EE5980 Homework #1

problem 1 solution:

(i) a) two poles at $p_{1,2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

TF: $H(s) = \dfrac{K}{(s-p_1)(s-p_2)} = \dfrac{K}{s^2+\sqrt{2}s+1}$

at $s=0$ $\quad H(s)\big|_{s=0} = 1 \Rightarrow \underline{K=1}$

$$\therefore \boxed{H(s) = \frac{1}{s^2+\sqrt{2}s+1}}$$

(b) two poles at $p_{1,2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

one zero at $z=0$

TF: $H(s) = \dfrac{K\,s}{s^2+\sqrt{2}s+1}$

at $\omega=1$: $\quad |H(s)|\big|_{s=1} = 1 \Rightarrow \underline{\dfrac{K}{\sqrt{2}} = }$

$$\therefore \boxed{H(s) = \sqrt{2}\,\frac{s}{s^2+\sqrt{2}s+1}}$$

(c) Two poles at $p_{1,2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

double zero at $z=0$,

TF: $H(s) = \dfrac{K\,s^2}{s^2+\sqrt{2}s+1}$

at $\omega=1$ $\quad |H(s)|\big|_{s=1} = 1 \Rightarrow \dfrac{K}{\sqrt{2}} = 1$

$$\therefore \boxed{H(s) = \frac{\sqrt{2}\,s^2}{s^2+\sqrt{2}s+1}}$$

(d) two poles ~~at~~ $p_{1,2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

two zeros at $z_{1,2} = \frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

TF: $H(s) = k \dfrac{s^2 - \sqrt{2}s + 1}{s^2 + \sqrt{2}s + 1}$

at $w=0$ $\quad |H(s)|\Big|_{s=0} = 1 \Rightarrow \underline{\underline{k=1}}$

$$\boxed{H(s) = \frac{s^2 - \sqrt{2}s + 1}{s^2 + \sqrt{2}s + 1}}$$

(e) two ~~pole~~ poles at $p_{1,2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

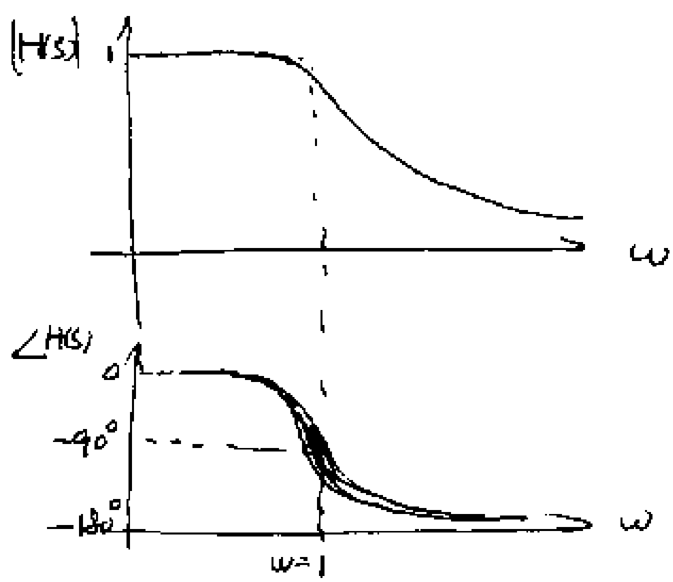
two zeros at $z_{1,2} = \pm j$

TF: $H(s) = k \dfrac{(s+j)(s-j)}{s^2 + \sqrt{2}s + 1} = k \dfrac{s^2+1}{s^2 + \sqrt{2}s + 1}$

at $w=0$ $\quad |H(s)|\Big|_{s=0} = 1 \Rightarrow k=1$

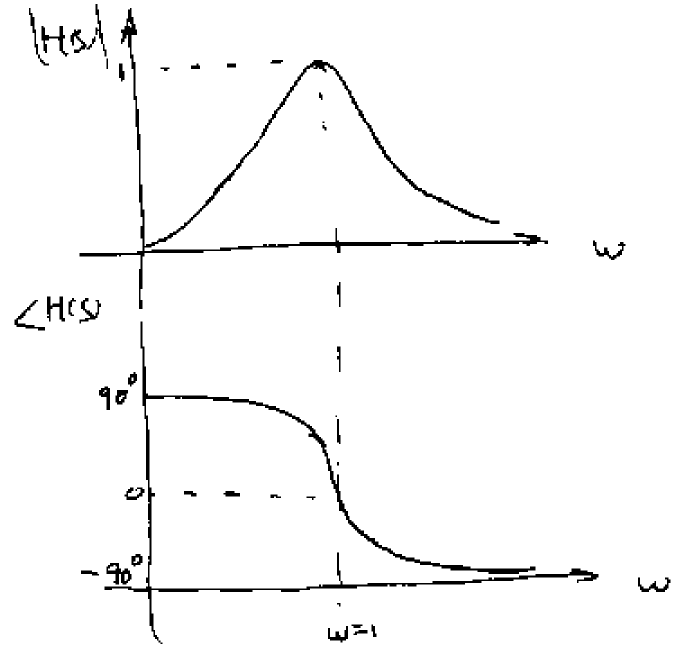
$$\therefore \boxed{H(s) = \frac{s^2+1}{s^2 + \sqrt{2}s + 1}}$$

(ii) a)



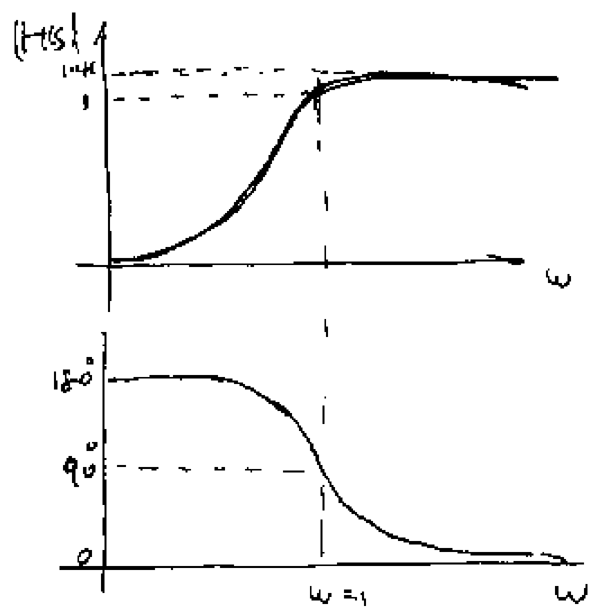
LP Filter

(b)



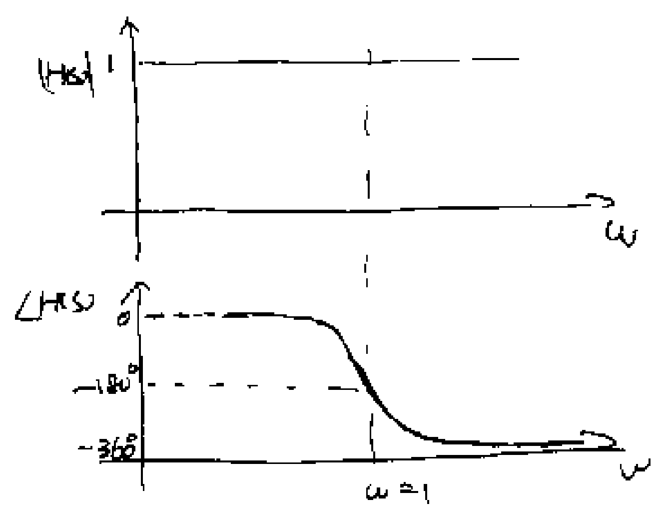
BP Filter

(C)



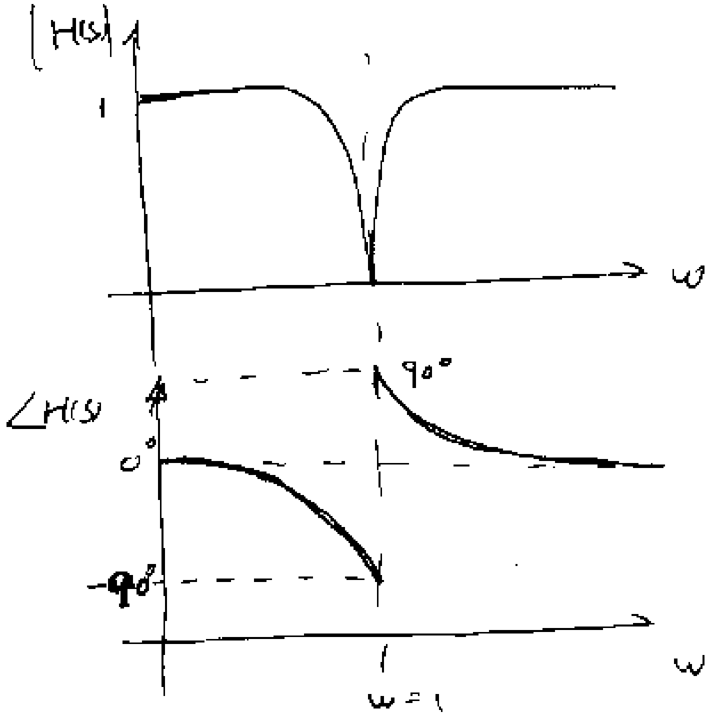
HP Filter

(d)



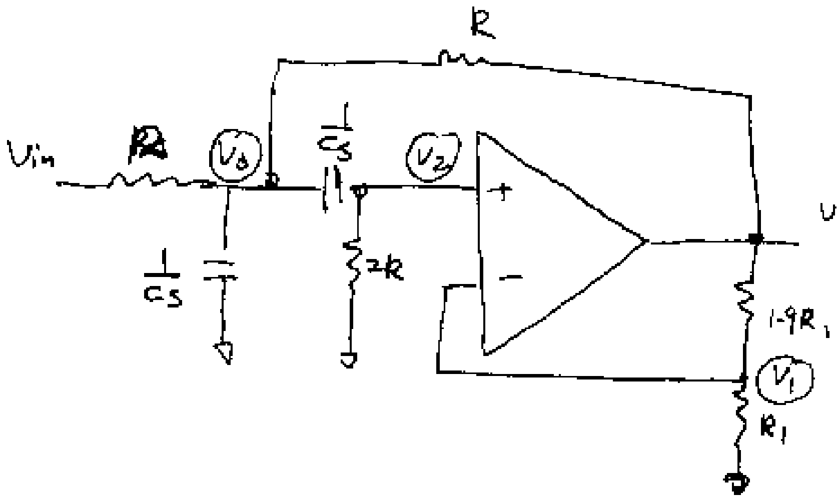
AP Filter

(e)



BS Filters

problem 2 solution



$$\begin{cases} R = 100k \\ C = 20p \\ R_1 = 10k \end{cases}$$

Set $\begin{cases} A \equiv \dfrac{R_1 + 1.9R_1}{R_1} = 2.9 \\ \beta \equiv RCS \end{cases}$

For ideal op. we have:

$$V_2 = V_1 = \frac{R_1}{R_1 + 1.9R_1} V_o = \frac{V_o}{A} \quad (A)$$

$$V_2 = V_3 \cdot \frac{2R}{2R + \frac{1}{Cs}} = V_3 \frac{2CRs}{2CR+1} = V_3 \frac{2\beta}{2\beta + 1}$$

or

$$\boxed{V_3 = \frac{2\beta+1}{2\beta} \cdot \frac{V_o}{A}} \quad (B)$$

For node V_3:

$$\sum i = 0 \Rightarrow \frac{V_i}{R} + \frac{V_o}{R} + \frac{V_2}{\frac{1}{Cs}} = V_3 \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{\frac{1}{sC}} + \frac{1}{\frac{1}{sC}} \right]$$

$$\Rightarrow \boxed{V_i = -V_o - \beta V_2 + V_3 2[1 + \beta]} \quad (C)$$

By (C) (A) & (B) we have,

$$V_i = -V_o - \frac{\beta}{A} V_o + \frac{2\beta+1}{\cancel{2}\beta} \frac{1}{A} \cancel{2} [1+\beta] V_o$$

$$\Rightarrow V_i = \frac{1}{\beta A} [-\beta A - \beta^2 + (1+\beta)(2\beta+1)] V_o$$

$$\therefore \frac{V_o}{V_i} = \frac{\beta A}{1+\beta^2+(3-A)\beta}$$

or
$$\boxed{\frac{V_o}{V_i} = \frac{ARC\ s}{1+(RC)^2 s^2+(3-A)RC\ s}}$$

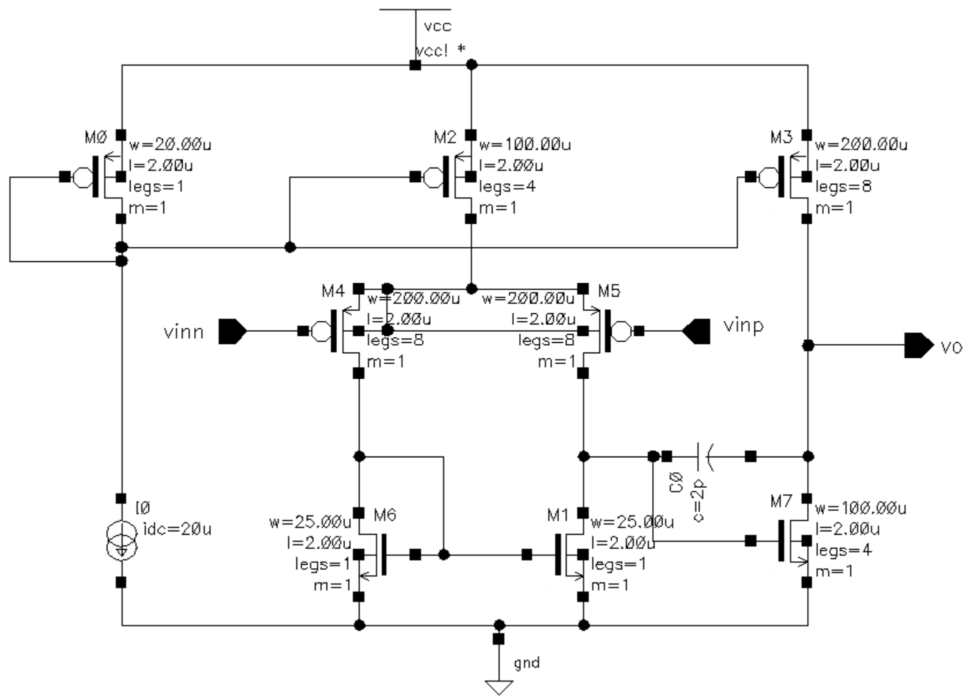
$$A = \frac{R_1+1.9R_1}{R_1} = 2.9$$

$$\boxed{\frac{V_o}{V_i} = \frac{2.9\ RC\ s}{1+(RC)^2 s^2 + \frac{RC}{10} s}}$$

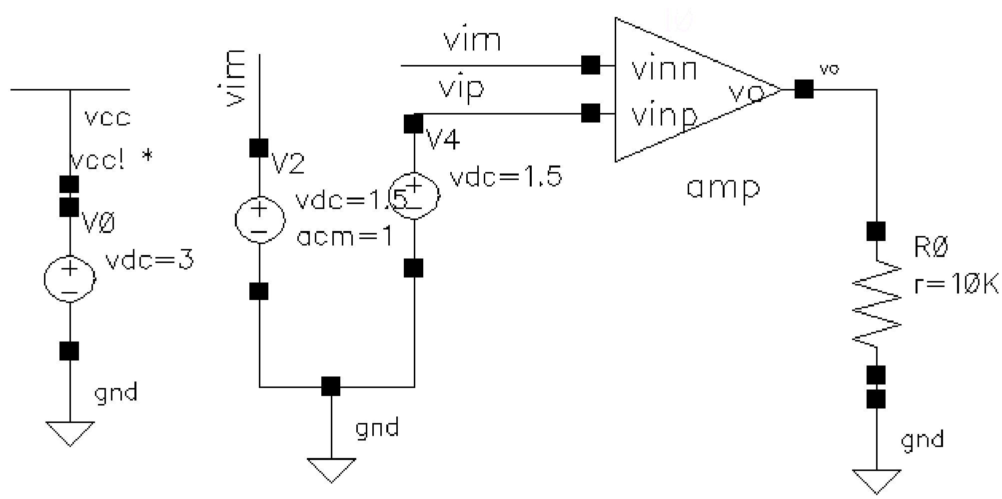
$$\omega_o = \left(\frac{1}{RC}\right) = \frac{1}{100k \times 20p} = 500k \text{ rad/s}$$

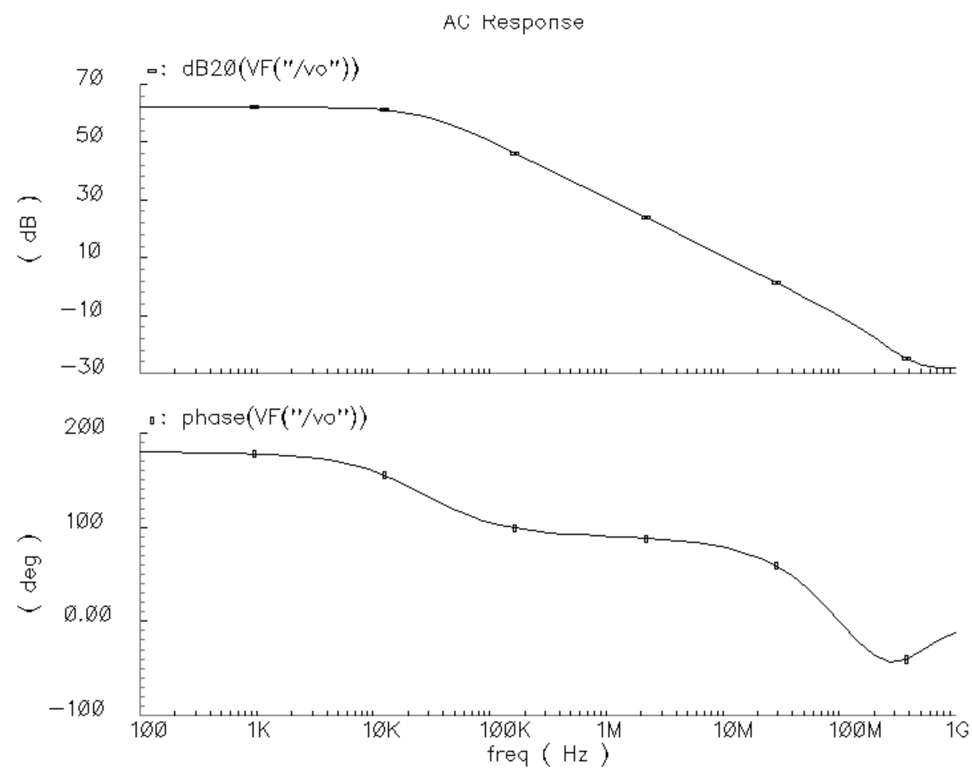
$$\simeq \frac{500k}{2\pi} \simeq 80 \text{ kHz}$$

$Q = 10$

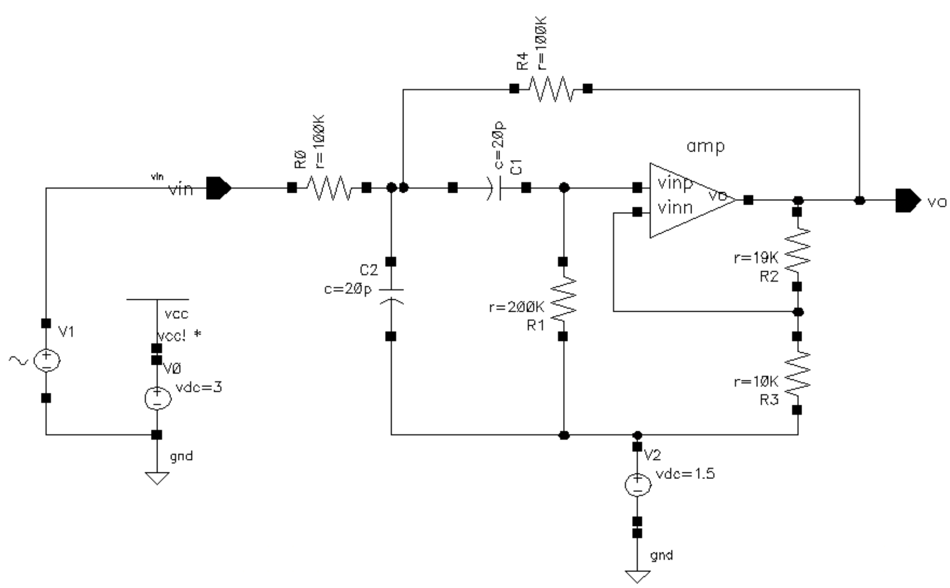


Note that there is no negative bias. The following figure shows the test circuit of the opamp. The power supply is 3V, and we use Vcc/2 as the common mode voltage.

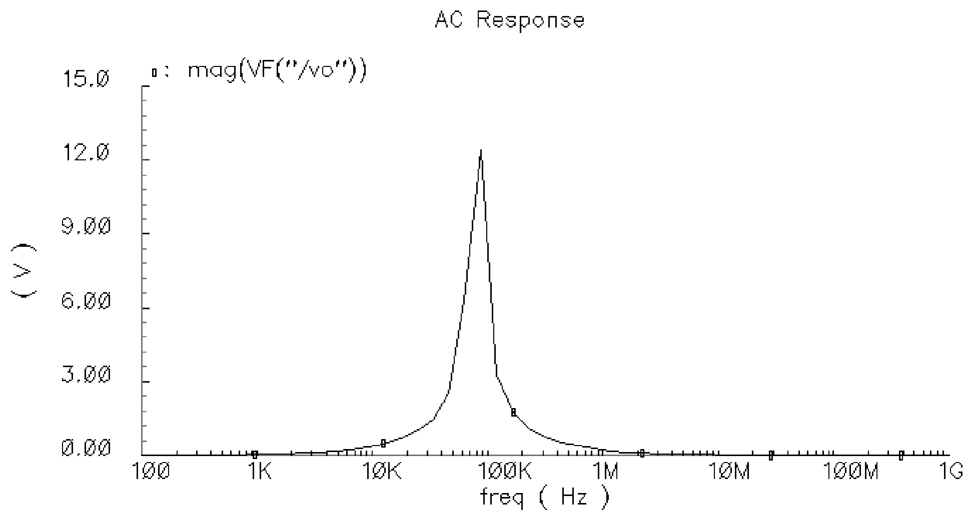




Bode plot of the opamp.



Simulation circuit of the filter. Note that the reference of the circuit is Vcc/2=1.5.



Output character of the filter. The ac magnitude of the input signal is 1V.