass (BP) filters
  - Bandstop (BS) filters

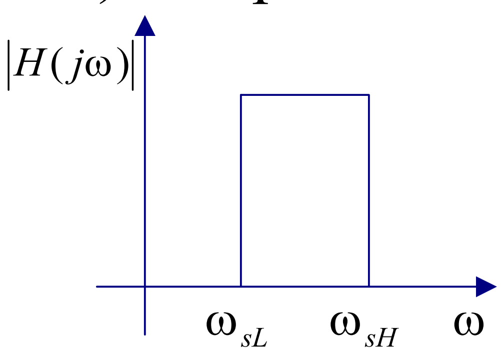
Ideal Filters Response



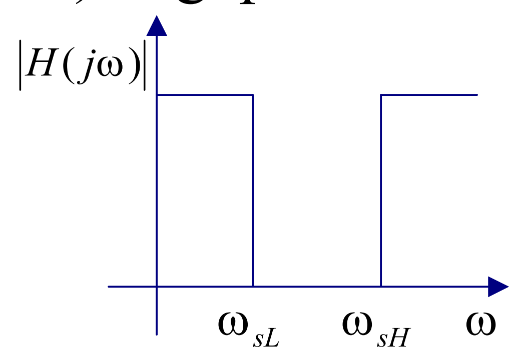
A) Lowpass Filter



B) Highpass Filter



C) Bandpass Filter



D) Bandstop Filter

Specification of Realistic Filter

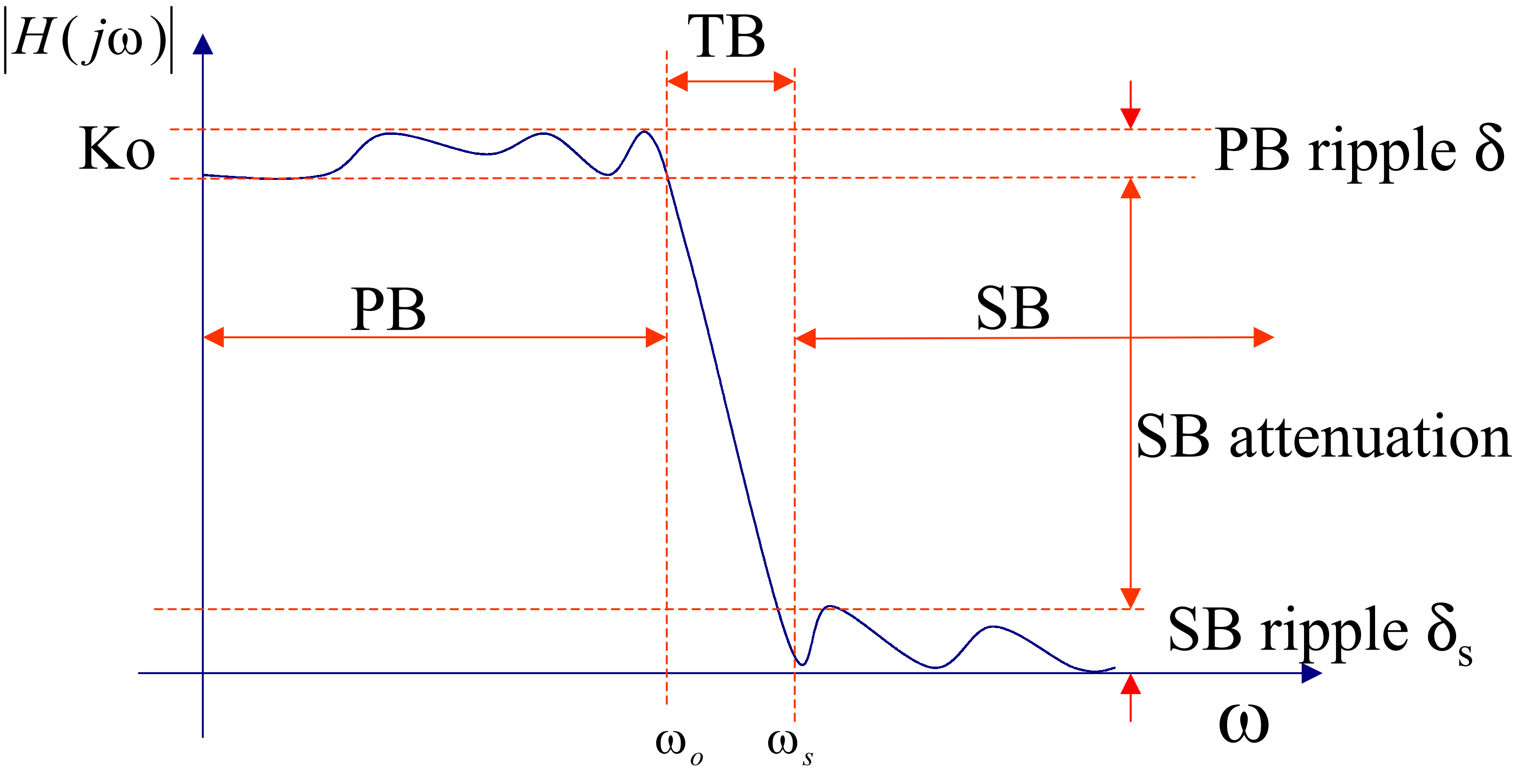
- In practice, it is not possible to realize the ideal transfer function with realistic filter consisting of a finite number of elements.
- A realistic filter is always described by real rational function of the complex frequency given as:

$$H(s) = \frac{\sum_{k=0}^{m} a_k s^k}{\sum_{k=0}^{n} b_k s^k} = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0} \quad (2.4)$$

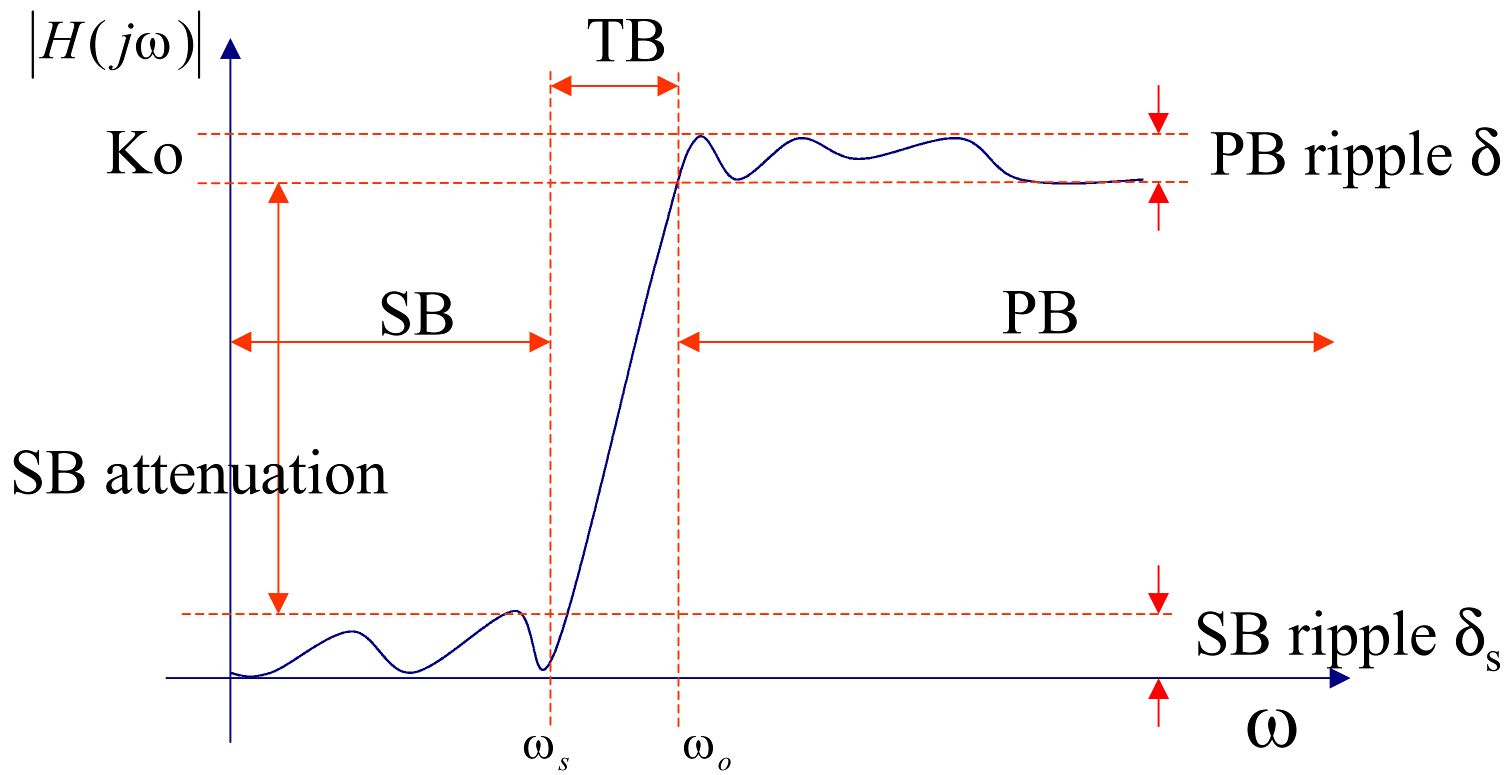
Specification of Realistic Filter

- Such a transfer function are seen have finite transition region (known as transition bands) between passbands and stopbands.
- Filters may be specified either in
  - Gain of transfer function, or
  - Attenuation of transfer function

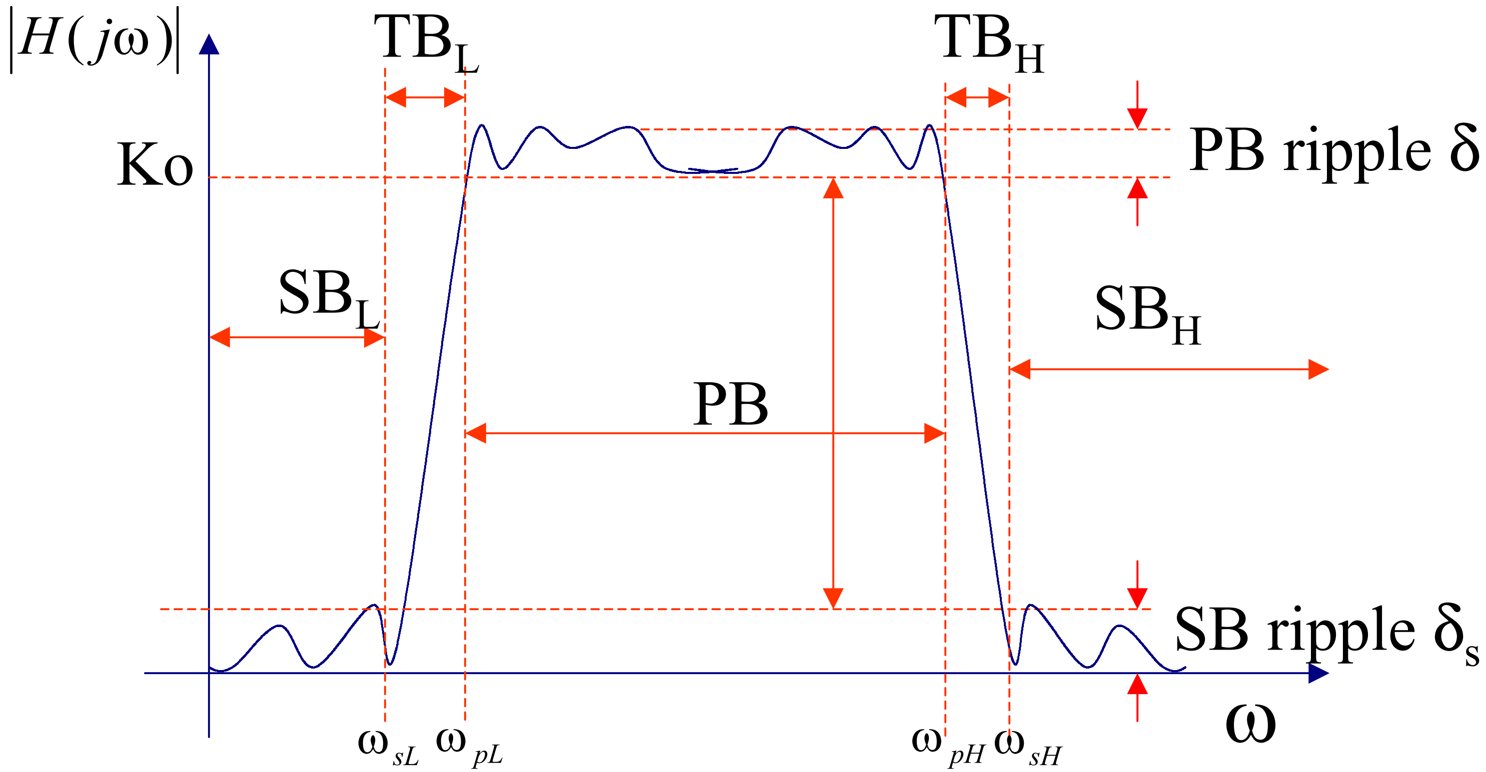
Specification of Realistic LP Filter



Specification of Realistic HP Filter



Specification of Realistic BP Filter



Specification of Realistic BS Filters