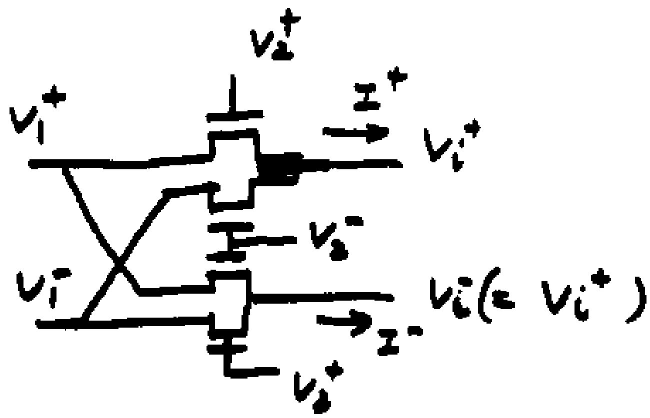
HW#2

problem 1 solution



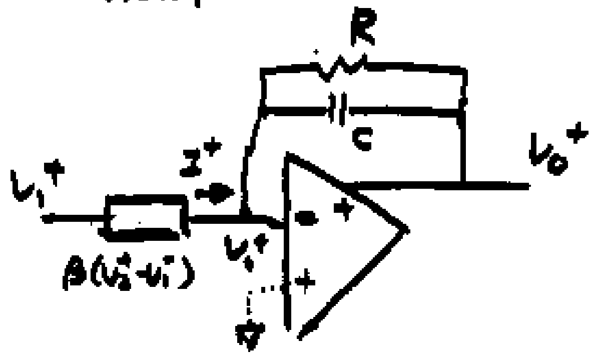
For the 4-T VCR, we have

$$
\begin{aligned}
I^+ - I^- = {} & \beta\left[V_2^+ - V_T - \frac{V_1^+ + V_i^+}{2}\right]\left[V_1^+ - V_i^+\right] \\
& + \beta\left[V_2^- - V_T - \frac{V_1^- + V_i^+}{2}\right]\left[V_1^- - V_i^+\right] \\
& - \beta\left[V_2^+ - V_T - \frac{V_1^- + V_i^-}{2}\right]\left[V_1^- - V_i^-\right] \\
& - \beta\left[V_2^- - V_T - \frac{V_1^+ + V_i^-}{2}\right]\left[V_1^+ - V_i^-\right] \\
= {} & \beta\left[V_2^+ - V_2^-\right]\left[V_1^+ - V_1^-\right] \qquad (1)
\end{aligned}
$$

Since V_{cm} (common mode voltage) $= 0$

$$
\begin{cases}
I^+ = -I^- \\
V_1^+ = -V_1^- \\
V_0^+ = V_0^- \\
V_i^+ = V_i^-
\end{cases}
\Rightarrow
\begin{cases}
I^+ - I^- = 2I^+ \\
V_1^+ - V_1^- = 2V_1^+ \\
V_0^+ - V_0^- = 2V_0^+ \\
V_i^+ = V_i^- = 0
\end{cases}
\qquad (2)
$$

we may use half circuit to simplify the calculation:



$$\underline{V_o^+-V_o^-} = -\beta\,[V_2^+-V_2^-]\,[V_1^+-V_1^-]\,[R\,//\,\tfrac{1}{sC}]$$

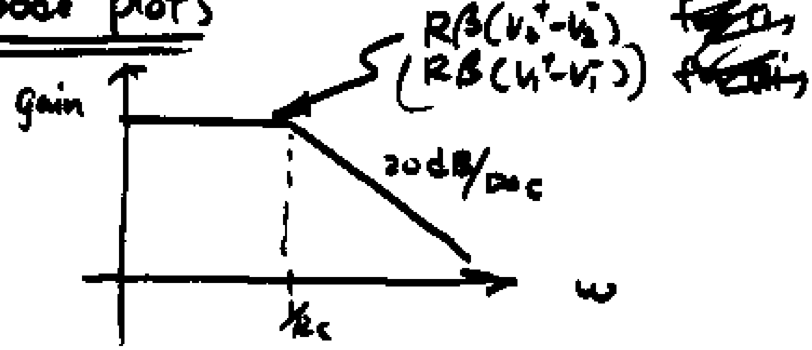
$$= -\beta(V_2^+-V_2^-)(V_1^+-V_1^-)\,\frac{\frac{1}{C}}{\frac{1}{RC}+s}$$

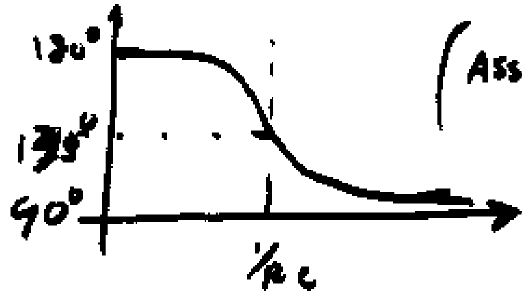
(a) $$\left.\frac{V_o^+-V_o^-}{V_1^+-V_1^-}\right|_{V_2^+-V_2^-=\text{const}} = \frac{-\beta(V_2^+-V_1^-)\cdot\frac{1}{C}}{\frac{1}{RC}+s}$$

$$\left.\frac{V_o^+-V_o^-}{V_2^+-V_2^-}\right|_{V_1^+-V_1^-=\text{const}} = -\frac{\beta(V_1^+-V_1^-)\,\frac{1}{C}}{\frac{1}{RC}+s}$$

(b) no finite zero. in TF
single pole at $\underline{\underline{s=-\frac{1}{RC}}}$

(c) Bode plots





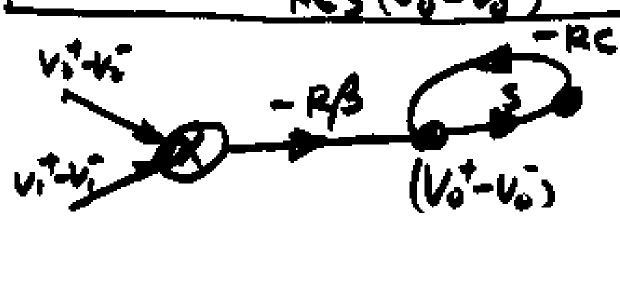
(Assuming $V_2^+-V_2^->0$ or $V_1^+-V_1^->0$)

(d) SFG

$$(V_0^+-V_0^-) = -\frac{\beta(V_2^+-V_2^-)(V_1^+-V_1^-)\frac{1}{C}}{\frac{1}{RC}+s}$$

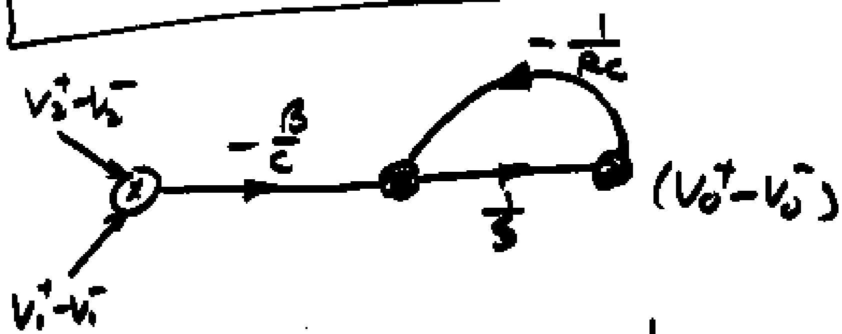
$$\Rightarrow \left(\frac{1}{RC}+s\right)(V_0^+-V_0^-) = -\frac{\beta}{C}(V_2^+-V_2^-)(V_1^+-V_1^-)$$

$$\boxed{(V_0^+-V_0^-) = -R\beta(V_2^+-V_2^-)(V_1^+-V_1^-) - RCs(V_0^+-V_0^-)}$$

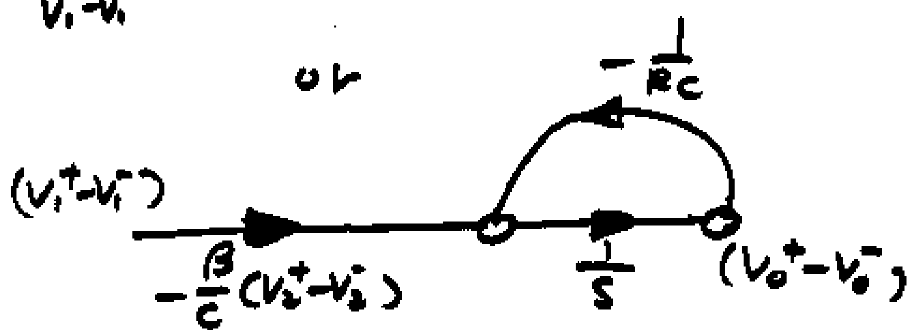


or in integrator version

$$\boxed{(V_o^+ - V_o^-) = -\frac{\beta}{C}(V_2^+ - V_2^-)(V_1^+ - V_1^-)\frac{1}{s} - \frac{1}{RC}(V_o^+ - V_o^-)\frac{1}{s}}$$

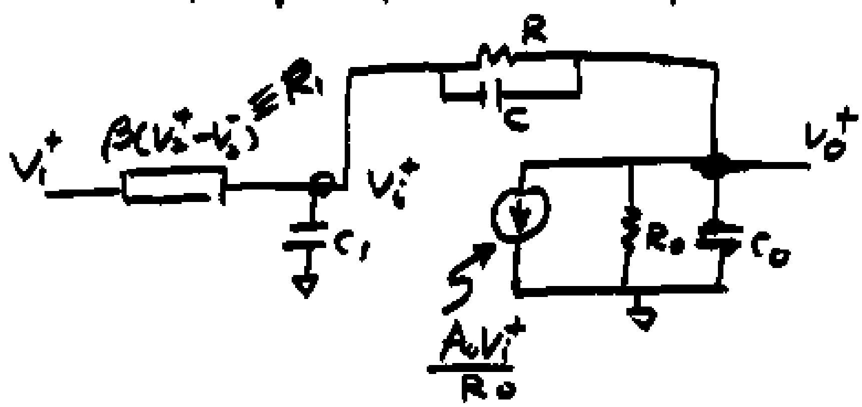


or



problem 2 Solution

Assuming V_i^+ & V_i^- still very close, the 4-T VCR equation ① still valid we have the half equivalent circuit w/ finite op gain & bandwidth.



we have:

$\Sigma I = 0$ @ V_i^+:

Ⓐ $V_i^+\left[\frac{1}{R_1} + C_1 s + \frac{1}{R} + Cs\right] = V_1^+\frac{1}{R_1} + V_o^+\left[\frac{1}{R} + Cs\right]$

$\Sigma I = 0$ @ V_o^+:

Ⓑ $V_o^+\left[\frac{1}{R} + Cs + \frac{1}{R_o} + C_o s\right] = V_i^+\left[\frac{1}{R} + Cs\right] - \frac{A_o}{R_o} V_i^+$

From (B)

$$V_i^+ = -V_0^+ \frac{\frac{1}{R} + \frac{1}{R_0} + (C + C_0)S}{\frac{A_0}{R_0} - \left(\frac{1}{R} + CS\right)} \qquad (C)$$

insert (C) into (A)

$$-V_0^+ \frac{\left(\frac{1}{R} + \frac{1}{R_0} + (C + C_0)S\right)\left(\frac{1}{R_1} + \frac{1}{R} + S(C_1 + C)\right)}{\frac{A_0}{R_0} - \left(\frac{1}{R} + CS\right)}$$

$$= V_i^+ \frac{1}{R_1} + V_0 \left[\frac{1}{R} + CS\right]$$

$$\Rightarrow -V_0^+ \left[\frac{\left(\frac{1}{R} + \frac{1}{R_0} + (C + C_0)S\right)\left(\frac{1}{R_1} + \frac{1}{R} + S(C_1 + C)\right)}{\frac{A_0}{R_0} - \left(\frac{1}{R} + CS\right)} + \frac{1}{R} + CS \right]$$

$$= V_i^+ \frac{1}{R_1}$$

$$\Leftrightarrow \boxed{\frac{V_0^+}{V_i^+} = - \frac{\frac{1}{R_1}}{\frac{\left(\frac{1}{R} + \frac{1}{R_0} + (C + C_0)S\right)\left(\frac{1}{R_1} + \frac{1}{R} + S(C_1 + C)\right)}{\frac{A_0}{R_0} - \left(\frac{1}{R} + CS\right)} + \frac{1}{R} + CS}}$$

$$\Rightarrow \frac{V_o^+}{V_i^+} = - \frac{\left(\frac{A_o}{R_o} - \left(\frac{1}{R} + Cs\right)\right)\frac{1}{R_1}}{\left[\left(\frac{1}{R} + \frac{1}{R_o}\right) + (C + C_o)s\right]\left[\frac{1}{R_1} + \frac{1}{R} + s(C_1 + C)\right] + \frac{A_o}{R_o}\left(\frac{1}{R} + Cs\right) - \left(\frac{1}{R} + Cs\right)^2}$$

$$\boxed{\frac{V_o^+}{V_i^+} = - \frac{K\,(z - s)}{s^2 + as + b}}$$

is a 2nd order filter

where K, z, a, b, are constants.

There some special cases:

① at low frequency & high A_o

$$\frac{V_o^+}{V_i^+} = \frac{V_o^+ - V_o^-}{V_i^+ - V_i^-} \simeq - \frac{\cancel{\frac{A_o}{R_o}}\ \frac{1}{R_1}}{\cancel{\frac{A_o}{R_o}}\left(\frac{1}{R} + Cs\right)} = - \frac{\beta(V_o^+ - V_o^-)\frac{1}{C}}{\frac{1}{RC} + s}$$

same as ideal op.

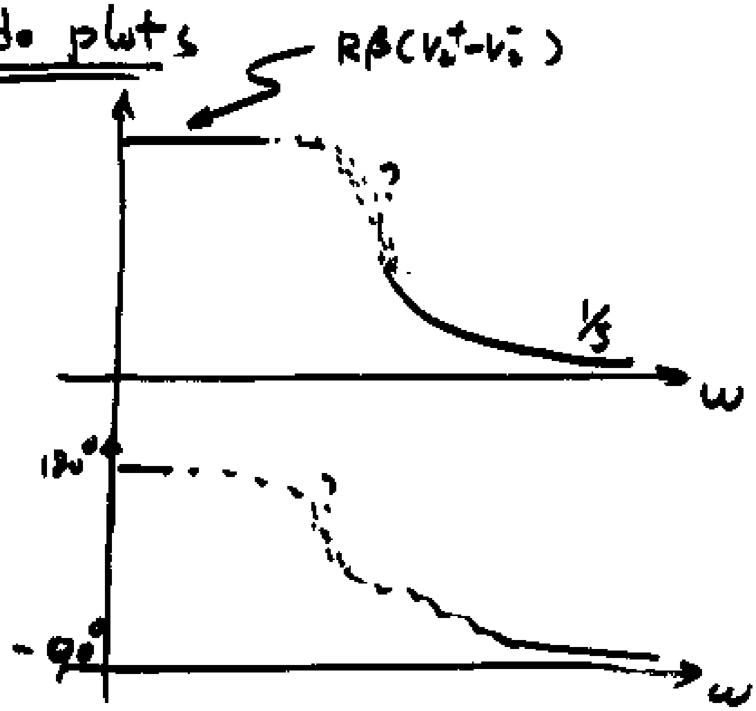
this is used to keep low frequency band of the mixer.

3 at high frequency

$$\frac{V_o^+}{V_i^+} = \frac{V_o^+ - V_o^-}{V_i^+ - V_i^-} \simeq \frac{\frac{C}{R_1} s}{(C+C_o)(C_1+C) - C^2} \frac{1}{s} \to 0$$

the mixer use this to filter out the high frequency (2x) band.

Bode plots



SFG

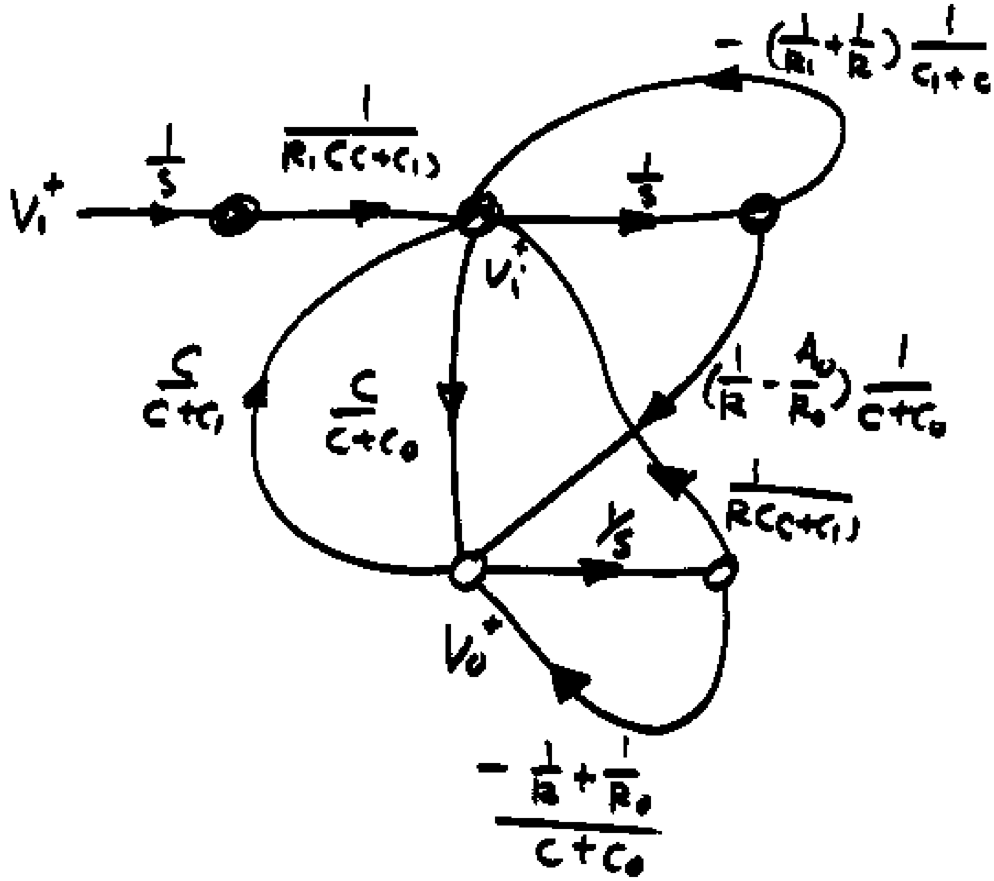
From A

$$V_i^+ = V_i^+ \frac{1}{R_1(C+C_1)}\frac{1}{s} + V_o^+\left[\frac{1}{R(C+C_1)}\frac{1}{s} + \frac{C}{C+C_1}\right] - V_i^+\left(\frac{1}{R_1}+\frac{1}{R}\right)\frac{1}{C_1+C}\cdot\frac{1}{s}$$

From B

$$V_o^+ = \frac{V_i^+\left[\frac{1}{R} + Cs - \frac{A_o}{R_o}\right] - \left(\frac{1}{R}+\frac{1}{R_o}\right)V_o^+}{(C+C_o)s}$$

$$= V_i^+\left[\left(\frac{1}{R}-\frac{A_o}{R_o}\right)\frac{1}{C+C_o}\cdot\frac{1}{s} + \frac{C}{C+C_o}\right] - \frac{\frac{1}{R}+\frac{1}{R_o}}{C+C_o}\frac{1}{s}V_o^+$$



V_1^+
$\frac{1}{s}$
$\frac{1}{R_1 C(C+C_1)}$
$-\left(\frac{1}{R_1}+\frac{1}{R}\right)\frac{1}{C_1+C}$
$\frac{1}{s}$
V_1^+
$\frac{C}{C+C_1}$
$\frac{C}{C+C_0}$
$\left(\frac{1}{R}-\frac{A_0}{R_0}\right)\frac{1}{C+C_0}$
$\frac{1}{RC(C+C_1)}$
$\frac{1}{s}$
V_0^+
$\frac{-\frac{1}{R}+\frac{1}{R_0}}{C+C_0}$