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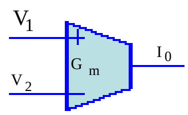
TRANSCONDUCTANCE AMPLIFIER TOPOLOGIES AND NON-IDEALITIES

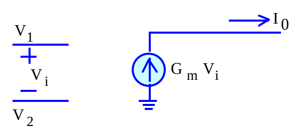
- **Architectures and Macromodels***
- **Linearized OTAs**
- OTA Non-Idealities
  - Input impedance
  - Frequency dependent transconductance
  - Non-linear transconductance
  - Output impedance
- Applications, i.e., OTA-C Filters, Oscillators, and VGA

* Reference :
E. Sánchez-Sinencio and M.L. Majewski, " A Nonlinear Macromodel of Operational Amplifiers in the Frequency Domain", *IEEE Trans. Circuits and Systems*, Vol. CAS-26,No.6, pp 395-402, June 1979

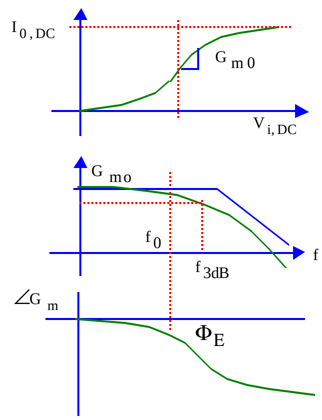
OPERATIONAL TRANSCONDUCTANCE AMPLIFIER



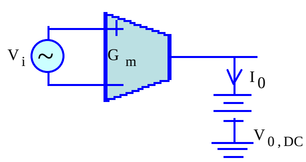
$$G_m = \frac{I_0}{V_i}\Big|_{V_{0(AC)=0}}$$



IDEAL MODEL



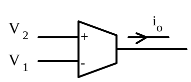
How do you simulate G_m?



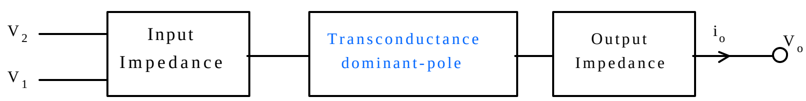
Φ_E is the phase difference between the actual G_m phase and the ideal phase.

$$\omega_p = 2\pi f_{3dB}$$

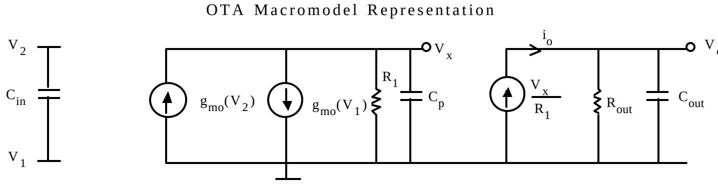
A Linear CMOS OTA Macromodel



$$g_m = \frac{g_{mo}}{1+\frac{s}{w_p}} \quad ; \quad w_p = \frac{1}{R_1 C_p}$$



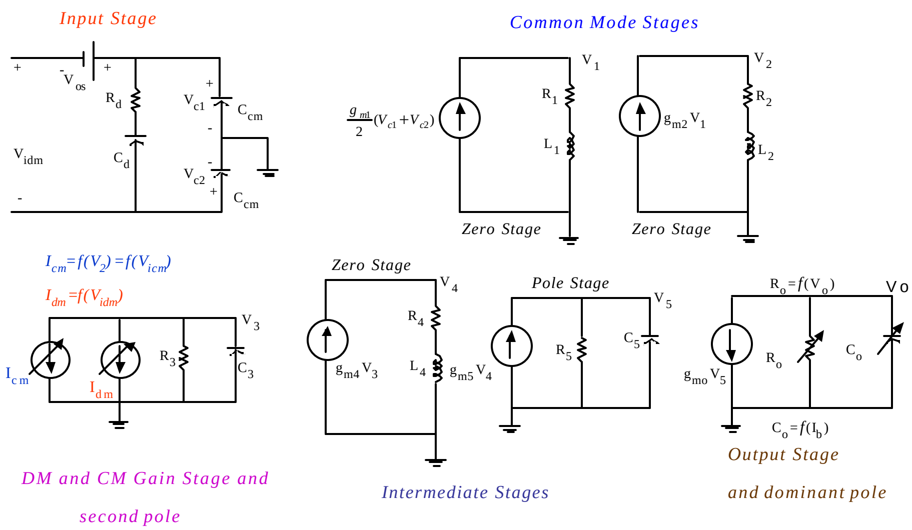
OTA Macromodel Representation



Real OTA Macromodel

Reference:

A nonlinear macromodel for CMOS OTAs *Gomez, G.J.; Embabi, S.H.K.; Sanchez-Sinencio, E.; Lefebvre, M.* Circuits and Systems, 1995. ISCAS '95., 1995 IEEE International Symposium on Volume: 2 , 1995 Page(s): 920 -923 vol.2



A Non-Linear CMOS OTA Macromodel

A curve fitting is required to satisfy *Idm=f(Vidm)*

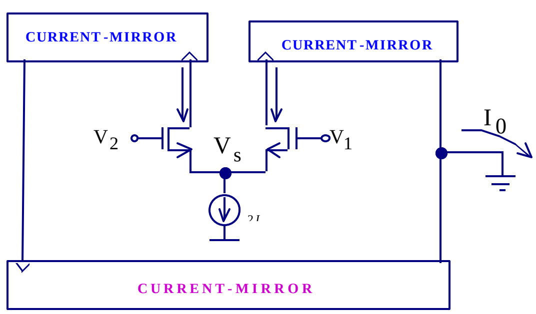
CALCULATED PARAMETER VALUES FOR NON-LINEAR MACROMODEL

Element	Value
C_{cm}	27.88 fF
C_d	72.70 fF
R_d	2128.50Ω
$R_1=R_2=R_3=R_4=R_5$	1KΩ
$g_{m1}=g_{m2}=g_{m3}=g_{m4}=g_{mo}$	1mmhos
L_1	125.60μH
L_2	57.38μH
C_3	2.83pF
L_4	Not used
C_5	2.05pF
R_o	2.17MΩ
C_o	109.88fF
I_{dm}	g_{mdm}=590.04μA/V
I_{cm}	g_{mcm}=56.41ηA/V
V_{os}	-1.44mV

SIMULATION DATA

Parameter		Simulated Value
G_{mdn}	DM Transconductance	596.04 mA/V
G_{mcm}	CM Transconductance	56.41 hA/V
A_{vdm}	DM V/V Gain @ DC	1239.40 ® 62.2dB
CMRR	CM Rejection Ratio	10565.95 ® 80.4dB
R_o	Output Resistance	2.17M W
f_e	Phase Error @ 1MHz	1.75^o
w_{p1}	Dom. Pole ($2\,pf_{p1}$)	2p(666.95 KHz)
w_{p2}	2^{nd} pole ($2\,pf_{p2}$)	2p(56.16 MHz)
w_{p3}	3^{rd} pole ($2\,pf_{p3}$)	2p(77.69 MHz)
w_{z1}	Zero ($2\,pf_{z1}$)	2p(77.94 MHz)
w_{zcm1}	1^{st} CMZero ($2\,pf_{zcm1}$)	2p(813.67 KHz)
w_{zcm2}	2^{nd} CMZero ($2\,pf_{zcm2}$)	2p(2.77 MHz)
Z_{idm}	DM Z_{in} ® Real part	1112.10W
Z_{idm}	DM Z_{in} ® Imag.	-1.58 E09
Z_{icm}	CM Z_{in} @ LF	2.85 E09
Z_{icm}	CM Z_{in} : Phase	90^o

Basic OTA (three current-mirrors): Non-linearity



$$\left.\begin{aligned} I_1 &= \beta(V_1 - V_s - V_T)^2 \\ I_2 &= \beta(V_2 - V_s - V_T)^2 \\ I_1 + I_2 &= I_{ss} \end{aligned}\right\} \Rightarrow V_s = \frac{V_1 + V_2 - 2V_T}{2} \pm \sqrt{\frac{I_{ss}}{2\beta} - \left(\frac{\Delta V}{2}\right)^2}$$

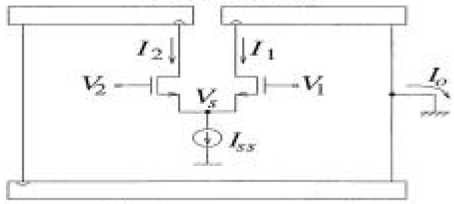
$$\beta = \frac{\mu C_{ox}}{2}\frac{W}{L}$$

$$g_m = \sqrt{2 I_{ss}\beta}$$

$$\Delta V = V_1 - V_2$$

$$I_0 = I_1 - I_2 \quad \Rightarrow \quad I_0 = \Delta V \sqrt{2 I_{ss}\beta}\sqrt{1 - \frac{\beta}{[illegible]}(\Delta V)^2}$$

Basic OTA



$$\left.\begin{aligned} I_1 &= \beta(V_1 - V_s - V_T)^2 \\ I_2 &= \beta(V_2 - V_s - V_T)^2 \\ I_1 + I_2 &= I_{ss} \end{aligned}\right\} \Rightarrow V_s = \frac{V_1 + V_2 - 2V_T}{2} \pm \sqrt{\frac{I_{ss}}{2\beta} - \left(\frac{\Delta V}{2}\right)^2}$$

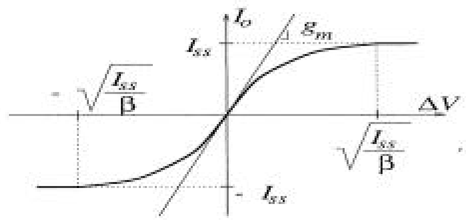
$$\Delta V = V_1 - V_2$$

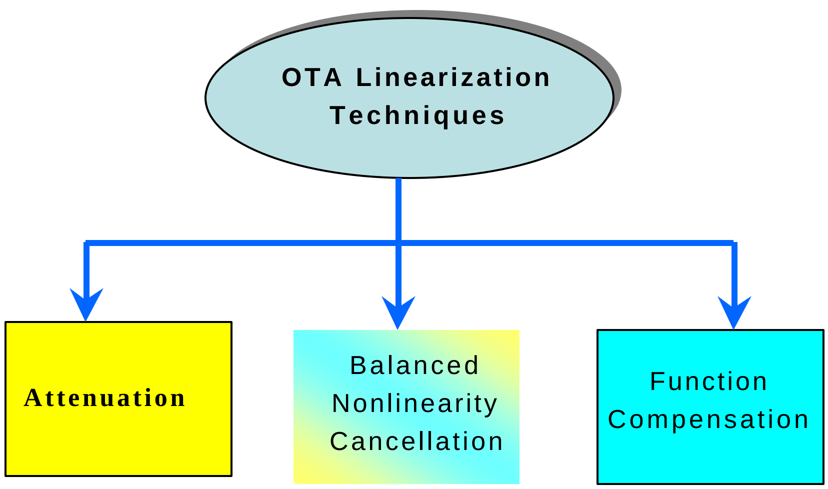
$$I_o = I_1 - I_2 \rightarrow I_o = \Delta V \sqrt{2 I_{ss}\beta}\sqrt{1 - \frac{\beta}{2 I_{ss}}(\Delta V)^2}$$

Linearity considerations

$$\beta = \frac{\mu C_{ox}}{2}\frac{W}{L}$$

$$g_m = \sqrt{2 I_{ss}\beta}$$





Remarks on Linearization Techniques

ATTENUATION

- Overall Gm is reduced by the attenuation factor a
- Non-idealities of the attenuation network affect the overall transconductance amplifier performance
- Relative input noise increase by a
- Input offset is proportional to a
- Dynamic range can be modified via tradeoffs.

Balanced Nonlinearity Cancellation

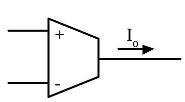
Type I :

$$I_0(V_1,V_2) = \sum_{i=1}^{\infty} a_i(V_1^i - V_2^i)$$

$$= a_1(V_1 - V_2) + a_2(V_1^2 - V_2^2) + a_3(V_1^3 - V_2^3) + \dots$$

$V_{in}^{+} = V_1 = V_A + V/2$

$V_{in}^{-} = V_2 = V_A - V/2$



$$I_0(V) = V(a_1 + 2a_2V_A + 3a_3V_A + \dots) + \frac{1}{4}V^3(a_3 + 4a_4V_A + 10a_5V_A + \dots) + \frac{1}{16}V^5(a_5 + 6a_6V_A + \dots)$$

For small $a_3, a_4, a_5, \dots$

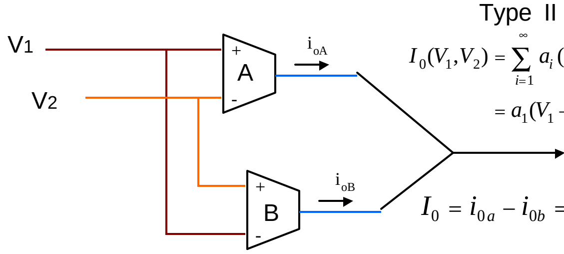
$$I_0 \cong g_m V$$

where

$$g_m = a_1 + 2a_2V_A^1 + 3a_3V_A^2 + \dots$$

Balanced Nonlinearity Cancellation

Type II



$$I_0(V_1,V_2) = \sum_{i=1}^{\infty} a_i (V_1^i - V_2^i)$$

$$= a_1(V_1 - V_2) + a_2(V_1^2 - V_2^2) + a_3(V_1^3 - V_2^3) + \ldots$$

$$I_0 = i_{0a} - i_{0b} = \sum_{i=1}^{\infty} (a_{ai}V_a^i - a_{bi}V_b^i)$$

Particular cases:

For matched transconductors

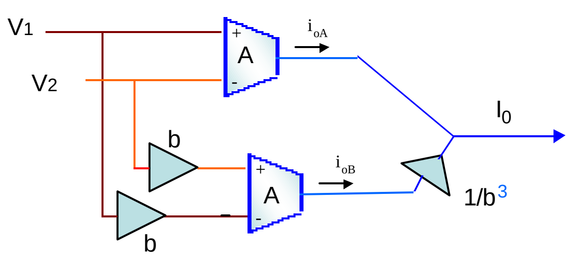
$$a_{ai} = a_{bi} = a_i$$

$$V \equiv V_1 - V_2$$

$$I_0 = 2(a_1 V + a_3 V^3 + a_5 V^5 + \ldots)$$

Assume each OTA has the following type II non-linearity description:

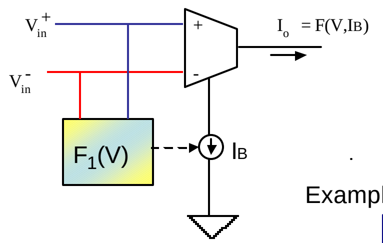
$i_{oA} = A\ V_{in}\ (1\text{-}a\ V^2_{in}) = a_1 V_{in} - a_2 V_{in}^{\ 3}$



Equal Transconductance gains can be used, but one of then modified at the input and output. Then the linearized output current yields:

$$I_o = A(1 - \frac{1}{b^2})V_{in}$$

Function Compensation



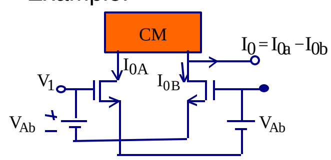
Let $I_B = F_1(V) + I_o$

Then $I = F[V, F_1(V) + I_o]$

Pick F1 such that

$I = V\, F_2(i_o)$

Example:

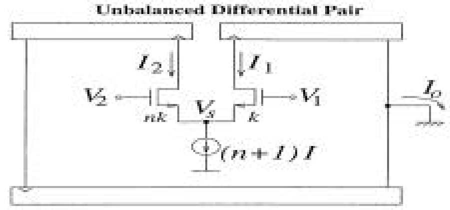


Balanced Configuration

Nedungadi Transconductor

[A. Nedungadi and T. R. Viswanathan, "Design of Linear CMOS Transconductor Elements," *IEEE Transactions on Circuits and Systems*, vol. CAS-31, No. 10, pp. 891-894, October 1984]

Unbalanced Differential Pair



$$\left.\begin{aligned} I_1 &= k(V_1 - V_s - V_T)^2 \\ I_2 &= nk(V_2 - V_s - V_T)^2 \\ I_1 + I_2 &= (n+1)I \end{aligned}\right\} \Rightarrow V_s^2 - 2V_s\left(\frac{V_1 + nV_2}{n+1} - V_T\right) + \frac{(V_1 - V_T)^2 + n(V_2 - V_T)^2}{n+1} = \frac{I}{k}$$

$$V_s = \frac{V_1 + nV_2 - (n+1)V_T}{n+1} - \sqrt{\frac{I}{k} - \frac{n\Delta V^2}{(n+1)^2}} \Rightarrow$$

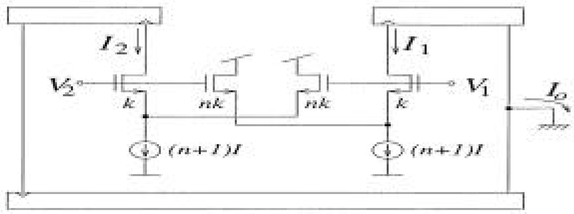
$$\Rightarrow \begin{cases} I_1 = k\left[\dfrac{n\Delta V}{n+1} + \sqrt{\dfrac{I}{k}}\sqrt{1 - \dfrac{kn\Delta V^2}{(n+1)^2 I}}\right]^2 \\ I_2 = nk\left[\dfrac{-\Delta V}{n+1} + \sqrt{\dfrac{I}{k}}\sqrt{1 - \dfrac{kn\Delta V^2}{(n+1)^2 I}}\right]^2 \end{cases}$$

Range:

$$\left.\begin{aligned} I_1 &= (n+1)I \\ I_2 &= 0 \end{aligned}\right\} \Rightarrow \Delta V = \sqrt{\frac{I}{k}(n+1)}$$

$$\left.\begin{aligned} I_1 &= 0 \\ I_2 &= (n+1)I \end{aligned}\right\} \Rightarrow \Delta V = -\sqrt{\frac{I}{k}\left(1 + \frac{1}{n}\right)}$$

The Cross-Coupled Quad Transconductor



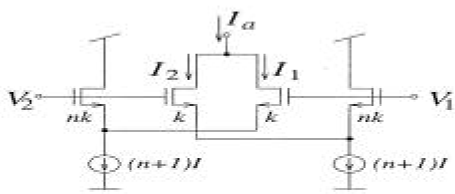
$$\left.\begin{aligned} I_1 &= k\left[\frac{n\Delta V}{n+1} + \sqrt{\frac{I}{k}}\sqrt{1-\frac{kn\Delta V^2}{(n+1)^2 I}}\right]^2 \\ I_2 &= k\left[\frac{-n\Delta V}{n+1} + \sqrt{\frac{I}{k}}\sqrt{1-\frac{kn\Delta V^2}{(n+1)^2 I}}\right]^2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow I_o = I_1 - I_2 \Rightarrow \begin{cases} I_o = g_m \Delta V\sqrt{1-\beta\Delta V^2} \\ g_m = \frac{4n}{n+1}\sqrt{Ik} \quad , \quad \beta = \frac{k}{I}\frac{n}{(n+1)^2} \end{cases}$$

Range: $|\Delta V| \le \sqrt{\frac{I}{k}\left(1+\frac{1}{n}\right)}$

For $\Delta V = \sqrt{\frac{I}{k}\left(1+\frac{1}{n}\right)} \quad \Rightarrow \quad I_o = \frac{4n}{n+1}I$.

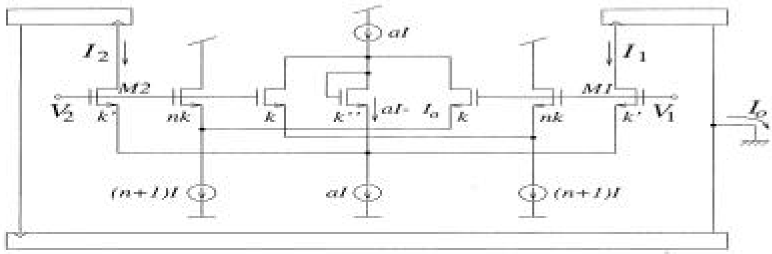
Squarer



$$\left.\begin{aligned} I_1 &= k\left[\frac{n\Delta V}{n+1} + \sqrt{\frac{I}{k}}\sqrt{1-\frac{kn\Delta V^2}{(n+1)^2 I}}\right]^2 \\ I_2 &= k\left[\frac{-n\Delta V}{n+1} + \sqrt{\frac{I}{k}}\sqrt{1-\frac{kn\Delta V^2}{(n+1)^2 I}}\right]^2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \quad I_a = I_1 + I_2 = 2k\frac{n(n-1)}{(n+1)^2}\Delta V^2 + 2I \quad , \quad \Delta V \le \sqrt{\frac{I}{k}\left(1+\frac{1}{n}\right)}$$

Nedungadi Transconductor



Transistors M1 and M2 form a simple differential pair biased by

$$I_{ss} = aI - (aI - I_o) = I_o = 2k\frac{n(n-1)}{(n+1)^2}\Delta V^2 + 2I$$

Consequently,

$$I_o = \Delta V\sqrt{2k'I_{ss} - k'^2\Delta V^2} = \Delta V\sqrt{k'^2\Delta V^2\left(4\gamma\frac{n(n-1)}{(n+1)^2}\right) + 4k'I} \quad , \quad \gamma = \frac{k}{k'}$$

$$\text{If} \quad 4\gamma n(n-1) - (n+1)^2 = 0 \quad \Rightarrow \quad I_o = 2\Delta V\sqrt{k'I} \quad \begin{cases} g_m = 2\sqrt{k'I} \\ |\Delta V| \le min\left(\sqrt{\frac{I}{k}\left(1+\frac{1}{n}\right)}, 2\sqrt{\frac{I}{k'}}\right) \end{cases}$$

Examples:

$$\gamma = 1 \quad \Rightarrow \quad n = 1 + \frac{2\sqrt{3}}{3} = 2.155$$

$$n = 3 \quad \Rightarrow \quad \gamma = \frac{(n+1)^2}{4n(n-1)} = \frac{2}{3}$$

$$aI - I_a \ge 0 \quad \Leftrightarrow \quad \begin{cases} \Delta V_{max}^2 = \frac{I}{k}\left(\frac{n+1}{n}\right) \quad \rightarrow \quad a \ge \frac{4n}{n+1} \\ \Delta V_{max}^2 = \frac{I_a}{k'} \quad \rightarrow \quad a \ge 2\left[\frac{(1+4\gamma)n^2 + 2n(1-2\gamma) + 1}{(n+1)^2}\right] \end{cases}$$

Distortion Analysis

$i_d = i_d(v_{in})$ Suppose that i_d is a non-linear function, then

$$i_d = I_D + \frac{\partial i_d}{\partial v_{in}}\Big|_Q v_{in} + \frac{1}{2}\frac{\partial^2 i_d}{\partial v_{in}^2}\Big|_Q v_{in}^2 + \frac{1}{6}\frac{\partial^3 i_d}{\partial v_{in}^3}\Big|_Q v_{in}^3 + \bullet\bullet$$

for $\cdot\, v_{in} = V\sin(\omega t)$

$$i_d \cong I_D + \frac{1}{4}\frac{\partial^2 i_d}{\partial v_{in}^2}\Big|_Q V^2 + \frac{\partial i_d}{\partial v_{in}}\Big|_Q + \frac{\partial^3 i_d}{\partial v_{in}^3}\Big|_Q \frac{V^2}{8} + \cdots V\sin(\omega t) +$$

$$+ \frac{1}{4}\frac{\partial^2 i_d}{\partial v_{in}^2} + \cdots \Big|_Q V^2\sin(2\omega t) + \frac{1}{24}\frac{\partial^3 i_d}{\partial v_{in}^3} + \cdots \Big|_Q V^3\sin(3\omega t) + \bullet\bullet$$

CONTINUOUS-TIME FILTERS

Transconductance

$$g_m = \left.\frac{\partial i_d}{\partial v_{in}}\right|_Q + \left.\frac{\partial^3 i_d}{\partial v_{in}^3}\right|_Q \frac{V^2}{8} + \cdots$$

Second Harmonic Distortion

$$HD2 \cong \frac{1}{4}\frac{\left.\frac{\partial^2 i_d}{\partial v_{in}^2}\right|_Q}{\left.\frac{\partial i_d}{\partial v_{in}}\right|_Q} V$$

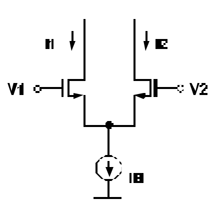
Third Harmonic Distortion

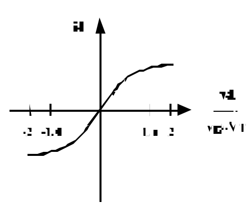
$$HD3 \cong \frac{1}{24}\frac{\left.\frac{\partial^3 i_d}{\partial v_{in}^3}\right|_Q}{\left.\frac{\partial i_d}{\partial v_{in}}\right|_Q} V^2$$

Third Intermodulation Distortion

$$IM3 = 3 * HD3$$

Differential Pair as a V-I converter





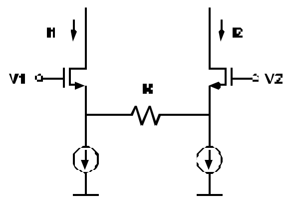
$$i_d = \left(\frac{\sqrt{2b_1 I_B}}{2}\right) v_d \sqrt{1 - \left(\frac{v_{in}}{2(v_{GS} - v_T)}\right)^2}$$

$$g_m = \sqrt{2 b_1 I_B}$$

$$HD2 = 0$$

$$HD3 = \frac{1}{32}\left(\frac{v_d}{2(v_{GS} - v_T)}\right)^2$$

Differential Pair with Source Degeneration



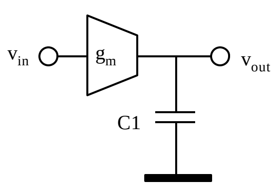
$$n = \frac{1}{1 + g_m R}$$

$$i_d = \left(\frac{\sqrt{2 b_1 I_B}}{2}\right) \frac{v_d}{n} \sqrt{1 - \left(\frac{v_{in}}{2n(v_{GS} - v_T)}\right)^2}$$

$HD2 = 0$ Ideally

$$HD3 = \frac{1}{32 n^2} \frac{v_{in}^2}{(v_{GS} - v_T)^2}$$

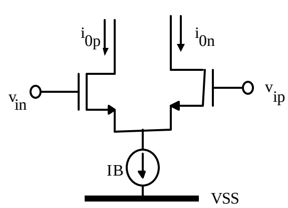
BASIC INTEGRATOR: Fundamental Equations



$$g_m \cong \mu_n C_{OX} \frac{W}{L}(V_{GS} - V_T) \cong \sqrt{2\mu_n C_{OX} \frac{W}{L} I_D}$$

$$HD3 \cong \frac{1}{32}\left(\frac{V_{in}}{V_{GS} - V_T}\right)^2$$

Mobility degradation:



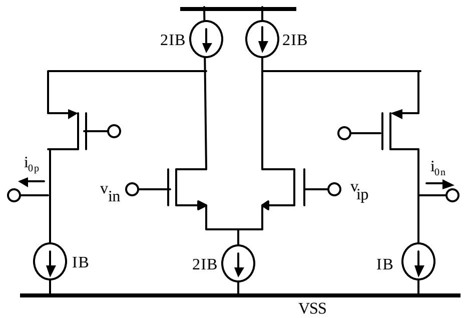
$$\mu_n \cong \left(\frac{\mu_0}{1 + \theta(V_{GS} - V_T)}\right)\left\{\frac{1}{1 + \frac{\theta V_{in}}{1 + \theta(V_{GS} - V_T)}}\right\}$$

$$HD3 \cong \frac{1}{4}\left(\frac{\theta V_{in}}{1 + \theta(V_{GS} - V_T)}\right)^2$$

NOISE:

$$v_n^2 = \frac{16kT}{3g_m}(1 + NF)$$

FOLDED-CASCODE OTA: Design Equations



POWER: $P = 4I_B(VDD + VSS)$

NOISE: $v_n^2 = \frac{16kT}{3g_m}(1+NF)$

TRANSCONDUCTANCE:

$$g_m = \frac{g_{mo}}{1 + \frac{s}{C_p / g_{mp}}}$$

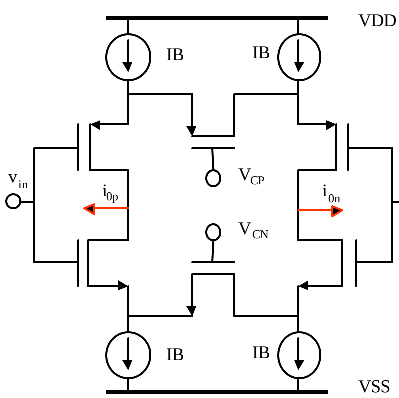
INPUT SWING: $V_{SS} + 2V_{DSAT} + V_T > V_{IN}$

OUTPUT RESISTANCE:

$$R_O \approx g_{mn} R_{ON}^2 \,\|\, g_{mp} R_{OP}^2$$

OUTPUT SWING: $V_{SS} + 2V_{DSAT} < V_{OUT} < V_{DD} - 2V_{DSATP}$

Single stage OTA based on complementary differential pairs



$$P = I_B(VDD + VSS)$$

$$G_m = \frac{g_{mn}}{1 + N_N} + \frac{g_{mp}}{1 + N_P}$$

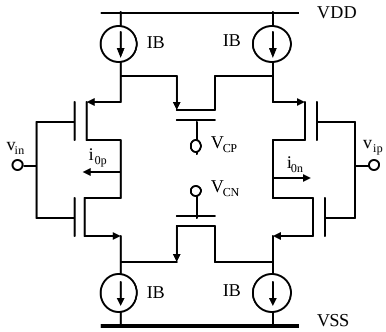
$$v_n^2 = \frac{16kT}{3G_m}(1 + NF)$$

OUTPUT SWING IS LIMITED:

$$V_{IN} - V_{TN} < V_{OUT} < V_{IN} + V_{TP}$$

Need to make sure no drivers operate in the ohmic region.

OTA based on Complementary Differential Pairs



TRANSCONDUCTANCE:

$$G_m = \frac{g_{mn}}{1+N_N} + \frac{g_{mp}}{1+N_P}$$

DISTORTION:

$$HD3 \cong \frac{1}{32}\left(\frac{V_{in}}{(V_{GS}-V_T)(N+1)}\right)^2$$

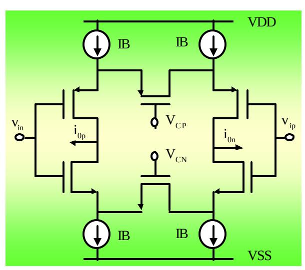
OUTPUT RESISTANCE:

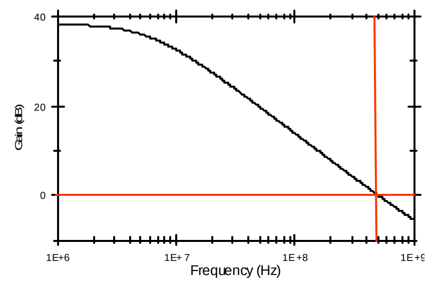
$$R_{out} \approx \frac{g_{mn} r_{on} R_N}{2} \Bigg\| \frac{g_{mp} r_{op} R_P}{2}$$

DC GAIN:

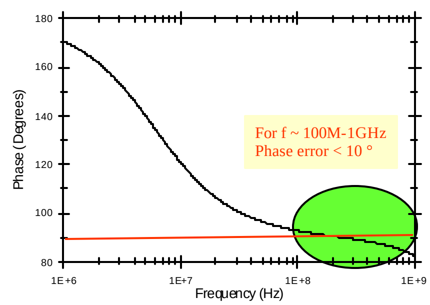
$$A_V \approx A_{VN}$$

OTA Results: 0.35 μm Process

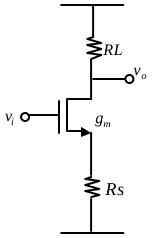




Transistor	W, L	Bias current
MN	200, 0.6 μm	200 μA
MCN	15, 0.6 μm	0 μA
MN	200, 0.6 μm	200 μA
MCN	30, 0.6 μm	0 μA



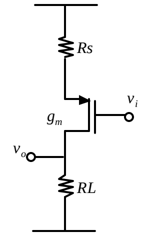
Wide Output and Good Linearity Observations



N-driver Transconductor

$$G_m = \frac{g_m}{1+g_m Rs} = \frac{1}{\frac{1}{g_m} + Rs}$$

$$A_V = -G_m R_L = \frac{-R_L}{\frac{1}{g_m} + Rs}$$

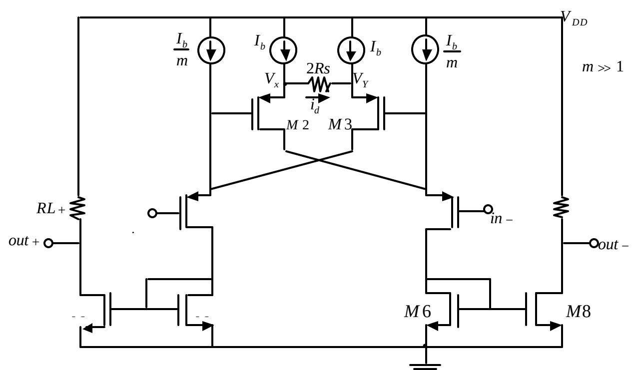


P-driver Transconductor

- In order to make A_v or Gm independent of g_m, it is required that $R_s >> 1/g_m$
- Large R_s take up large area and have large parasitic capacitances which limit the amplifier bandwidth.
- It would be desirable to have A_v (Gm) independent of g_m with small R_s

Next we introduce a wideband voltage amplifier, however a similar approach might be used to derive a transconductance amplifier whose Gm could be nearly 1/R without requiring a large gm associated with driver transistor.

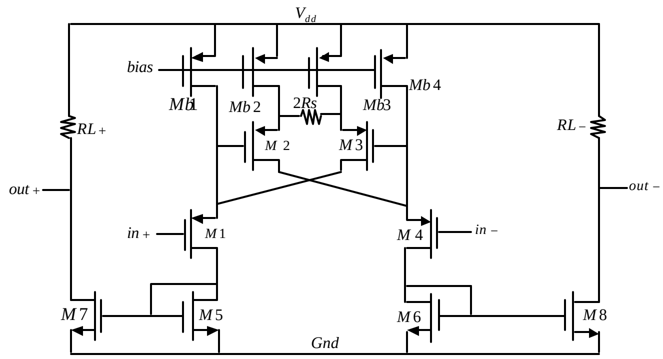
Wide Output Swing and Improved Linearity Amplifier



- This circuit is capable to cancel the effect of g_{m1}, g_{m4}
- We need to force the DC components of V_x and V_Y to be equal, such that the current through $2R_S$ is only a function of the differential input $v_{in}^+ - v_{in}^- = v_d$, thus i_d becomes $v_d/2R_S$.
- Condition to satisfy

$$v_{gs_1} + v_{gs_2} = v_{gs_4} + v_{gs_3} \Rightarrow V_x = V_Y$$

Amplifiers with Wide Output Swing and Good Single-Ended Linearity A Circuit Implementation.



Resistively loaded MOS amplifier with g_m cancellation.

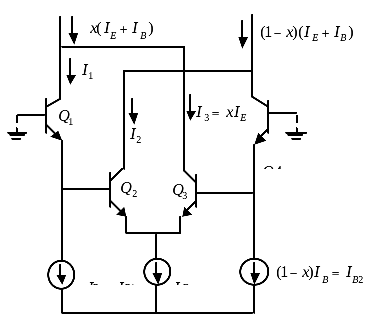
$$V_{gs_1} + V_{gs_2} = V_{gs_3} + V_{gs_4}$$

$$i_{2RS} = (v_{in}^{+} - v_{in}^{-}) / 2R_s = v_d / 2R_S$$

$$\text{If } I_{b1,b4} << I_{b2,b3} \quad \text{then } A_v \cong \frac{R_L}{R_S} \frac{(W/L)_7}{(W/L)_5}$$

A Bipolar Amplifier version is next shown.

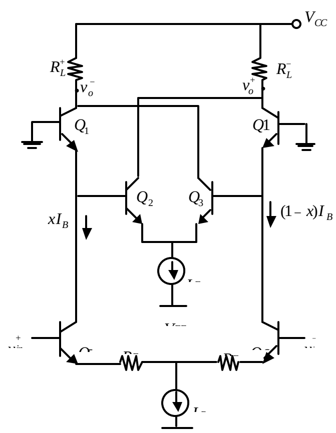
This Circuit is called Gilbert Gain Cell.



$$\frac{I_2}{I_3} = \frac{I_{B2}}{I_{B1}} = \frac{I_4}{I_1}$$

$$I_{o1} = (I_1 + I_3) - (I_2 + I_4)$$

$$A_I = \frac{I_o}{I_{in}} = \frac{I_B + I_E}{I_B}$$



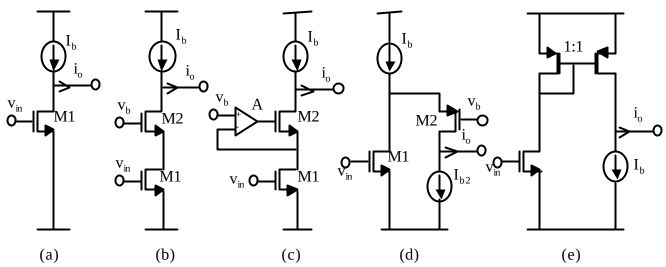
$$A_V = \frac{v_o}{v_{in}} = \frac{R_L}{R_E}(1 + \frac{I_E}{I_B})$$

The dominant pole is placed at

Reference

A.B. Grebene, “Bipolar and MOS Analog Integrated Circuit Design”, John Wiley & Sons, New York 1984.

Single-input Transconductor Implementations



Single Input (a) Negative Simple Transconductor, (b) Cascode Transconductor, (c) Enhanced Transconductor, (d) Folded-Cascode Transconductor, (e) Positive Simple Transconductor.

- Observe that:

  $g_m = f(I_b)$, the exact relation is a function of the transistor region of operation.
- Note that output impedance of (a) is only $1/g_{ds}$ and (b) and (c) implementations have larger output impedances.

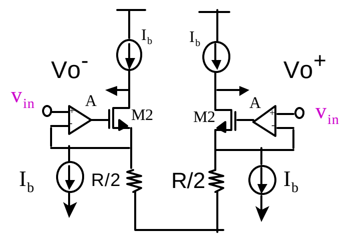
Properties of Simple Transconductors

Structure/ Figure	R_{out}	Min V_{DD} *
Simple/1(a)	$\frac{1}{g_{ds1}}$	$\sqrt{\frac{2I_B}{k}} + V_{sat,I_B}$
Cascode/l(b)	$\frac{g_{m2}}{g_{ds1}\ g_{ds2}}$	$(1+m)\sqrt{\frac{2I_B}{k}} + V_{sat,I_B}$
Enhanced/1(c)	$\frac{A g_{m2}}{g_{ds1}\ g_{ds2}}$	$(1+m)\sqrt{\frac{2I_B}{k}} + V_{sat,I_B}$
Folded/1(d)	$\frac{g_{m2}}{g_{ds1}\ g_{ds2}}$	$\sqrt{\frac{2I_B}{k}} + V_{Tp} + V_{sat,I_B}$

* The bottom devices of the cascode pairs have an aspect ratio of $(W/L)_1/(W/L)_2=m^2$. *k* is a technological parameter determined by the mobility, and the gate oxide; V_{sat,I_B} is the saturation voltage for the I_B current source.

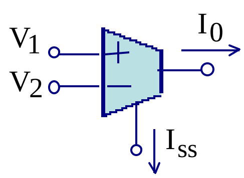
A Transconductance Amplifier with Gm = 1/R

An option to have a transconductance proportional to 1/R without requiring a large g_{m2}



Active Frequency Compensation Transconductor

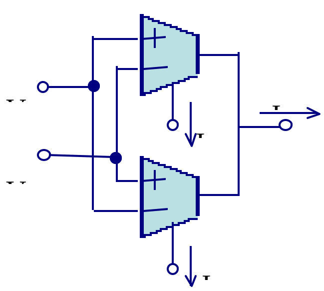
[J. Ramfrez-Angulo and E. Sanchez-Sinencio, "Active Compensaton of Operational Transconductance Amplifier Filters Using Partial Positive Feedback," IEEE Journal of Solid-State Circuits, vol. 25, No. 4, pp. 1024-1028, August 1990]



$$I_0 = g_m (V_1 - V_2)$$

$$g_m(s) = g_{mo}\left(1 - \frac{s}{\omega}\right)$$

ω depends on I_{SS}



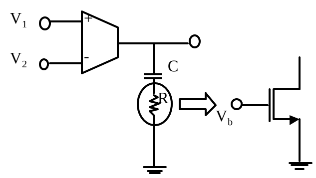
$$I_0 = (g_{mp}(s) - g_{mN}(s))\Delta V = g_{meff}(s)\Delta V$$

$$g_{meff}(s) = g_{meffo}\left[1 - \frac{s}{\omega_{eff}}\right]$$

$$g_{meffo} = g_{mPo} - g_{mNo} \quad , \quad \omega_{eff} = \frac{g_{meffo}}{\frac{g_{mPo}}{\omega_p} - \frac{g_{mNo}}{\omega_N}}$$

It is possible to make

Passive OTA Excess Phase Compensation

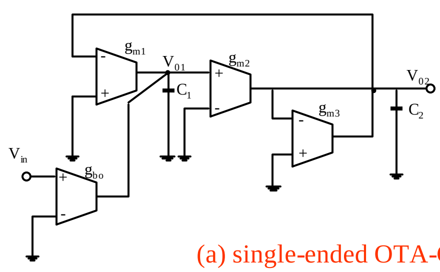


How to determine the value of RC ?

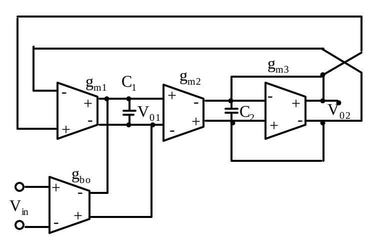
The R is implemented with a transistor operating in the triode (ohmic) region.

The zero generated by the RC should cancel the dominant pole of Gm(s).

Two-integrator biquads



(a) single-ended OTA-C Biquad with one input

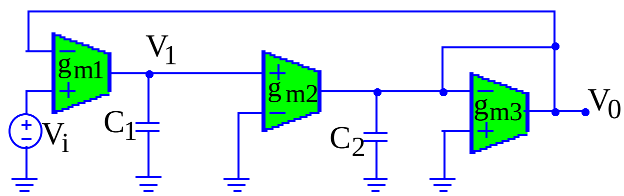


(b)

Fully differential OTA-C Biquad

Analog and Mixed Signal Center, TAMU

OTA Specifications in Open Loop Applications



$$V_1 = \frac{1}{sC_1} g_{m1}(V_i - V_0) \qquad (1)$$

$$V_0 = (g_{m2}V_1 - g_{m3}V_0)\frac{1}{sC_2} \qquad (2)$$

(1) into (2)

$$(sC_2)V_0 = \left[g_{m2}\frac{g_{m1}}{sC_1}(V_i - V_0) - g_{m3}V_0 \right]$$

$$V_0\left[sC_2 + \frac{g_{m1}g_{m2}}{sC_1} + g_{m3} \right] = \frac{g_{m1}g_{m2}}{sC_1} V_i$$

$$H_{LP}(s) = \frac{V_0}{V_i} = \frac{g_{m1}g_{m2}}{s^2C_1C_2 + sC_1g_{m3} + g_{m1}g_{m2}} = \frac{\frac{g_{m1}g_{m2}}{C_1C_2}}{s^2 + s\frac{g_{m3}}{C_2} + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

$$\omega_o^2 = \frac{g_{m1}g_{m2}}{C_1C_2} \quad , \quad BW = \frac{\omega_o}{Q} = \frac{g_{m3}}{C_2}$$

$$Q = \frac{1}{g_{m3}}\sqrt{\frac{g_{m1}g_{m2}C_2}{C_1}} = \frac{C_2\omega_o}{g_{m3}}$$

Now let's assume the transconductance is characterized by:

$$g_m = g_{mo} e^{-s/\omega_p} \cong g_{mo}(1 - s/\omega_p) \text{ for } \omega_p << \omega_o .$$

Under this condition the excess phase can be expressed as $\phi_E \cong \omega_o / \omega_p$.

Note that ideally $\phi_E = 0^0$.

then,

$$H_{LP}(s) = \frac{g_{mo1} g_{mo2}(1 - s/\omega_{p1})(1 - s/\omega_{p2})}{s^2 C_1 C_2 + s C_1 g_{mo3}(1 - s/\omega_{p3}) + g_{mo1} g_{mo2}(1 - s/\omega_{p1})(1 - s/\omega_{p2})}$$

$$D(s) = s^2 C_1 C_2 + s C_1 g_{mo3} - s^2 \frac{C_1 g_{mo3}}{\omega_{p3}} + g_{mo1} g_{mo2}\left\{1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}\right\}$$

$$D(s) = s^2 \left\{ C_1 C_2 - \frac{C_1 g_{mo3}}{\omega_{p3}} + \frac{g_{mo1} g_{mo2}}{\omega_{p1}\omega_{p2}} \right\} + s\left\{ C_1 g_{mo3} - \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) g_{mo1} g_{mo2} \right\} + g_{mo1} g_{mo2}$$

Then the actual ω_{oa} and BW_a become

$$\omega_{oa}^2 = \frac{g_{mo1} g_{mo2}}{C_1 C_2 + \frac{g_{mo1} g_{mo2}}{\omega_{p1}\omega_{p2}} - \frac{C_1 g_{mo3}}{\omega_{p3}}}$$

$$BW_a = \frac{\omega_{oa}}{Q_a} = \frac{C_1 g_{mo3} - \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) g_{mo1} g_{mo2}}{C_1 C_2 - \frac{C_1 g_{mo3}}{\omega_{p3}} + \frac{1}{\omega_{p1}\omega_{p2}}}$$

Let us also assume that $\omega_{p1} = \omega_{p2} = \omega_p$, then $\omega_{oa} \cong \omega_o$, thus,

$$BW_a = \frac{\omega_{oa}}{Q_a} = \frac{C_1 g_{mo3} - \frac{2}{\omega_{p1}} g_{mo1} g_{mo2}}{C_1 C_2 - \frac{C_1 g_{mo3}}{\omega_p} + \frac{1}{\omega_p^2}} \cong \frac{C_1 g_{mo3} - \frac{2}{\omega_p} g_{mo1} g_{mo2}}{C_1 C_2}$$

$$BW_a \cong \frac{g_{mo3}}{C_2} - \frac{g_{mo1} g_{mo2}}{C_1 C_2}\frac{2}{\omega_{p1}} = BW - \omega_{oa}^2 \cdot \frac{2}{\omega_{p1}} = BW - \frac{2\omega_{oa}^2}{\omega_{p1}}$$

$$Q_a = \frac{\omega_{oa}}{BW_a} \cong \frac{1 \cdot \omega_{oa}}{\frac{g_{mo3}}{C_2} - \omega_{oa}^2 \frac{2}{\omega_{p1}}}$$

$$Q_a = \frac{\frac{C_2 \omega_{oa}}{g_{m3}}}{1 - \frac{C_2 \omega_{oa}}{g_{m3}} \frac{2\omega_{oa}}{\omega_{p1}}} = \frac{Q}{1 - \frac{Q 2 \omega_{oa}}{\omega_{p1}}} = \frac{Q}{1 - \frac{2\omega_{oa}}{\omega_{p1}} Q}$$

Alternatively, Q_a can be expressed in terms of the excess phase $\phi_E = \tan^{-1}\frac{\omega_o}{\omega_p} \cong \frac{\omega_o}{\omega_p}$ then

$$Q_a \cong \frac{Q}{1 - 2\phi_E Q} \cong Q(1 + 2\phi_E Q)$$

Furthermore, if $A_{vo} = g_m R_o$ is taken into account, then

$$Q_a = \frac{Q}{1+\frac{2Q}{A_{vo}}}$$

If $A_{vo} = 500$

$$Q_a = \frac{Q}{1+4 \times 10^{-3} Q}$$

Q	Q_a
1	0.996
5	4.902
10	9.6
50	41.667

Note that:

$$Q_a \downarrow \quad \text{when } A_{vo} \downarrow$$

$$BW_a \downarrow \quad Q_a \uparrow \quad \text{when } \phi_E \uparrow$$