Spring 2002

EEE598D: Analog Filter & Signal Processing Circuits

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Today: s- to z- Transformations

- Standard s-z- Transformation
- Matched s- z- Transformation
- Numerical s-z- Transformations

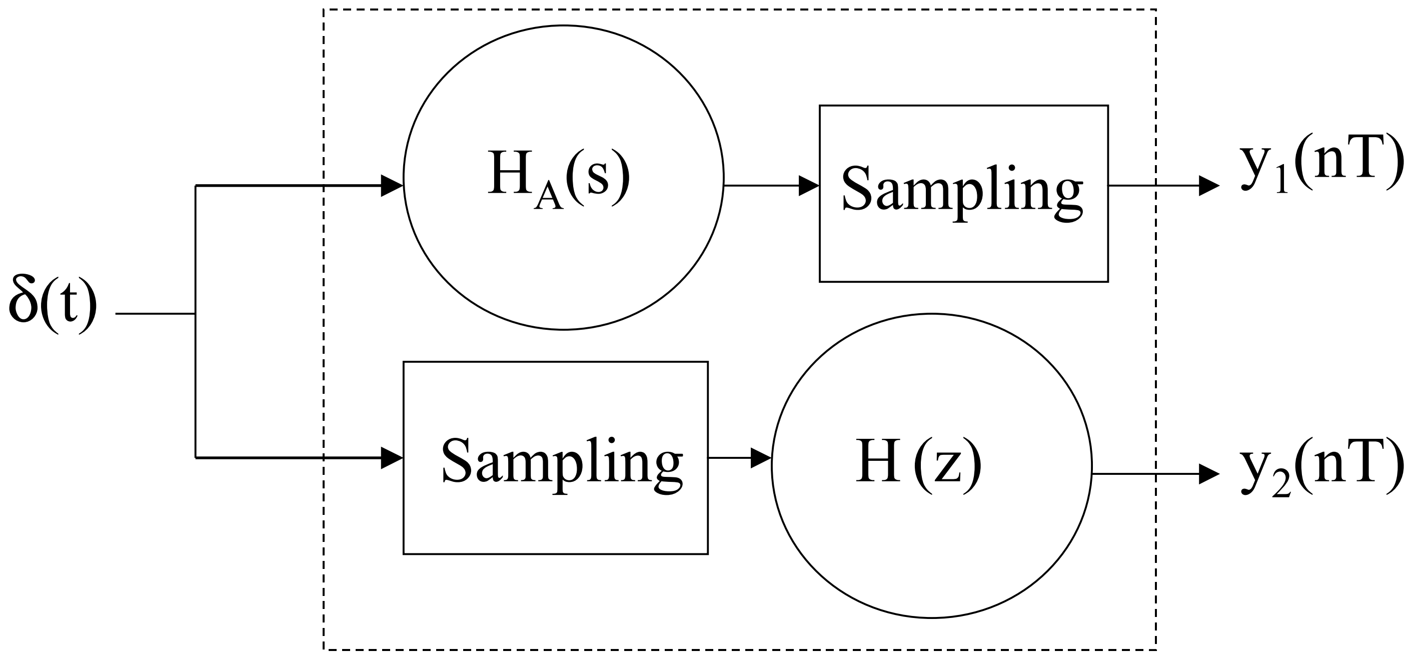
S- To Z- Transformations

- Approximation of differential equation by appropriate difference equation
- Why?
  – Take advantages of continuous-time system design techniques
  – Signal processing systems are often specified in continuous-time domain
  – Even though they may be realized in sampled-data systems.

General Approaches

- Standard (or Impulse Invariant) Approach
- Numerical Approximation Approaches

Standard S-Z- Transformation



If $y_1(nT) = y_2(nT) \qquad n = 0,1,2,,,$

Then H(z) is the standard s-z- transform (or equivalent) of $H_A(s)$

Standard S-Z- Transformation

- For standard s-z- transform, we have

$$H_A(s) = \frac{A}{s+p} \quad \Rightarrow \quad H(z) = \frac{A}{1 - e^{-pT} z^{-1}}$$

- Or for multiple poles :

$$H_A(s) = \frac{A}{(s+p)^n} \quad \Rightarrow \quad H(z) = \frac{(-1)^{n-1} A}{(n-1)!} \frac{d^{n-1}}{dp^{n-1}} \left(\frac{1}{1 - e^{-pT} z^{-1}} \right)$$

- Standard s-z- transform is also called impulse invariant transform

Matched-z Transformation

- It is closely related to standard s-z-transform
- It simply maps the s-domain pole and zero to z-domain by:

$$s + p \quad \Rightarrow \quad 1 - e^{-pT} z^{-1}$$

$$s + z \quad \Rightarrow \quad 1 - e^{-zT} z^{-1}$$

$$H_A(s) = C \frac{\prod_{k=1}^{M} (s + z_k)}{\prod_{k=1}^{N} (s + p_k)} \quad \Rightarrow \quad H(z) = K \frac{\prod_{k=1}^{M} (1 - e^{-z_k T} z^{-1})}{\prod_{k=1}^{N} (1 - e^{-p_k T} z^{-1})}$$

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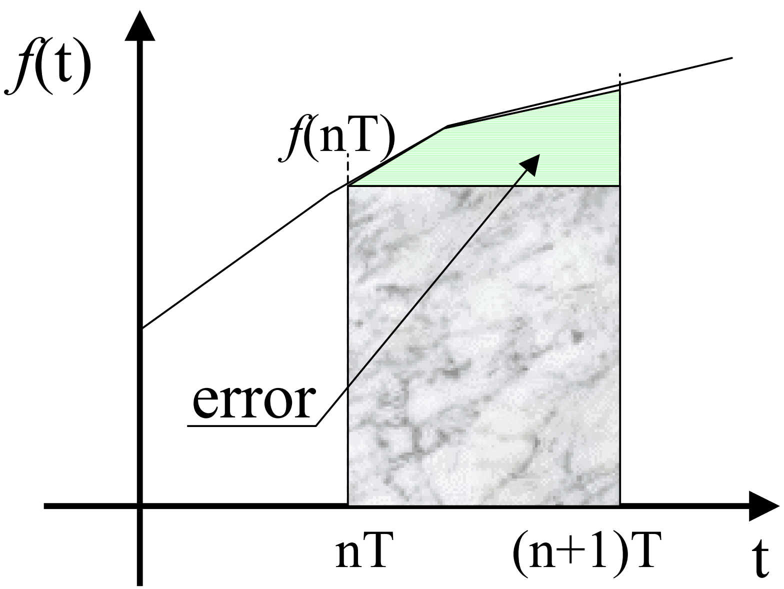
Forward Euler Transformation

- Forward Euler Integration Rule

$$y(t) = \int^{t} f(t)dt$$



$$\Rightarrow \qquad Y(s) = \frac{1}{s}F(s) \qquad (1)$$

$$\int_{t=nT}^{(n+1)T} f(t)dt = y((n+1)T) - y(nT)$$

$$\approx f(nT)\cdot T$$

$$\Rightarrow Y(z) \approx \frac{T}{(z-1)}F(z) \qquad (2)$$

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Forward Euler Transformation

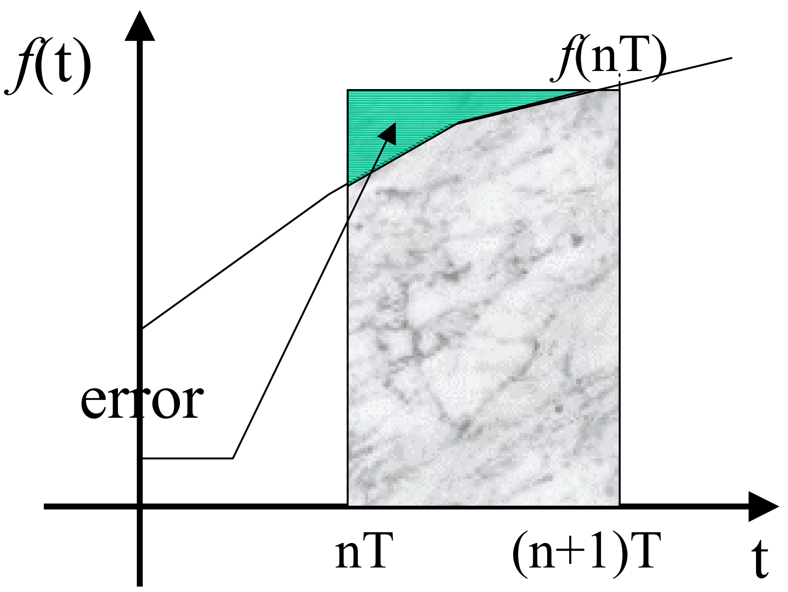
- Forward Euler Integration Rule

$$\frac{1}{s} = T\frac{z^{-1}}{1-z^{-1}} \quad or \quad z = 1 + sT$$

- An approximation of integration

Backward Euler Transformation

- Backward Euler Integration Rule



$$y(t) = \int^{t} f(t)dt$$

$$\Rightarrow \quad Y(s) = \frac{1}{s} F(s) \qquad (3)$$

$$\int_{t=nT}^{(n+1)T} f(t)dt = y((n+1)T) - y(nT)$$

$$\approx f((n+1)T) \cdot T$$

$$\Rightarrow Y(z) \approx \frac{Tz}{(z-1)} F(z) \qquad (4)$$

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Backward Euler Transformation

- Backward Euler Integration Rule

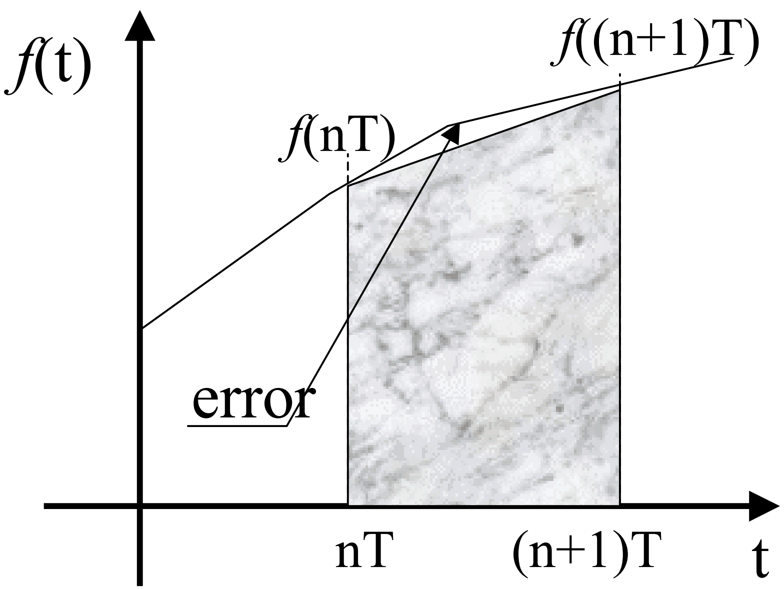
$$\frac{1}{s} = T\frac{1}{1-z^{-1}} \quad or \quad z = \frac{1}{1-sT}$$

- An approximation of integration

Bilinear Transformation

- Bilinear Integration Rule

$$y(t) = \int^{t} f(t)dt$$



$$\Rightarrow \qquad Y(s) = \frac{1}{s}F(s) \qquad (5)$$

$$\int_{t=nT}^{(n+1)T} f(t)dt = y((n+1)T) - y(nT)$$

$$\approx \frac{f((n+1)T) + f(nT)}{2} \cdot T$$

$$\Rightarrow Y(z) \approx \frac{T(z+1)}{2(z-1)}F(z) \qquad (6)$$

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Bilinear Transformation

- Forward Euler Integration Rule

$$\frac{1}{s} = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \quad or \quad z = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}}$$

- An approximation of integration

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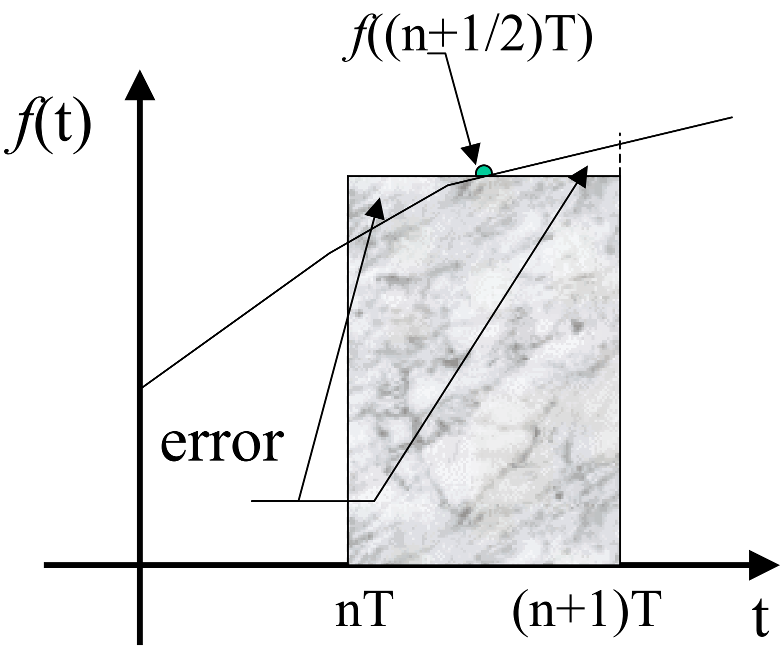
LDI Transformation

- LDI (midpoint) Integration Rule



$$y(t) = \int^{t} f(t)dt$$

$$\Rightarrow \qquad Y(s) = \frac{1}{s}F(s) \qquad (5)$$

$$\int_{t=nT}^{(n+1)T} f(t)dt = y((n+1)T) - y(nT)$$

$$\approx f((n+1/2)T)\cdot T$$

$$\Rightarrow Y(z) \approx \frac{T(z^{1/2})}{2(z-1)}F(z) \qquad (6)$$

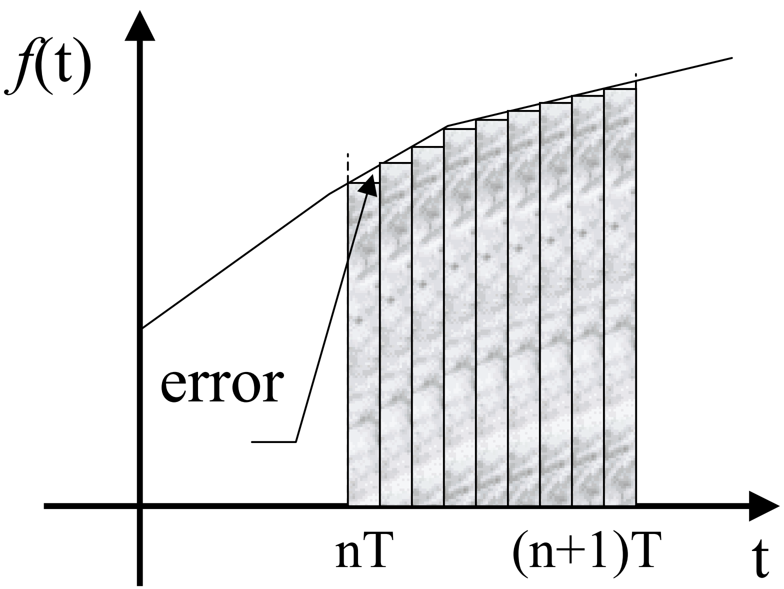
LDI Transformation

$$\frac{1}{s} = T \frac{z^{-1/2}}{1 - z^{-1}} \quad or \quad z = \left(\frac{T}{2}s \pm \sqrt{1 + \left(\frac{T}{2}s\right)^2}\right)$$

- An approximation of integration

Multi-Step Integration Transformation

- Multi-Step Integration Rule



$$y(t) = \int^{t} f(t)dt \quad \Rightarrow \quad Y(s) = \frac{1}{s}F(s) \qquad (7)$$

$$\int_{t=nT}^{(n+1)T} f(t)dt = y((n+1)T) - y(nT)$$

$$\approx \sum_{k=0}^{m} c_m f((n+1-k/m)T) \cdot T / \sum_{k=0}^{m} c_m$$

$$\Rightarrow Y(z) \approx \frac{Tz\sum_{k=0}^{m} c_m z^{k/m}}{(z-1)\sum_{k=0}^{m} c_m} F(z) \qquad (8)$$

Multi-Step Integration Transformation

- We have

$$\frac{1}{s} \approx \frac{T\sum_{k=0}^{m} c_m z^{-k/m}}{(1-z^{-1})}$$

- Let $T \Rightarrow mT$ (slower clock) we have:
- Multi-Step Transformation:

$$\frac{1}{s} = \frac{mT}{\sum_{k=0}^{m} c_k} \frac{c_0 + c_1 z^{-1} + \ldots + c_m z^{-m}}{1 - z^{-m}}$$

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Multi-Step Integration Transformation

- Special cases:
  - Simpson's Rule:
    - m =2, $c_0 = c_2$ =1/3, c_1 = 1/4
  - Tick's Rule:
    - m =2, $c_0 = c_2$ =0.3584, c_1 = 1.2832
  - ...

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ψ Transformation

- Although $z=e^{i\omega T}$ appears quite natural. It is often more convenient to use another complex variable as intermediate step

$$\psi = \frac{1 - z^{-1}}{1 + z^{-1}} \quad , \quad z^{-1} = e^{-sT}$$

or

$$\psi = \tanh(\frac{sT}{2}), \qquad z^{-1} = \frac{1-\psi}{1+\psi} = e^{-sT}$$