Spring 2002

EEE598D: Analog Filter & Signal Processing Circuits

Instructor:

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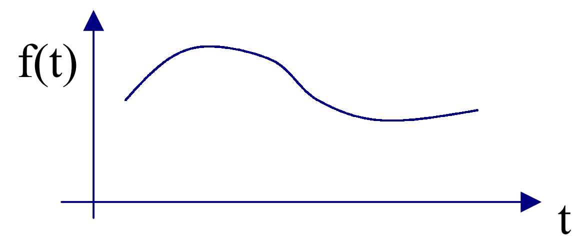
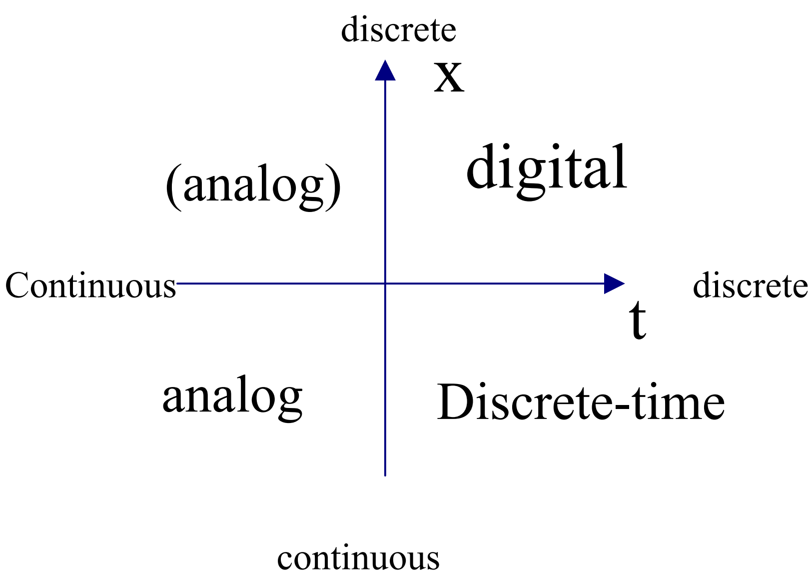
Today: Discrete-Time Filter Fundamental

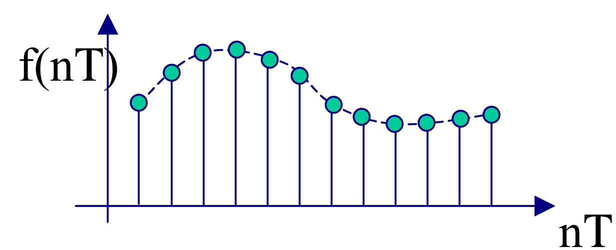
- Classification of Signals
- Sampling of Signals
- Z- Transformation
- Z-Domain Transfer Function
- Block Diagram and SFG

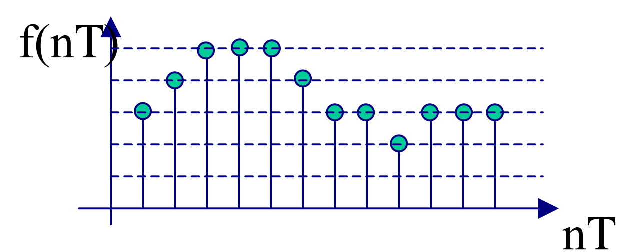
Classification of Signals

- Continuous-Time Signal
- Discrete-Time Signals
- Digital Signals









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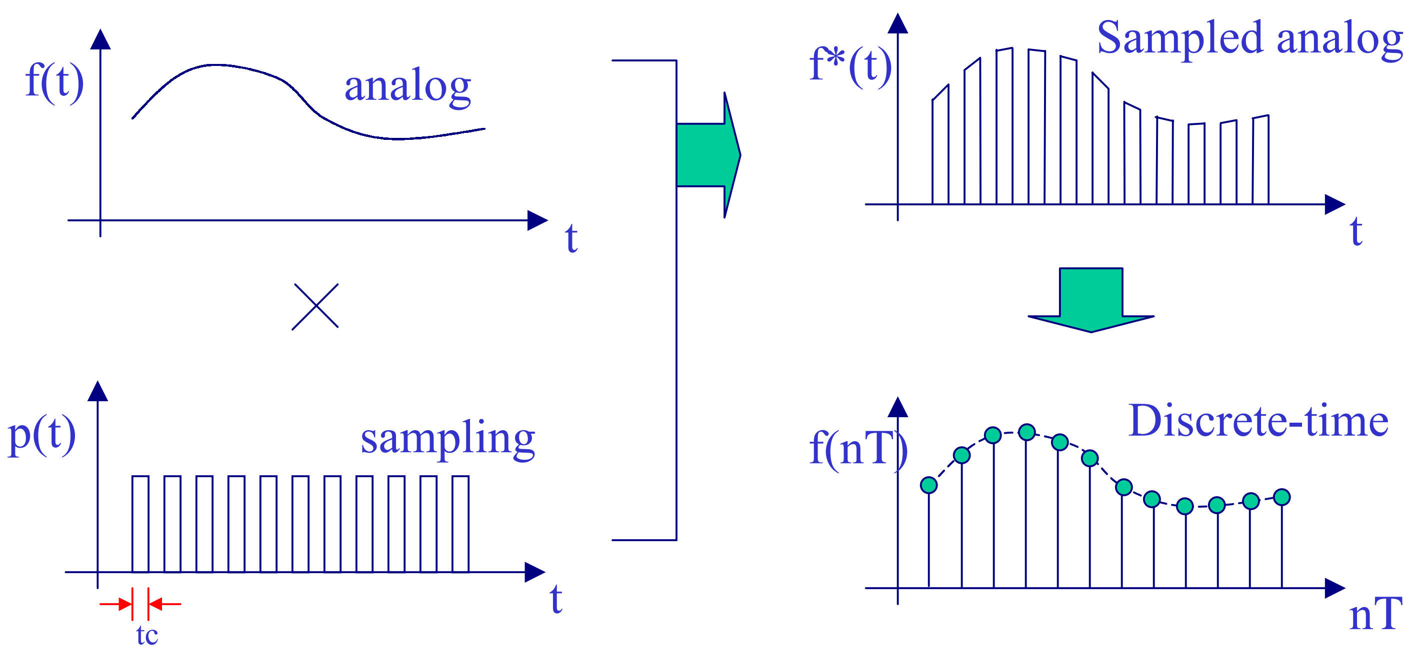
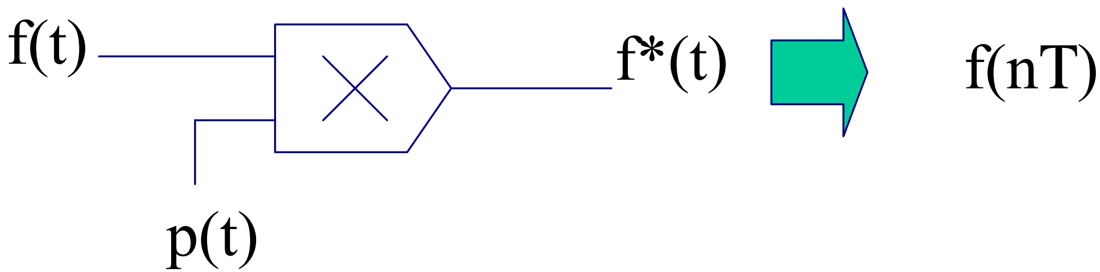
Sampling of Analog Signal

- A process converting continuous-time analog signal to discrete-time signal

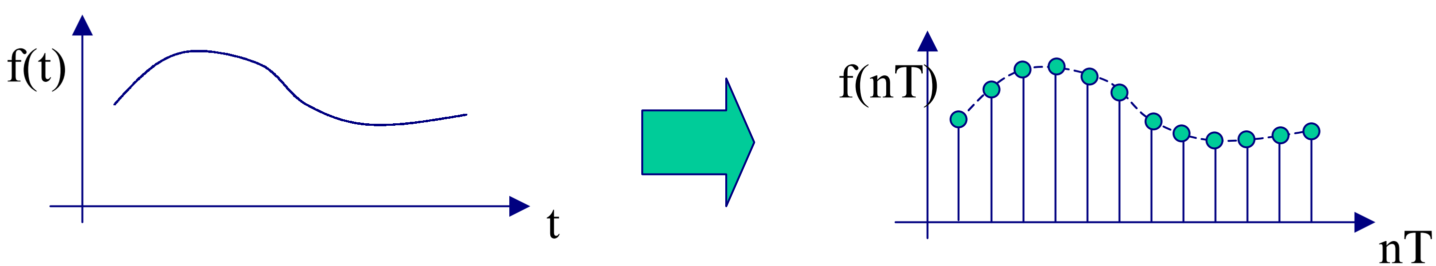


Sampling of Analog Signal

- Sampling can be modeled as the modulation of the analog signal f(t) with a sampling function p(t)



$$f^*(t) = f(t) \cdot p(t)$$ ➡ f(nT)



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Some Special Functions

- Unit Impulse Function (Dirac Delta Function):

$$\delta(t) \equiv \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \& \quad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Discrete Unit Sampling Function:

$$\delta(nT) \equiv \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

- Unit Step Function

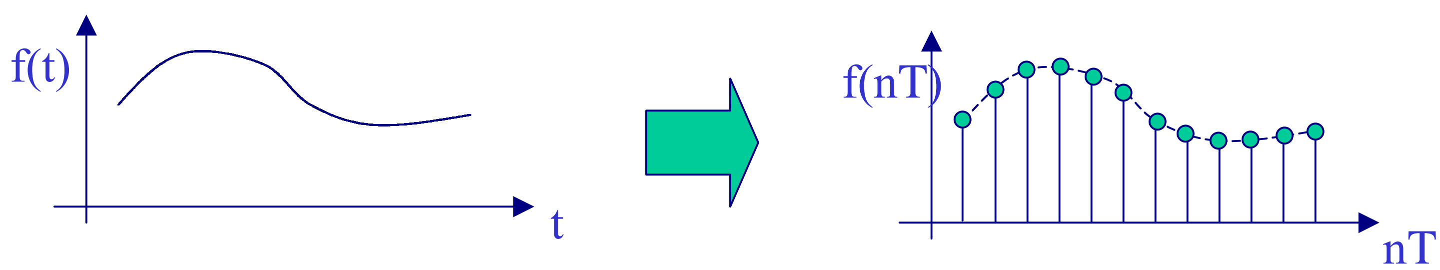
$$u(nT) \equiv \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

Ideal Sampling of Analog Signal

- Ideal Sampling (tc -> 0):

$$f^*(t) = f(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} f(nT) \cdot \delta(t - nT)$$

$$f(nT) = f(t)\big|_{t=nT} = \sum_{k=-\infty}^{\infty} f(kT) \cdot \delta((n-k)T)$$



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Sampling Theory

- Sampling can be viewed as a data compression process.
- To keep useful information after compression, sampling rate must be at least twice of the useful signal frequency

$$f_{sample} \geq 2 \cdot f_{signal} \equiv f_{Niquist}$$

- Signal frequency higher than 1/2 sample rate (if not interested) must be filtered out to avoid aliasing

Z-Transformation

- (One-side) Laplace Transform

$$F(s) \equiv L\{f(t)\} \equiv \int_0^\infty f(t)e^{-st}dt$$

- (One-side) Z-Transform

$$F(z) \equiv Z\{f(nT)\} \equiv \sum_{n=0}^{\infty} f(nT) \cdot z^{-n}$$

Useful Z-Transform Pairs

$f(nT)$	$F(z)$
$f(nT - kT),\ k = 0, 1, 2, \ldots$	$z^{-k}F(z)$
$f(nT + kT),\ k = 1, 2, \ldots$	$z^{k}F(z) - \sum_{m=0}^{k-1} f(mT)z^{-m}$
$\sum_{m=0}^{n-1} f(mT)$	$(z-1)^{-1}F(z)$
$a^{-n}f(nT)$	$F(az)$
$nf(nT)$	$-z\dfrac{dF(z)}{dz}$
$f(-nT)$	$F(1/z)$
$\sum_{k=0}^{n} f_1(kT)f_2(nT-kT)$	$F_1(z)F_2(z)$
$k_1 f_1(nT) + k_2 f_2(nT)$	$k_1F_1(z) + k_2F_2(z)$

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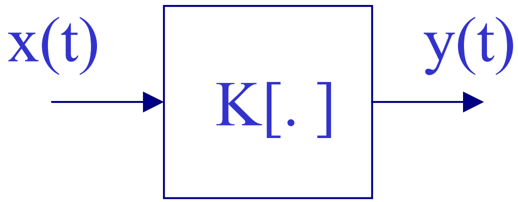
Useful Z-Transform Pairs

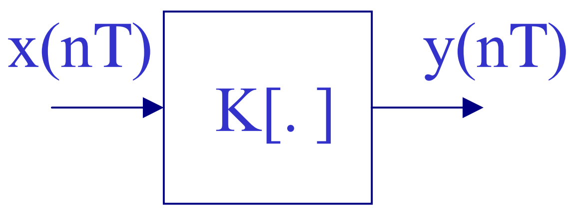
$\delta(nT)$	1
$Ku(nT),\ K$	$\dfrac{Kz}{z-1}$
a^{nT}	$\dfrac{z}{z-a^T}$
$a^{nT}\sin n\omega_0 T$	$\dfrac{\left(a^T \sin \omega_0 T\right) z}{z^2-\left(2a^T \cos \omega_0 T\right) z + a^{2T}}$
$a^{nT}\cos n\omega_0 T$	$\dfrac{z\left(z - a^T \cos \omega_0 T\right)}{z^2-\left(2a^T \cos \omega_0 T\right) z + a^{2T}}$
nTa^{nT-T}	$\dfrac{zT}{\left(z-a^T\right)^2}$

System Model of Filters



$$y(t) = K(x(t))$$

A) Continuous-Time System



$$y(nT) = K(x(nT))$$

B) Discrete-Time System

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Linear Time Invariant (LTI) Models

- Linear Systems

constant constant

$$K(a_1x_1(t)+a_2x(t))=a_1K(x_1(t))+a_2K(x_2(t))$$

$$K(a_1x_1(n\tau)+a_2x(n\tau))=a_1K(x_1(n\tau))+a_2K(x_2(n\tau))$$

- Time- (shift-) Invariant Systems

$$if \quad y(t)=K(x(t)) \quad Then \quad y(t-t_o)=K(x(t-t_o))$$

$$if \quad y(n\tau)=K(x(nT)) \quad Then \quad y((n-k)T)=K(x((n-k)T))$$

S-Domain Representation of (LTI) Systems

$$L\{y(t)\} = L\{K(x(t))\} = L\{K(\int_{-\infty}^{\infty} x(\zeta)\delta(t-\zeta)d\zeta)\}$$

$$= L\{\int_{-\infty}^{\infty} x(\zeta)K(\delta(t-\zeta))d\zeta\} = L\{x(t)\}\cdot L\{K(\delta(t))\}$$

Let:

$$Y(s) \equiv L\{y(t)\}$$

$$X(s) \equiv L\{x(t)\}$$

$$h(t) \equiv K(\delta(t))$$

$$H(s) \equiv L\{h(t)\}$$

Then:

$$\frac{Y(s)}{X(s)} = H(s)$$

Transfer Function (TF) of system

Impulse response of system

Note: Continuous-time LTI System can be completely determined by its impulse response.

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Z-Domain Representation of (LTI) Systems

$$Z\{y(nT)\} = Z\{K(x(nT))\} = Z\{K(\sum_{k=-\infty}^{\infty} x(kT)\delta((n-k)T)\}$$

$$= Z\{\sum_{k=-\infty}^{\infty} x(kT)K(\delta((n-k)T)) = Z\{x(nT)\} \cdot Z\{K(\delta(nT))\}$$

Let:

$$Y(z) \equiv Z\{y(nT)\}$$

$$X(z) \equiv L\{x(nT)\}$$

$$h(n\tau) \equiv K(\delta(nT))$$

$$H(z) \equiv Z\{h(nT)\}$$

Impulse response of system

Then:

$$\frac{Y(z)}{X(z)} = H(z)$$

Transfer Function (TF) of system

Note: Discrete-time LTI System can be completely determined by its unit sample function response.

Z-Domain Transfer Function of the System

- In general a DT system can be expressed as a linear difference equation:

$$\sum_{i=0}^{m} a_i y((n-i)T) = \sum_{i=0}^{k} b_i x((n-i)T))$$

- Z-domain transfer function of the system is defined as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_k z^{-k} + b_{k-1} z^{-(k-1)} + \dots + b_0}{a_m z^{-m} + a_{m-1} z^{-(m-1)} + \dots + 1}$$

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Steady State Response of CT LTI Systems

$$x(t) = Au(t)e^{j\omega t} \quad \Rightarrow X(s) = A\int_0^{\infty} e^{-(s-j\omega)t}dt = \frac{A}{s-j\omega}$$

$$Y(s) = H(s)\cdot X(s) = \frac{H(s)}{s-j\omega} = \frac{H(j\omega)}{s-j\omega} + Other \quad Terms$$

$$y(t)\Big|_{t\to\infty} = Au(t)H(j\omega)e^{j\omega t} + y_h(t) = H(j\omega)x(t)$$

Approach zero for stable system

Steady State Response of the system

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Steady State Response of DT LTI Systems

$$x(t) = Au(t)e^{j\omega t} \quad \Rightarrow X(z) = \sum_{n=0}^{\infty} Ae^{j\omega T} z^{-n} = \frac{A}{1 - z^{-1}e^{j\omega T}}$$

$$Y(z) = \frac{AH(z)}{1 - z^{-1}e^{j\omega T}} = \frac{AH(e^{j\omega T})}{1 - z^{-1}e^{j\omega T}} + Y_h(z)$$

$$y(nT)\Big|_{n\to\infty} = Au(t)H(e^{j\omega T})e^{j\omega tTn} + y_h(t) = H(e^{j\omega T})x(nT)$$

Approach zero for stable system

Steady State Response of the system

Frequency Response of the System

- System frequency response can be calculated by replace Z by EXP(jωT):

$$H(z)\big|_{z=e^{j\omega T}}$$

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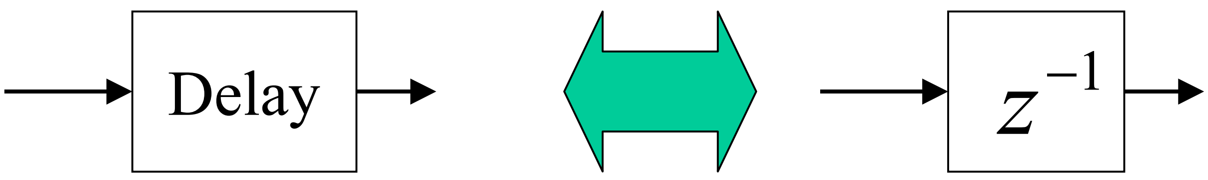
Mapping From Real Space To S-Domain



Block Diagram & Signal-Flow Graph

- Each Continuous-Time or Discrete-Time System Can be represented by its
  - Schematic
  - Differential or Difference Equations
  - Transfer Function
  - Block Diagram, or
  - Signal-Flow Graph

Block Diagram & Signal-Flow Graph

- Basic Elements (Continuous-Time)

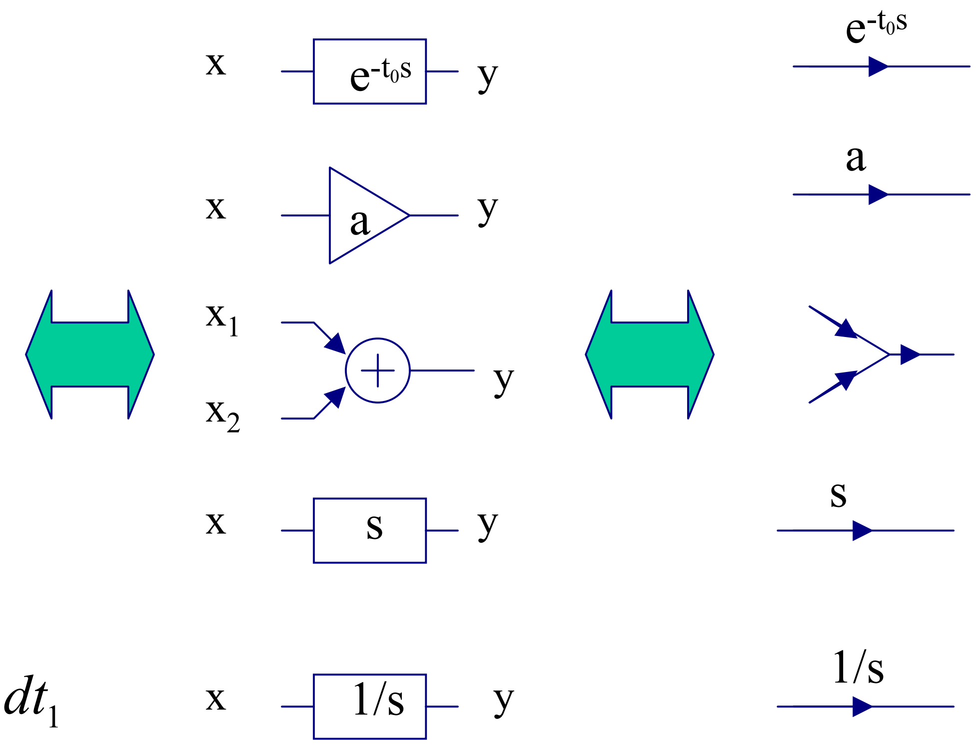
$$y(t) = x(t - t_0)$$

$$y(t) = ax(t)$$

$$y(t) = x_1(t) + x_2(t)$$

$$y(t) = \frac{dx(t)}{dt}$$

$$y(t) = y(0) + \int_0^t x(t_1)dt_1$$



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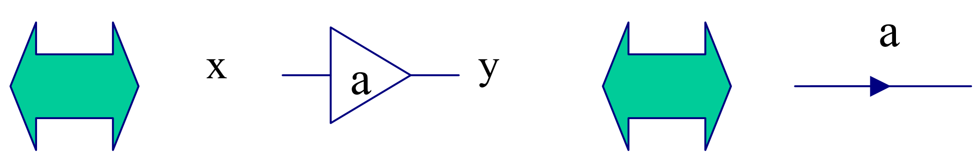
Block Diagram & Signal-Flow Graph

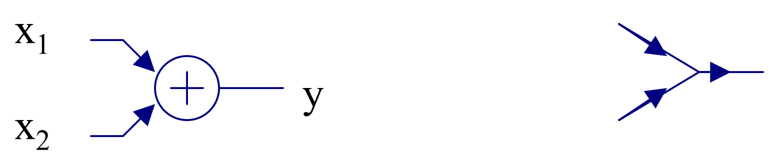
- Basic Elements (Discrete-Time)

$$y(nT) = x((n-1)T)$$

$$y(nT) = ax(nT)$$



$$y(nT) = x_1(nT) + x_2(nT)$$



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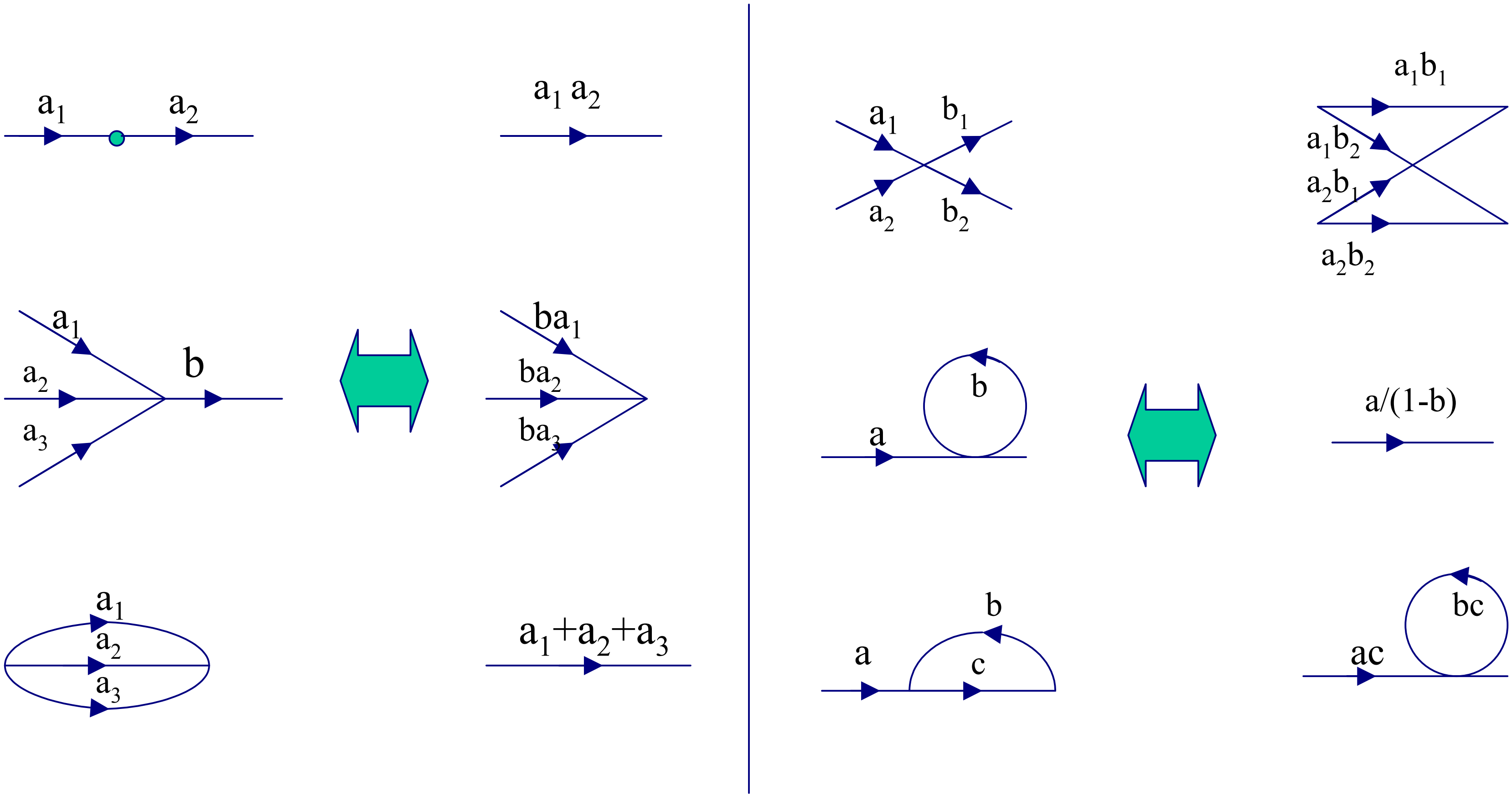
Rules for Signal-Flow Graph Reduction



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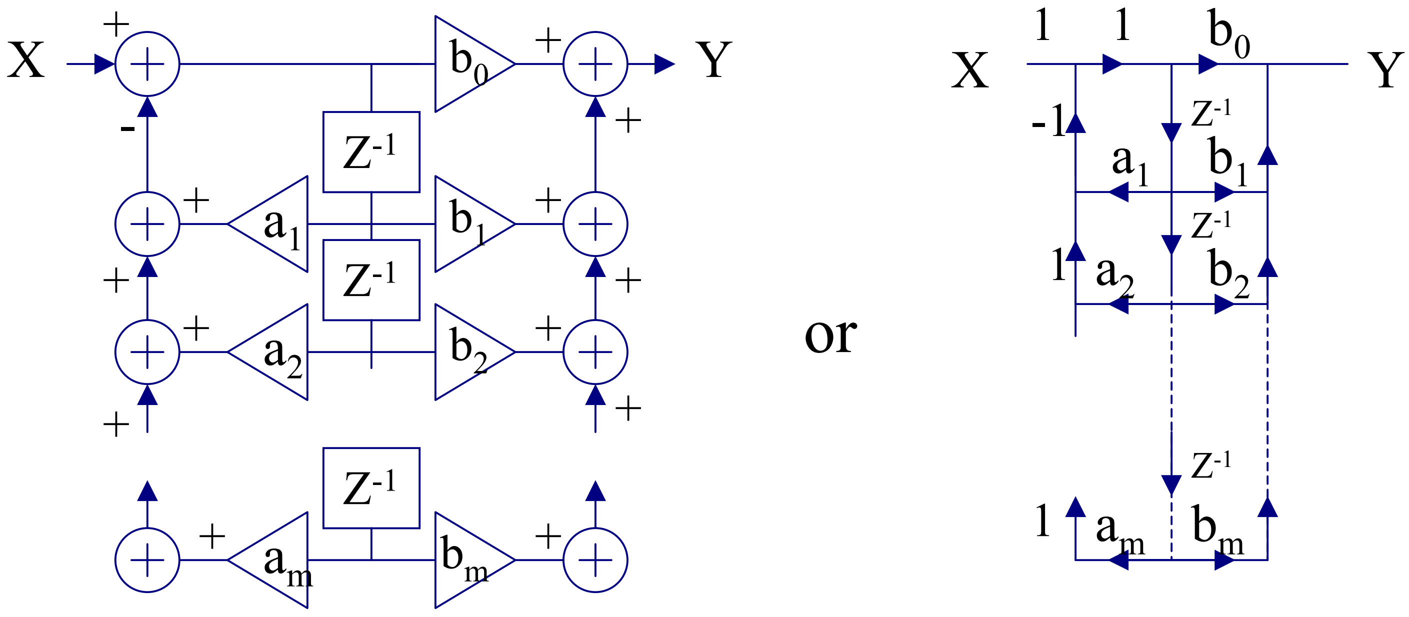
Block Diagram & Signal-Flow Graph

- Example

$$y(nT) + \sum_{i=1}^{m} a_i y((n-i)T) = \sum_{i=0}^{m} b_i x((n-i)T))$$



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