BEng in Civil and Structural Engineering:
Subject (CSE307)

Soil Mechanics
土力學

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Outline of Lectures by JH YIN:

Lecture 1: Basic characteristics of soils (Chapter 1)
Lecture 2: Seepage (Chapter 2)
Lecture 3: Effective stress (Chapter 3)
Lecture 4: Shear strength (Chapter 4)
Lecture 5: Stresses and displacements (Chapter 5)
Lecture 6: Lateral earth pressure (Chapter 6)
Lecture 7: Consolidation theory (Chapter 7)
Lecture 8: Bearing capacity (Chapter 8 plus)
Lecture 9: Stability of slopes (Chapter 9)

Essential Reference:

- **Craig, R.F. (2004). Soil Mechanics, 7th edition (6th or 5th edition), Spon Press, London and New York (ISBN 04-415-32702-2)**

Lecture 6: Lateral earth pressure (側向土壓力)

6.1 Introduction

- For many structures (like retaining walls) , the pressure from soils at failure (extreme cases) is needed for design analysis
- Deformation is not a big concern (or done separately)
- Simple solution to the soil (earth) pressure can be obtained for rigid plastic soil behavior:

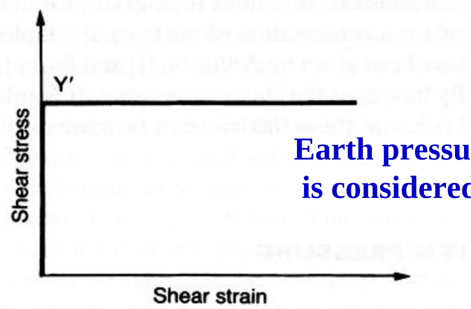


Figure 6.1 Idealized stress–strain relationship.

Earth pressure at failure (plastic collapse) is considered to be a problem in plasticity

Plasticity Lower Bound and Upper Bound approaches to calculation of earth pressure at failure (plastic collapse):

Lower Bound theorem - if the stresses assumed satisfy the equilibrium and yield equations (without consideration of a mode of deformation), so calculated external loads (earth pressures and bearing pressure) are <= the true external loads (collapse loads)

Upper Bound theorem - if a mechanism of failure (collapse or deformation mode) is assumed and the work done by external forces equals to the work done by stress acting on the assumed slip surface (without consideration of stress equilibrium), so calculated external loads (earth pressures and bearing pressure) are >= the true external loads (collapse loads)

6.2 Rankin's theory of earth pressure

Rankin's theory – considering the stress state in stress equilibrium and at plastic failure – a Lower Bound approach

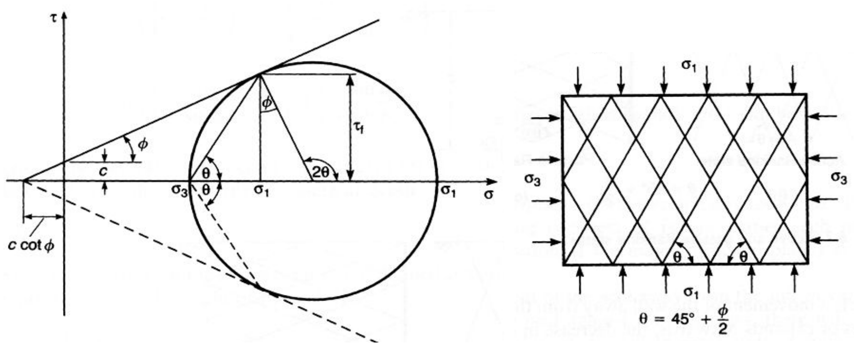


Figure 6.2 State of plastic equilibrium.

William John Maquorn Rankine (1820 - 1872)

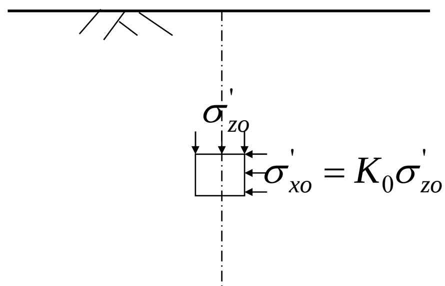
Earth pressure at rest (no lateral move) (no failure)

$$K_0 = \frac{\sigma'_{xo}}{\sigma'_{zo}} = \frac{p'_{ho}}{p'_{vo}}$$

$$K_0 = 1 - \sin\phi'$$

$$K_0 = (1 - \sin\phi')(OCR)^{\sin\phi'}$$

$$K_0 = (1 - \sin\phi')(OCR)^{0.5}$$



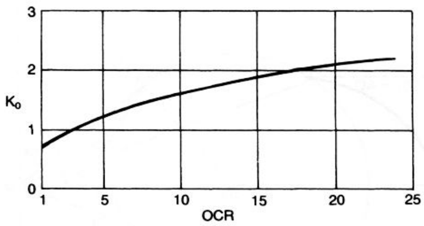


Figure 6.11 Typical relationship between K_0 and overconsolidation ratio for a clay.

Table 6.2 Coefficient of earth pressure at-rest

Soil	K_0
Dense sand	0.35
Loose sand	0.6
Normally consolidated clays (Norway)	0.5–0.6
Clay, OCR = 3.5 (London)	1.0
Clay, OCR = 20 (London)	2.8

Consider a vertical wall with smooth face (the wall has no friction)

$$p_x = \sigma_x$$

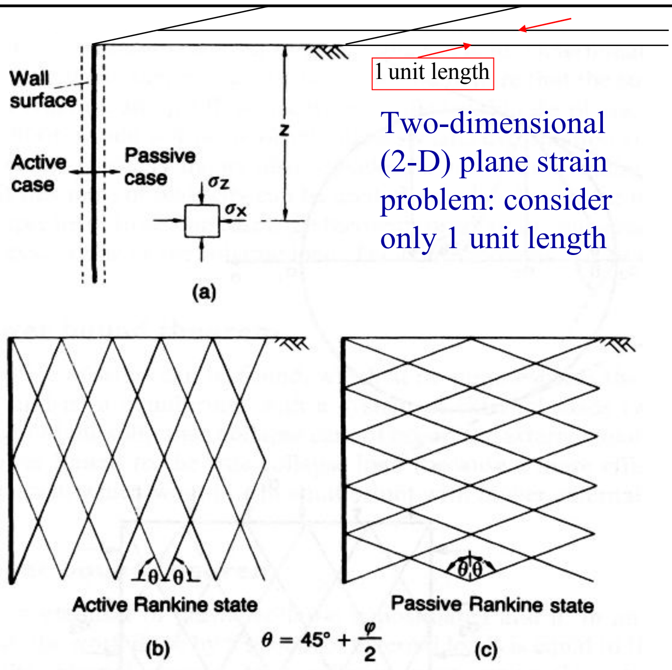


Figure 6.3 Active and passive Rankine states.

Two-dimensional (2-D) plane strain problem: consider only 1 unit length

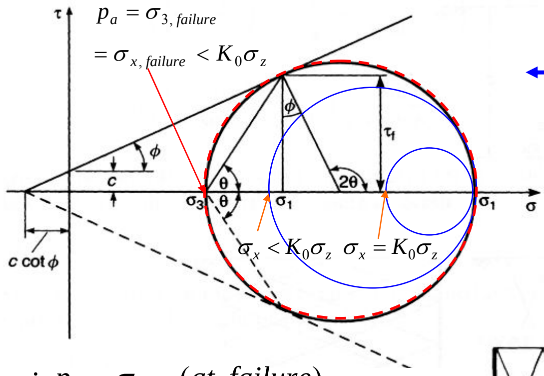
Rankine' theory - stress equilibrium is satisfied and a plastic (failure) state is reached - lower bound loads/pressures are then calculated.

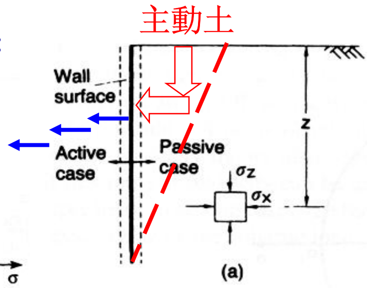
Active and Passive Rankine States:

Active State: a vertical wall (smooth face) moving away from the soil mass (like that the soil mass is actively pushing the wall away). The vertical stress σ_z is the major principal stress σ_1.

Passive State: a vertical wall (smooth face) moving toward the soil mass (like that the soil mass is compressed (passive) by the wall). The vertical stress σ_z is the minor principal stress σ_3.

Active state and active earth pressure:





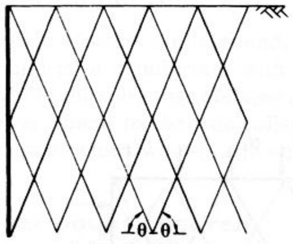
$$\sigma_x = K_0\sigma_z$$

$$\sigma_3 = K_0\sigma_1 < \sigma_1$$

$$\therefore p_a = \sigma_3 \quad (at\ \ failure)$$

Active earth pressure:
主動土壓力

$$\theta = 45^\circ + \frac{\varphi}{2}$$



Active Rankine state

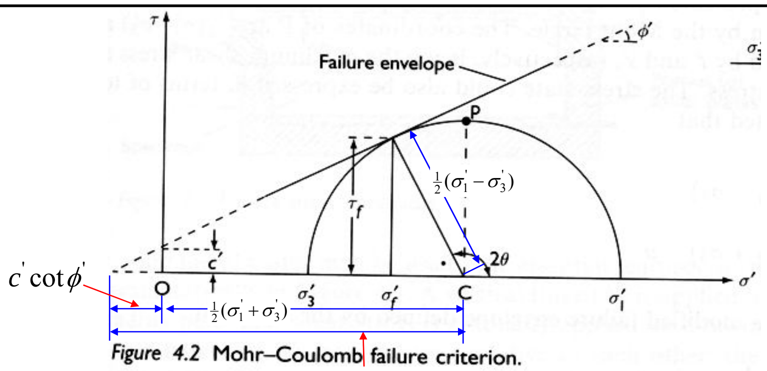


Figure 4.2 Mohr–Coulomb failure criterion.

$c'\cot\phi' + \frac{1}{2}(\sigma_1' + \sigma_3')$

From Fig.4.2:
$$\sin\phi' = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{c'\cot\phi' + \frac{1}{2}(\sigma_1' + \sigma_3')}$$

$$(\sigma_1' - \sigma_3') = (\sigma_1' + \sigma_3')\sin\phi' + 2c'\cos\phi' \qquad or$$

$$\sigma_1' = \sigma_3'\tan^2(45^o + \frac{\phi'}{2}) + 2c'\tan(45^o + \frac{\phi'}{2})$$

This is **Mohr-Coulomb failure criterion** !

$$(\sigma_1^{'} - \sigma_3^{'}) = (\sigma_1^{'} + \sigma_3^{'})\sin\phi^{'} + 2c^{'}\cos\phi^{'}$$

$$\sigma_1^{'}(1 - \sin\phi^{'}) = \sigma_3^{'}(1 + \sin\phi^{'}) + 2c^{'}\cos\phi^{'}$$

$$\sigma_1^{'} = \sigma_3^{'}\frac{(1 + \sin\phi^{'})}{(1 - \sin\phi^{'})} + 2c^{'}\frac{\cos\phi^{'}}{(1 - \sin\phi^{'})}$$

$$\sigma_1^{'} = \sigma_3^{'}\frac{(1 + \sin\phi^{'})}{(1 - \sin\phi^{'})} + 2c^{'}\frac{\sqrt{1 - \sin^2\phi^{'}}}{(1 - \sin\phi^{'})}$$

$$\sigma_1^{'} = \sigma_3^{'}\frac{(1 + \sin\phi^{'})}{(1 - \sin\phi^{'})} + 2c^{'}\frac{\sqrt{(1 + \sin\phi^{'})(1 - \sin\phi^{'})}}{\sqrt{(1 - \sin\phi^{'})^2}}$$

$$\sigma_1^{'} = \sigma_3^{'}\frac{(1 + \sin\phi^{'})}{(1 - \sin\phi^{'})} + 2c^{'}\sqrt{\frac{1 + \sin\phi^{'}}{1 - \sin\phi^{'}}}$$

This is **Mohr-Coulomb failure criterion** – another form !

$$(\sigma_1^{'} - \sigma_3^{'}) = (\sigma_1^{'} + \sigma_3^{'})\sin\phi^{'} + 2c^{'}\cos\phi^{'}$$

$$\sigma_3^{'}(1 + \sin\phi^{'}) = \sigma_1^{'}(1 - \sin\phi^{'}) - +2c^{'}\cos\phi^{'}$$

$$\sigma_3^{'} = \sigma_1^{'}\frac{(1 - \sin\phi^{'})}{(1 + \sin\phi^{'})} - 2c^{'}\frac{\cos\phi^{'}}{(1 + \sin\phi^{'})}$$

$$\sigma_3^{'} = \sigma_1^{'}\frac{(1 - \sin\phi^{'})}{(1 + \sin\phi^{'})} - 2c^{'}\sqrt{\frac{1 - \sin\phi^{'}}{1 + \sin\phi^{'}}}$$

$$\because K_a = \frac{(1 - \sin\phi^{'})}{(1 + \sin\phi^{'})}, \quad p_a^{'} = \sigma_3^{'}, \quad \sigma_z^{'} = \sigma_1^{'}$$

$$\therefore p_a^{'} = K_a\sigma_z^{'} - 2c^{'}\sqrt{K_a}$$

$$\because K_a = \frac{(1 - \sin\phi)}{(1 + \sin\phi)}, \quad p_a = \sigma_3$$

$$p_a = K_a\sigma_z - 2c\sqrt{K_a}$$

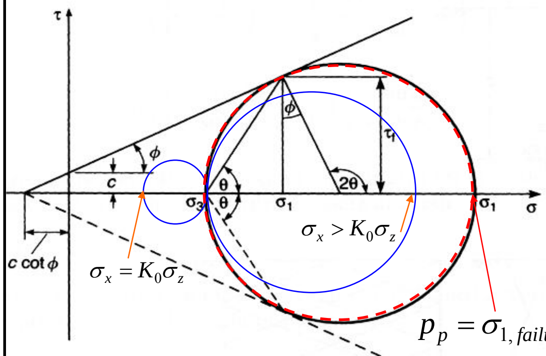
(1) Using effective stresses and effective stress parameters

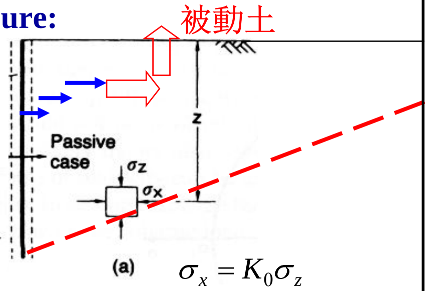
(2) Using total stresses and total stress parameters

(3) No mix up !

This is Rankine's active earth pressure theory (know how to derive)!

Passive state and passive earth pressure: 被動土





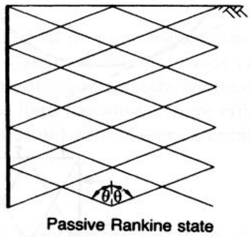
$\sigma_x = K_0\sigma_z$

$\sigma_3 = K_0\sigma_1 < \sigma_1$

$p_p = \sigma_{1,\,failure} =$

$= \sigma_{x,\,failure} > K_0\sigma_z$

$\therefore\ p_p = \sigma_1 \quad (at\ \ failure)$



Passive earth pressure:
被動土壓力

$\theta = 45^\circ + \dfrac{\varphi}{2}$

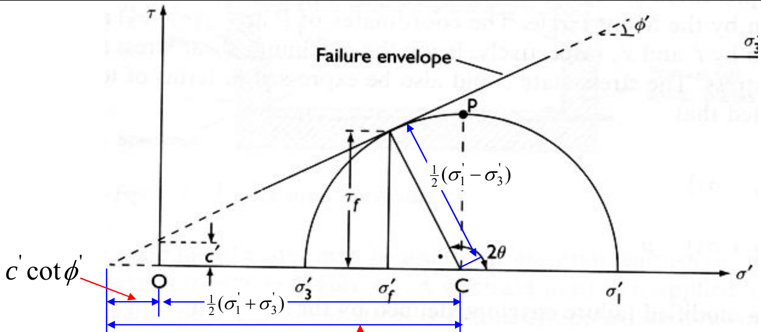


Figure 4.2 Mohr–Coulomb failure criterion.

$c'\cot\phi' + \frac{1}{2}(\sigma_1' + \sigma_3')$

From Fig.4.2:

$$\sin\phi' = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{c'\cot\phi' + \frac{1}{2}(\sigma_1' + \sigma_3')}$$

$$(\sigma_1' - \sigma_3') = (\sigma_1' + \sigma_3')\sin\phi' + 2c'\cos\phi' \qquad or$$

$$\sigma_1' = \sigma_3'\tan^2(45^o + \frac{\phi'}{2}) + 2c'\tan(45^o + \frac{\phi'}{2})$$

This is **Mohr-Coulomb failure criterion** !

$$(\sigma_1^{'} - \sigma_3^{'}) = (\sigma_1^{'} + \sigma_3^{'})\sin\phi^{'} + 2c^{'}\cos\phi^{'}$$

$$\sigma_1^{'}(1-\sin\phi^{'}) = \sigma_3^{'}(1+\sin\phi^{'}) + 2c^{'}\cos\phi^{'}$$

$$\sigma_1^{'} = \sigma_3^{'}\frac{(1+\sin\phi^{'})}{(1-\sin\phi^{'})} + 2c^{'}\frac{\cos\phi^{'}}{(1-\sin\phi^{'})}$$

$$K_p = \frac{1}{K_a}$$

$$\sigma_1^{'} = \sigma_3^{'}\frac{(1+\sin\phi^{'})}{(1-\sin\phi^{'})} + 2c^{'}\sqrt{\frac{1+\sin\phi^{'}}{1-\sin\phi^{'}}}$$

$$\because K_p = \frac{(1+\sin\phi^{'})}{(1-\sin\phi^{'})}, \quad p_a^{'} = \sigma_1^{'}, \quad \sigma_z^{'} = \sigma_3^{'}$$

$$\therefore p_p^{'} = K_p\sigma_z^{'} + 2c^{'}\sqrt{K_p}$$

(1) Using effective stresses and effective stress parameters

$$\because K_p = \frac{(1+\sin\phi)}{(1-\sin\phi)}, \quad p_p = \sigma_1$$

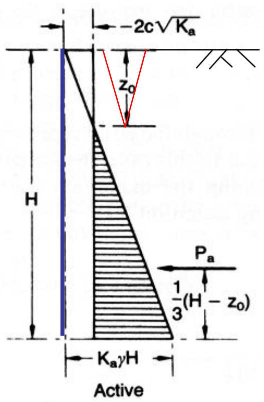
(2) Using total stresses and total stress parameters

(3) No mix up !

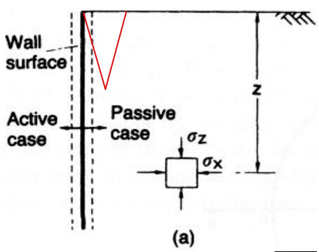
$$p_p = K_p\sigma_z + 2c\sqrt{K_p}$$

This is Rankine's passive earth pressure theory (know how to derive)!

Tension crack: (occurs in active state only)

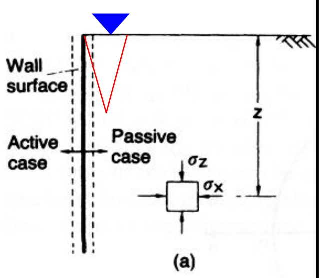


Dry crack



$$p_a = K_a\sigma_z - 2c\sqrt{K_a}$$

Crack filled with water



$$\because \sigma_z = \gamma z \quad (no\ \text{surcharge})$$

$$\therefore p_a = K_a\gamma z - 2c\sqrt{K_a} = 0 \quad (position) \Rightarrow \therefore z_o = \frac{2c}{\gamma\sqrt{K_a}}$$

Dry crack

$$\because \sigma_z = \gamma z \quad (no\ \text{surcharge})$$

$$\therefore p_a = K_a\gamma z - 2c\sqrt{K_a} = p_w = \gamma_w z \Rightarrow z_{ow} = \frac{2c\sqrt{K_a}}{K_a\gamma - \gamma_w}$$

Crack filled with water

Total active thrust (P_a) (force):

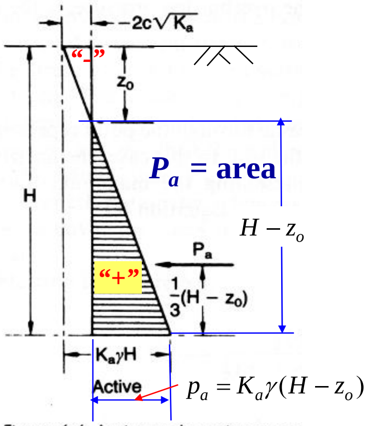
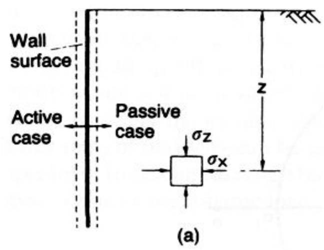


Figure 6.4 Active and passive press



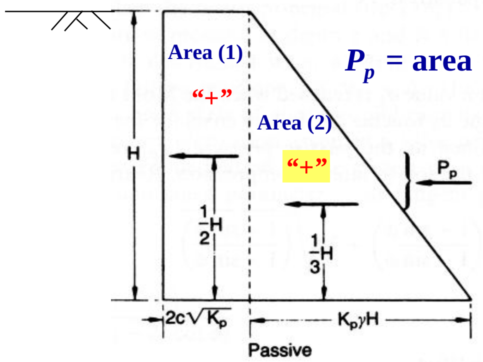
$$p_a = K_a \sigma_z - 2c\sqrt{K_a}$$

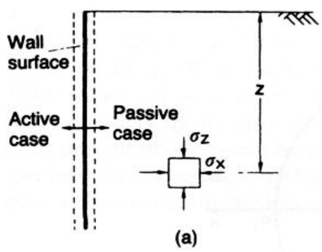
$$\therefore p_a = K_a \gamma H - 2c\sqrt{K_a} = K_a \gamma H - K_a \gamma z_o = K_a \gamma (H - z_o)$$

$$\therefore P_a = area = \tfrac{1}{2} K_a \gamma (H - z_o)(H - z_o) = \tfrac{1}{2} K_a \gamma (H - z_o)^2$$

Note: the negative pressure area is NOT included – assuming tension crack there

Total passive (P_p) thrust (force):





$$p_p = K_p \sigma_z + 2c\sqrt{K_p}$$

$$\therefore P_p = Area(1) + Area(2) = 2c\sqrt{K_p} \times H + \tfrac{1}{2} K_p \gamma H \times H$$

$$\therefore P_p = \tfrac{1}{2} K_p \gamma H^2 + 2c\sqrt{K_p} H$$

Note: (a) calculate p_p (or p_a)at key points, (b) plot the pressure distribution, and (c) calculate total area

Considering vertical pressure surcharge? – Easy!

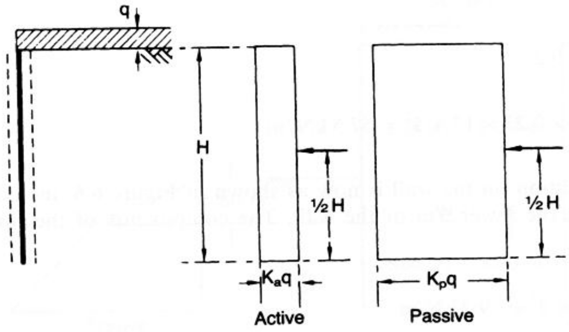


Figure 6.5 Additional pressure due to surcharge.

$$p_a = K_a\sigma_z - 2c\sqrt{K_a}$$

$$\sigma_z = \gamma z + q$$

$$p_p = K_p\sigma_z + 2c\sqrt{K_p}$$

Example 6.1

(a) Calculate the total active thrust on a vertical wall 5 m high retaining a sand of unit weight 17 kN/m^3 for which $\phi' = 35°$; the surface of the sand is horizontal and the water table is below the bottom of the wall. (b) Determine the thrust on the wall if the water table rises to a level 2 m below the surface of the sand. The saturated unit weight of the sand is 20 kN/m^3.

Notes: (a) If effective stress parameters are given, water pressure is calculated separately, (b) if total stress parameters are given, no need calculate water pressure (included in total stress). Why? $\sigma = \sigma' + u$

Solution:

(a) $K_a = \dfrac{1 - \sin 35°}{1 + \sin 35°} = 0.27$

$$P_a = \frac{1}{2}K_a\gamma H^2 = \frac{1}{2} \times 0.27 \times 17 \times 5^2 = 57.5\ \text{kN/m}$$

Key points:

(b) Pressure distribution is plotted.

(1) $\frac{1}{2} \times 0.27 \times 17 \times 2^2 = 9.2$ kN/m

(2) $0.27 \times 17 \times 2 \times 3 = 27.6$

(3) $\frac{1}{2} \times 0.27 \times (20 - 9.8) \times 3^2 = 12.4$

(4) $\frac{1}{2} \times 9.8 \times 3^2 = 44.1$

Total thrust = 93.3 kN/m

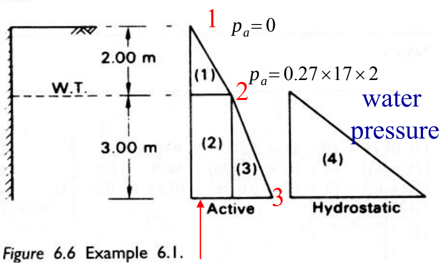


Figure 6.6 Example 6.1.

$p_a = 0.27[17 \times 2 + (20 - 9.8) \times 3]$
$= 0.27 \times 17 \times 2 + 0.27 \times (20 - 9.8) \times 3$

Example 6.2

The soil conditions adjacent to a sheet pile wall are given in Figure 6.7, a surcharge pressure of 50 kN/m^2 being carried on the surface behind the wall. For soil 1, a sand above the water table, $c' = 0$, $\phi' = 38°$ and $\gamma = 18\,\text{kN/m}^3$. For soil 2, a saturated clay, $c' = 10\,\text{kN/m}^2$, $\phi' = 28°$ and $\gamma_{sat} = 20\,\text{kN/m}^3$. Plot the distributions of active pressure behind the wall and passive pressure in front of the wall.

Solution:

Key points ?:

For soil 1,

$$K_a = \frac{1 - \sin 38°}{1 + \sin 38°} = 0.24, \quad K_p = \frac{1}{0.24} = 4.17$$

For soil 2,

$$K_a = \frac{1 - \sin 28°}{1 + \sin 28°} = 0.36, \quad K_p = \frac{1}{0.36} = 2.78$$

Jump points: at any location with different ***c*** and ***ϕ***

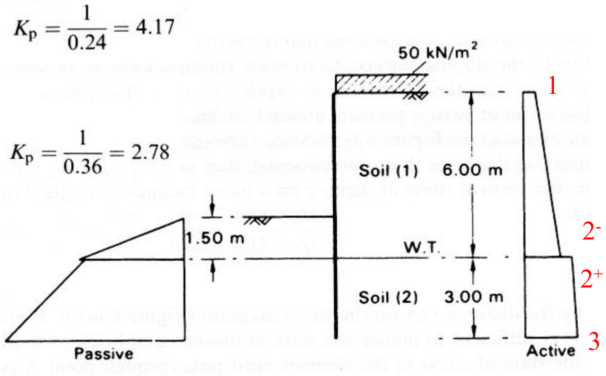
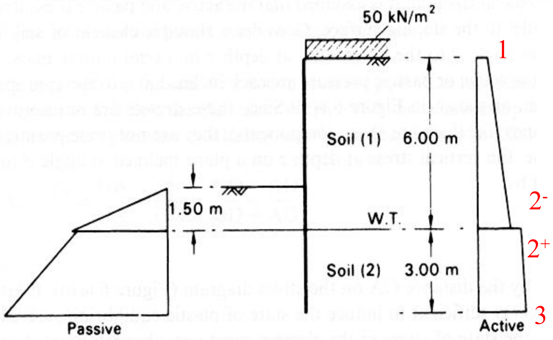


Figure 6.7 Example 6.2.

Table 6.1

Soil	*Depth* (m)	*Pressure* (kN/m^2)	
Active pressure			
1	0	0.24 × 50	= 12.0
1	6	(0.24 × 50) + (0.24 × 18 × 6) = 12.0 + 25.9	= 37.9
2	6	0.36[50 + (18 × 6)] − (2 × 10 × $\sqrt{0.36}$) = 56.9 − 12.0	= 44.9
2	9	0.36[50 + (18 × 6)] − (2 × 10 × $\sqrt{0.36}$) + (0.36 × 10.2 × 3) = 56.9 − 12.0 + 11.0	= 55.9
Passive pressure			
1	0	0	
1	1.5	4.17 × 18 × 1.5	= 112.6
2	1.5	(2.78 × 18 × 1.5) + (2 × 10 × $\sqrt{2.78}$) = 75.1 + 33.3	= 108.4
2	4.5	(2.78 × 18 × 1.5) + (2 × 10 × $\sqrt{2.78}$) + (2.78 × 10.2 × 3) = 75.1 + 33.3 + 85.1	= 193.5



Sloping soil surface:

$$K_a = \frac{p_a}{\sigma_z} = \frac{OB}{OA} = \frac{OB'}{OA} = \frac{OD - AD}{OD + AD}$$

Now

$$OD = OC \cos\beta$$

$$AD = \sqrt{(OC^2 \sin^2\phi - OC^2 \sin^2\beta)}$$

$$AD = \sqrt{CF^2 - CD^2}$$

$$= \sqrt{AC^2 - CD^2}$$

Cohesion is zero.
Know the direction.

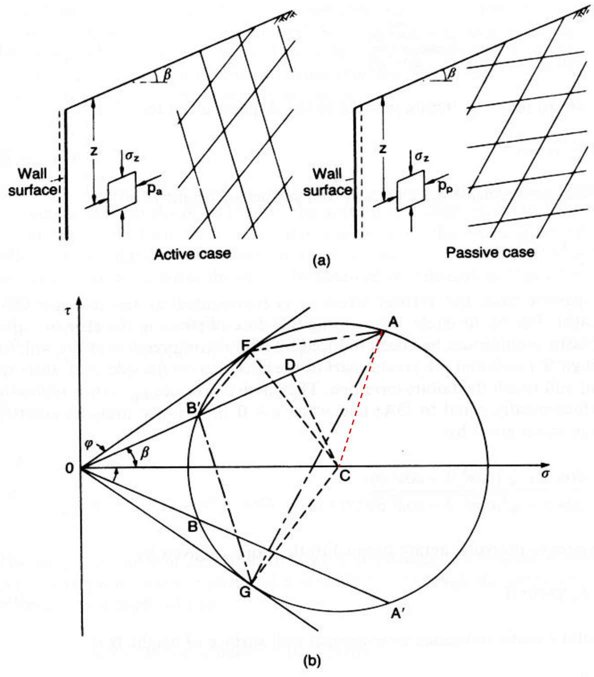


Figure 6.8 Active and passive states for sloping surface.

$$K_a = \frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi)}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi)}} \tag{6.8}$$

Thus the active pressure, acting parallel to the slope, is given by

$$p_a = K_a \gamma z \cos\beta \tag{6.9}$$

and the total active thrust on a vertical wall surface of height H is

$$P_a = \frac{1}{2} K_a \gamma H^2 \cos\beta \tag{6.10}$$

$$K_p = \frac{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi)}}{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi)}} \tag{6.11}$$

Then the passive pressure, acting parallel to the slope, is given by

$$p_p = K_p \gamma z \cos\beta \tag{6.12}$$

and the total passive resistance on a vertical wall surface of height H is

$$P_p = \frac{1}{2} K_p \gamma H^2 \cos\beta \tag{6.13}$$

Example 6.3

A vertical wall 6 m high, above the water table, retains a 20° soil slope, the retained soil having a unit weight of 18 kN/m^3; the appropriate shear strength parameters are $c' = 0$ and $\phi' = 40°$. Determine the total active thrust on the wall and the directions of the two sets of failure planes relative to the horizontal.

Solution:

$$K_a = \frac{\cos 20° - \sqrt{(\cos^2 20° - \cos^2 40°)}}{\cos 20° + \sqrt{(\cos^2 20° - \cos^2 40°)}} = 0.265$$

$$P_a = \frac{1}{2} K_a \gamma H^2 \cos\beta$$

$$= \frac{1}{2} \times 0.265 \times 18 \times 6^2 \times 0.940 = 81 \text{ kN/m}$$

horizontal. At a depth of 6 m,

$$\sigma_z = \gamma z \cos\beta = 18 \times 6 \times 0.940 = 102 \text{ kN/m}^2$$

and this stress is set off to scale (distance OA) along the 20° line. The Mohr circle is then drawn as in Figure 6.9 and the active pressure (distance OB or OB′) is scaled from the diagram, i.e.

$$p_a = 27 \text{ kN/m}^2$$

$$P_a = \frac{1}{2} p_a H = \frac{1}{2} \times 27 \times 6 = 81 \text{ kN/m}$$

The failure planes are parallel to B′F and B′G in Figure 6.9. The directions of these lines are measured as 59° and 71°, respectively, to the horizontal (adding up to 90° + ϕ).

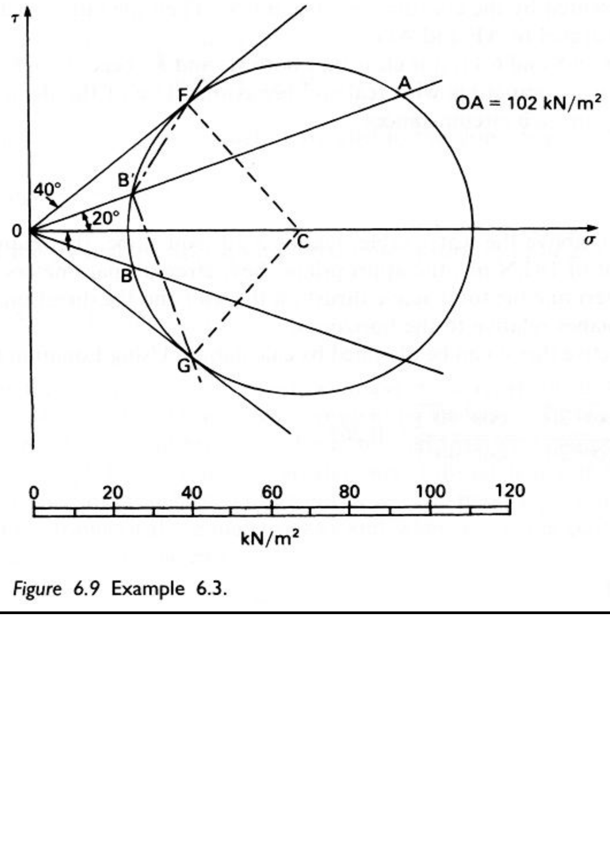


Figure 6.9 Example 6.3.

Earth pressure at rest (no lateral move) (no failure)

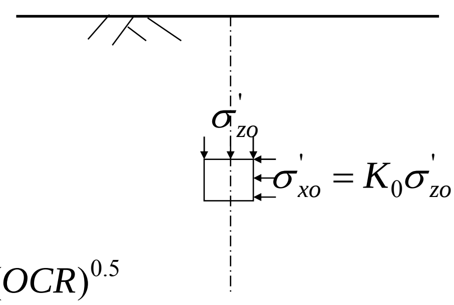
$$K_0 = \frac{\sigma'_{xo}}{\sigma'_{zo}} = \frac{p'_{ho}}{p'_{vo}}$$

Jaky: $K_0 = 1 - \sin\phi'$

Mayne and Kulhawy:

$$K_0 = (1 - \sin\phi')(OCR)^{\sin\phi'}$$

Eurocode 7: $K_0 = (1 - \sin\phi')(OCR)^{0.5}$



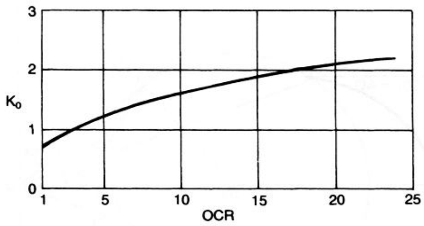


Figure 6.11 Typical relationship between K_0 and overconsolidation ratio for a clay.

Table 6.2 Coefficient of earth pressure at-rest

Soil	K_0
Dense sand	0.35
Loose sand	0.6
Normally consolidated clays (Norway)	0.5–0.6
Clay, OCR = 3.5 (London)	1.0
Clay, OCR = 20 (London)	2.8

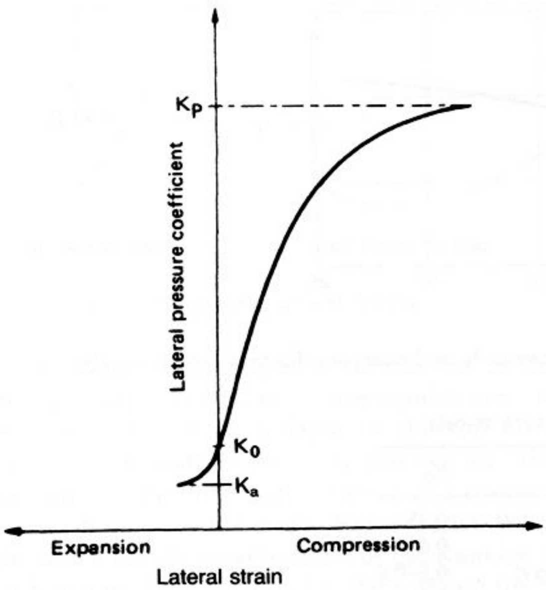


Figure 6.10 Relationship between lateral strain and lateral pressure coefficient.

6.3 Coulomb's theory of earth pressure

Rankine theory has limitations: wall must be vertical and perfect smooth (a lower bound approach – stress equilibrium).

Coulomb theory overcomes the limitations: wall may be inclined and not perfect smooth (δ>0) (an upper bound approach – force equilibrium). This theory has limitations: the passive pressure may be under-estimated.

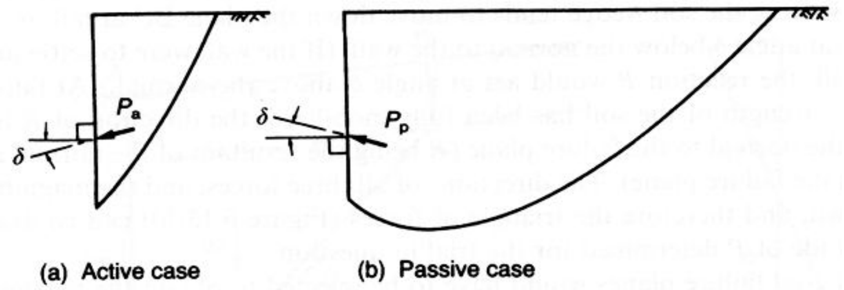


Figure 6.12 Curvature due to wall friction.

Coulomb active case and active earth pressure:

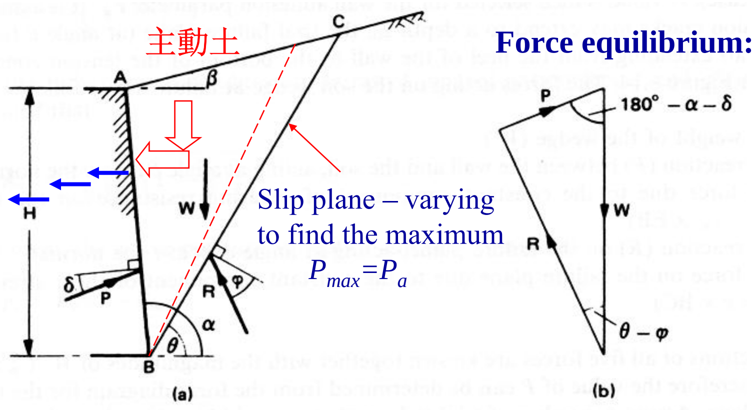


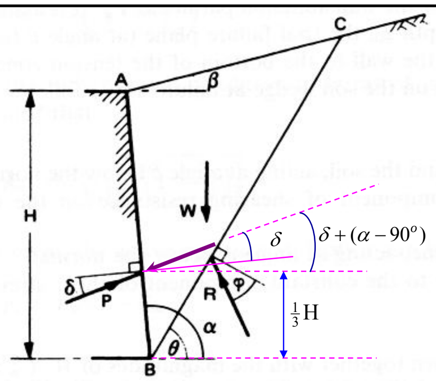
Figure 6.13 Coulomb theory: active case with $c = 0$.

Cohesion is zero.

Active earth pressure force – direction !

主動総土壓力 – 注意力的方向

Attention to:
Active force direction
and position !
主動総土壓力的方向和位置 ！



$$P_a = \frac{1}{2} K_a \gamma H^2 \qquad (6.16)$$

where

$$K_a = \left(\frac{\dfrac{\sin(\alpha - \phi)}{\sin\alpha}}{\sqrt{[\sin(\alpha+\delta)]} + \sqrt{\left[\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\beta)}\right]}} \right)^2 \qquad (6.17)$$

Extension of Coulomb active earth pressure theory:

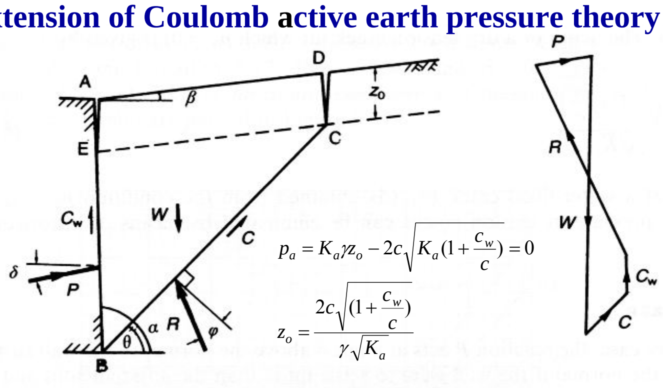


Figure 6.14 Coulomb theory: active case with c > 0.

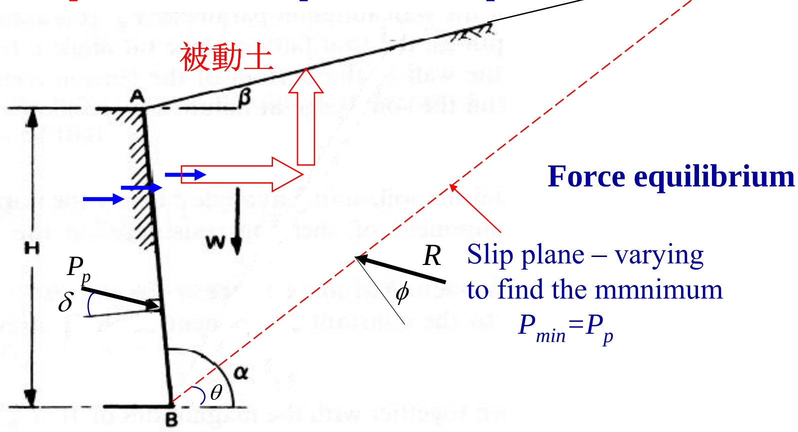
Consider force equilibrium and vary slip plane to find the maximum $P_{max}=P_a$

Coulomb theory: $p_a = K_a\sigma_z - K_{ac}c$; $K_{ac} = 2\sqrt{K_a(1+\frac{c_w}{c})}$

Rankine theory: $p_a = K_a\sigma_z - 2\sqrt{K_a}c$

Calculate p_a at key points, plot distribution, and calculate areas for P_a

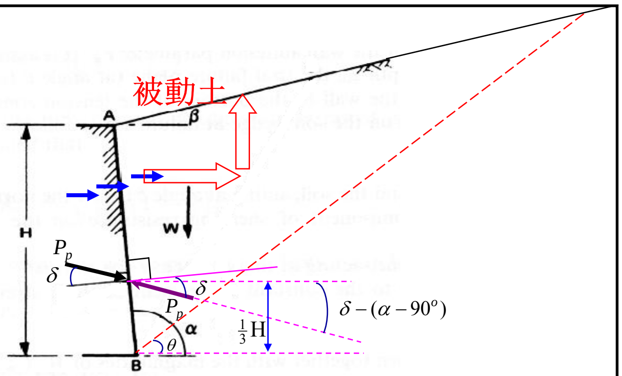
Coulomb passive case and passive earth pressure:



Cohesion is zero.
Passive earth pressure force – direction !
被動総土壓力 – 注意力的方向

Attention to:
Passive force direction
and position !
被動総土壓力的方向
和位置 ！

$$P_p = \frac{1}{2} K_p \gamma H^2$$



$$K_p = \left(\frac{\frac{\sin(\alpha+\phi)}{\sin\alpha}}{\sqrt{[\sin(\alpha-\delta)]} - \sqrt{\left[\frac{\sin(\phi+\delta)\sin(\phi+\beta)}{\sin(\alpha-\beta)}\right]}} \right)^2$$

Coulomb theory: $p_p = K_p \sigma_z + K_{pc} c; \quad K_{pc} = 2\sqrt{K_p (1+\frac{c_w}{c})}$

Rankine theory: $p_p = K_p \sigma_z - 2\sqrt{K_p} c$

Calculate p_p at key points, plot distribution, and calculate areas for P_p

Kerisel and Absi's calculation of active and passive earth pressures (more accurate):

$$\alpha = 90^o;\ \beta = 0$$

$$K_{ah} = K_a con\delta$$
$$K_{ph} = K_p con\delta$$

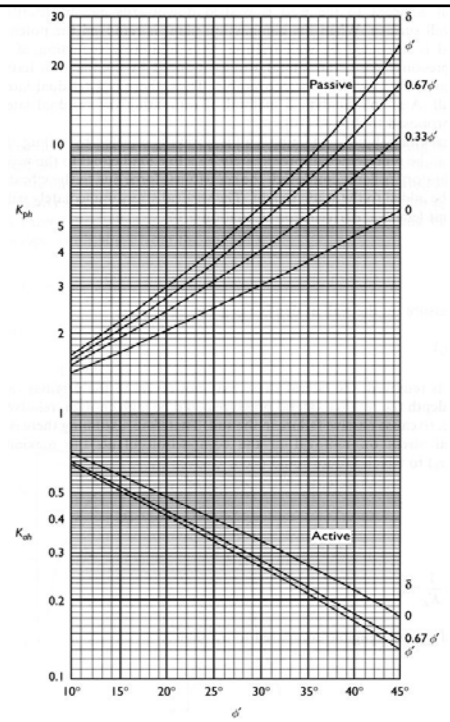
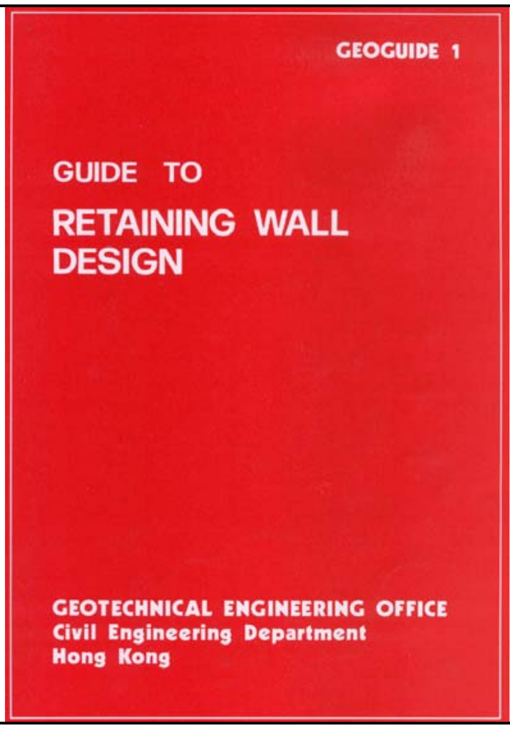
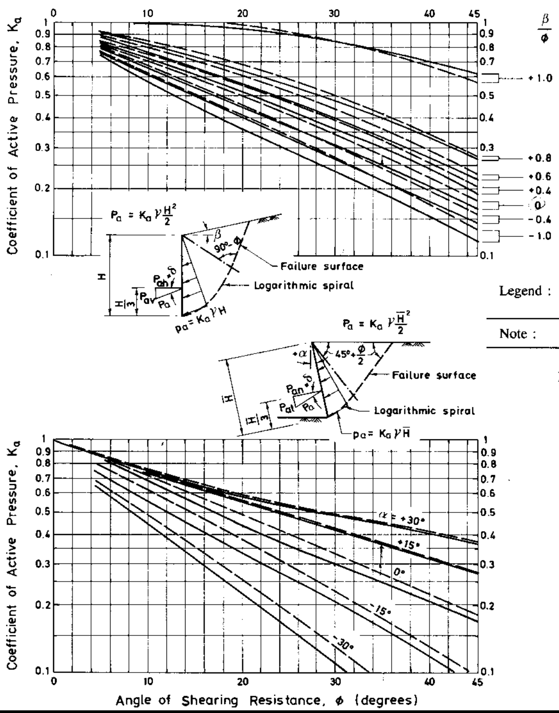


Figure 6.15 Coefficients for horizontal components of active and passive pressure.

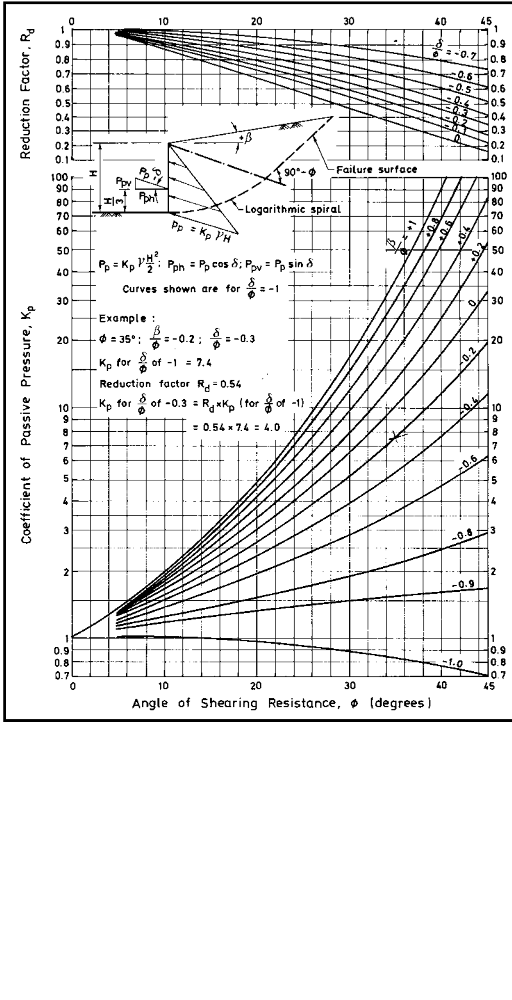




Legend : ——— $\frac{\delta}{\phi} = \frac{2}{3}$ – – – – $\frac{\delta}{\phi} = 0$

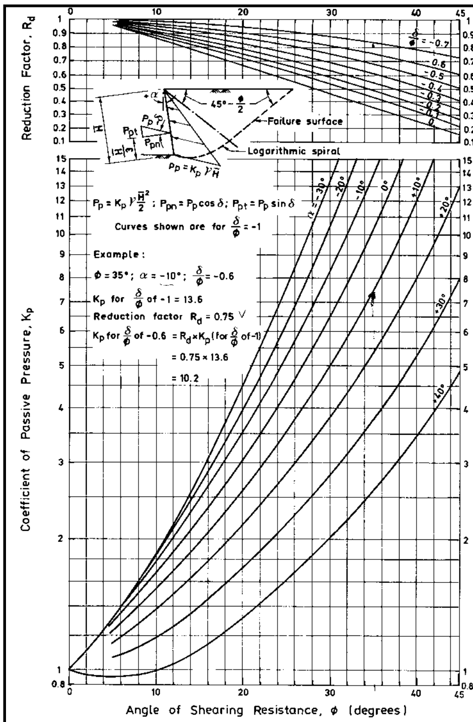
Note : Figure based on NAVFAC (1982b) and Caquot & Kerisel (1948).

Figure 18 - Active Earth Pressure Coefficients for Different Wall Configurations and Retained Slope Angles



Note : Figure based on NAVFAC (1982b) and Caquot & Kerisel (1948).

Figure 19 - Passive Earth Pressure Coefficients for a Vertical Wall Retaining Sloping Ground



Note : Figure based on NAVFAC (1982b) and Caquot & Kerisel (1948).

Figure 20 - Passive Earth Pressure Coefficients for a Sloping Wall Retaining Horizontal Ground

6.4 Application of earth pressure theory to retaining walls

The earth pressure is active, at-rest, or passive?

- Consider possible failure modes and make good engineering judgement !

Big force from an arched bridge
Move right: passive

No support
Move left:
active

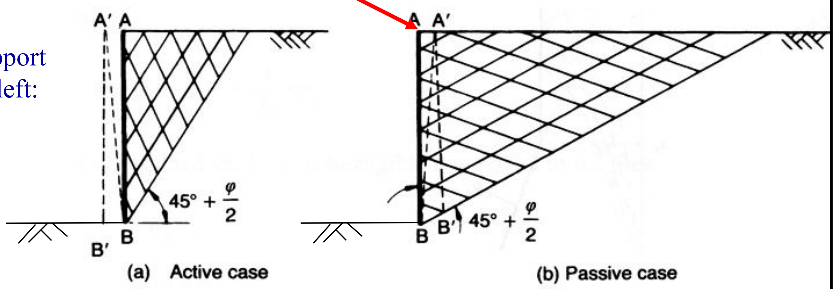


Figure 6.17 Minimum deformation conditions.

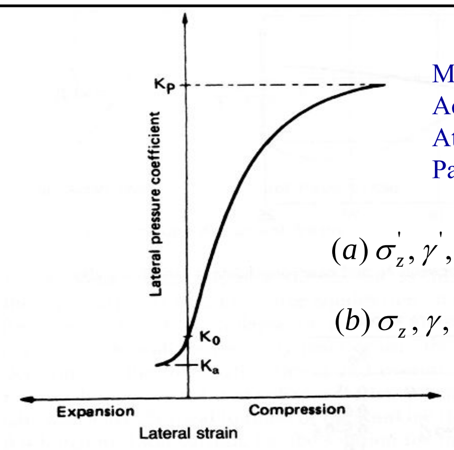


Figure 6.10 Relationship between lateral strain and lateral pressure coefficient.

Mobilization of pressures:
Active pressure: small displacement
At-resent pressure: zero displacement
Passive pressure: large displacement

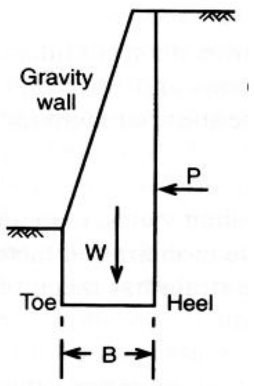
$(a)\ \sigma_z^{'}, \gamma^{'}, c^{'}, \phi^{'}; \quad p_a^{'}(or\ p_p^{'}) + u; \quad P_a^{'} + P_W$

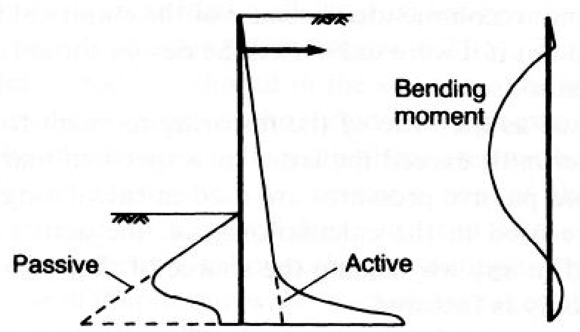
$(b)\ \sigma_z, \gamma, c, \phi; \quad p_a(or\ p_p); \quad P_a$

Soil parameters: appropriate selection of strength parameters
Use consistent soil parameters:
(a) Using effective stresses and effective stress parameters
(b) Using total stresses and total stress parameters
(c) No mix up !

6.5 Design of earth-retaining structures

Retaining structures:
(a) Gravity or freestanding walls
(b) Embedded walls (like sheet piles retained excavations)





Design methods :

(a) *Classic approach using* lumped factors (factor of safety *F*)
F shall be large enough to cover uncertainties.

(b) The *limit state approach* using partial factors (in Eurocode 7 and other codes)

- Limit states : *Ultimate limit state* and *Serviceability limit states*

(b) The *limit state* method for retaining wall design - Eurocode 7:

Three design cases for retaining walls:

Case A: is relevant to the overturning of the wall.

Case B: is relevant to the structural design of the wall.

Case C: is primarily concerned with uncertainties in soil properties. Therefore partial factors greater than 1 are applied to relevant soil parameters: 1.60 for c' ($c'_d=c'/1.60$); 1.25 for $\tan\phi'$ ($\phi'_d=\tan^{-1}(\tan\phi'/1.25)$); 1.40 for c_u ($c_{ud}=c_u/1.40$). Load factor 1.00 for all permanent action (load) and 1.30 for variable action (load)

Settlement: all partial factors are 1.

6.6 Gravity walls

δ=0, wall vertical – using Rankine theory

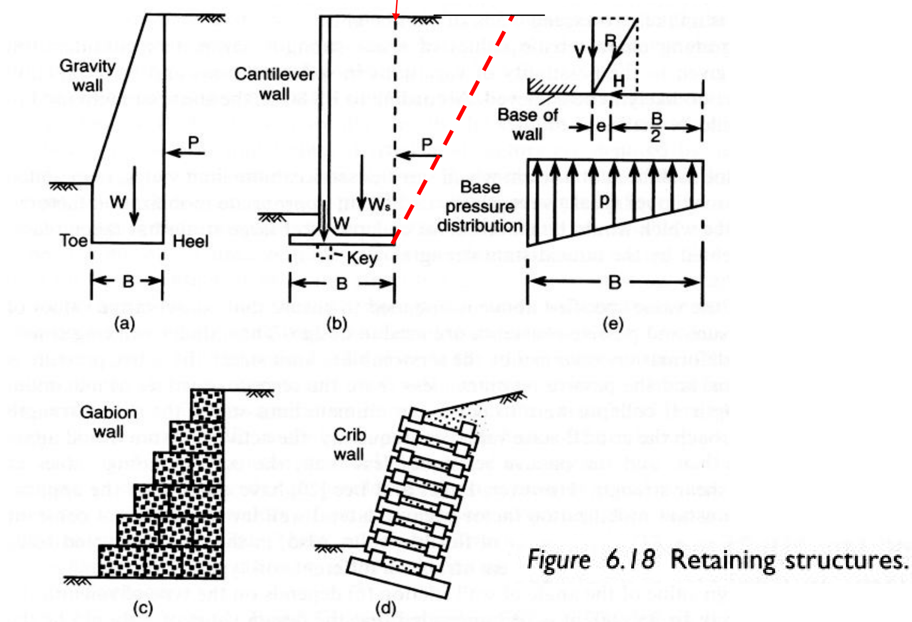


Figure 6.18 Retaining structures.

Design considerations of following limit states:

(1) Overturning failure of the wall at the right "point" (toe or heel?)

(2) Sliding failure at the base of the wall

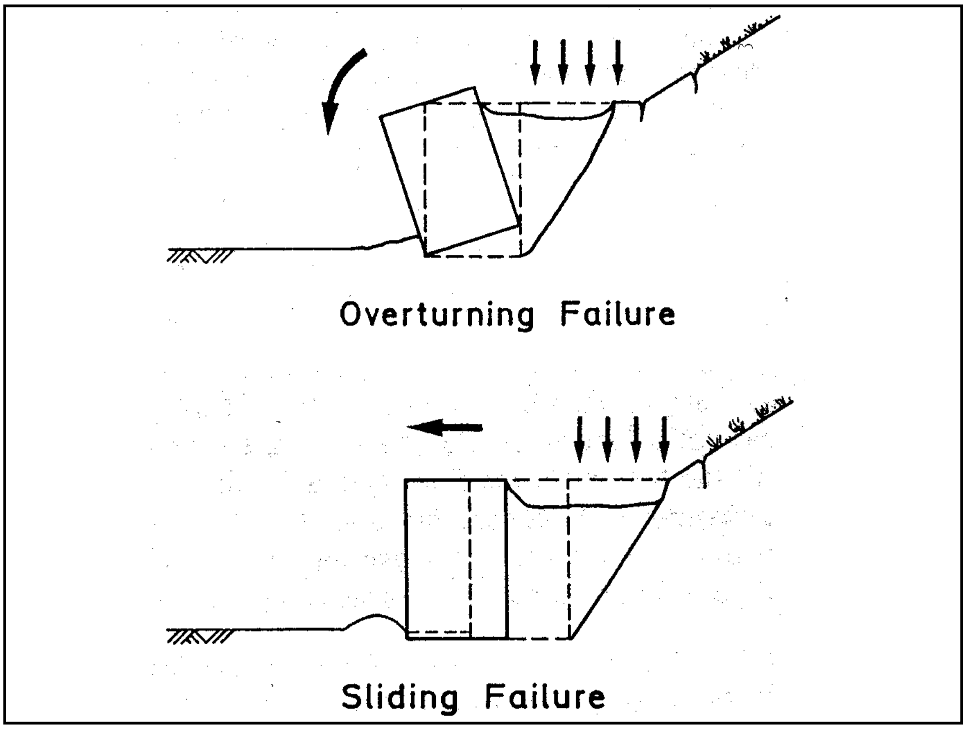
(3) Bearing capacity failure – the maximum base pressure < bearing capacity pressure (Chapter 8)

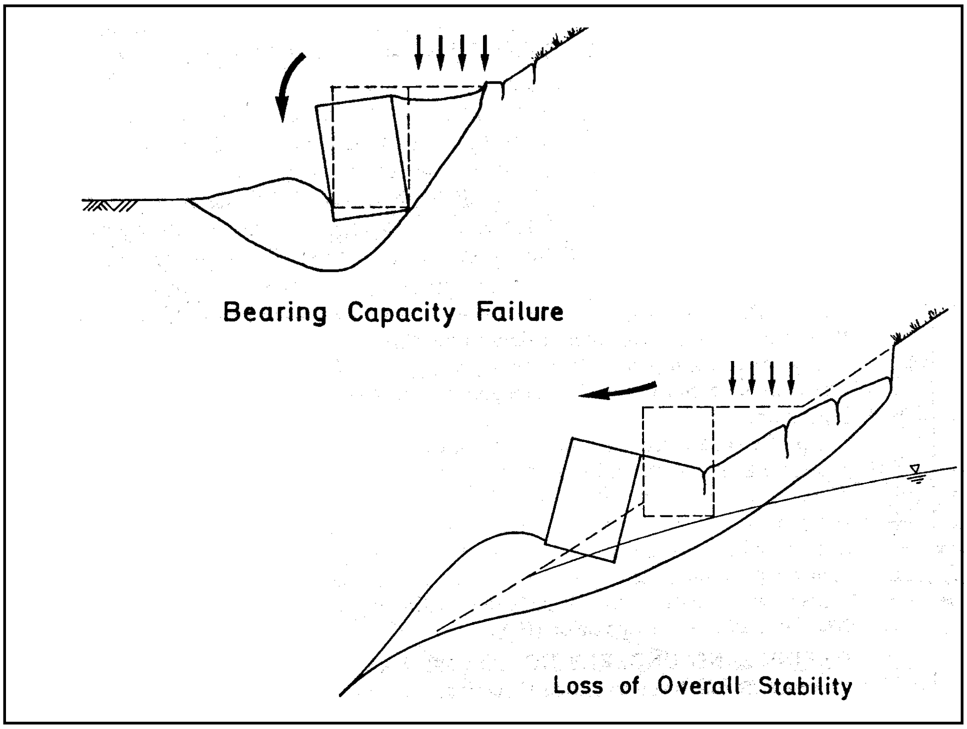
(4) Overall failure of the wall or deep slip (Chapter 9)

(5) Excessive soil and wall deformation causing problems near-by (Chapter 5)

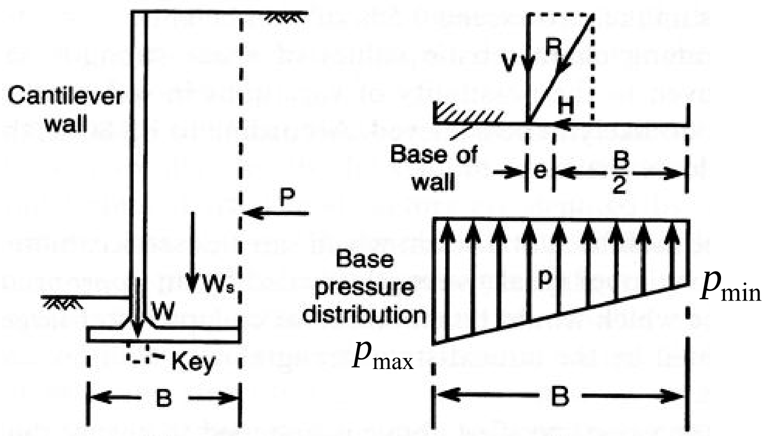
(6) Adverse seepage effects and internal erosion or leakage, failure of drainage system

(7) Structural failure of any element


Overturning Failure
Sliding Failure

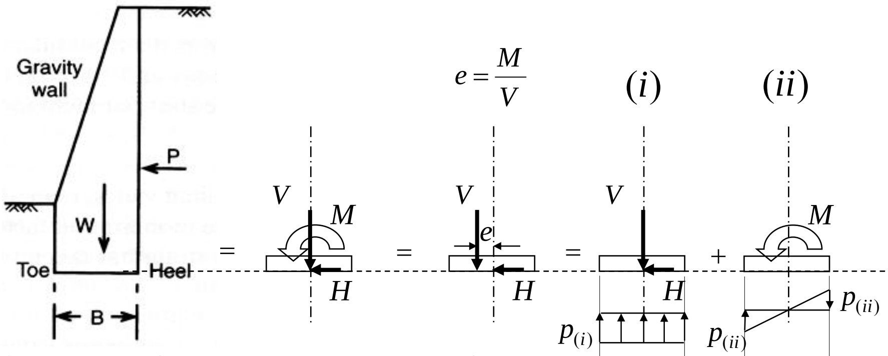

Bearing Capacity Failure
Loss of Overall Stability

Calculation of wall base pressure:



$$p = \frac{V}{B}(1 \pm \frac{6e}{B})$$

Calculation of wall base pressure:



$$\frac{1}{2} p_{(ii)} \times \frac{B}{2} \times \frac{2}{3} \times \frac{B}{2} \times 2 = M; \Rightarrow p_{(ii)} = \frac{6M}{B^2}$$

$$p = p_{(i)} + p_{(ii)} = \frac{V}{B} + \frac{6M}{B^2} = \frac{V}{B}(1 \pm \frac{6M}{BV}) = \frac{V}{B}(1 \pm \frac{6e}{B})$$

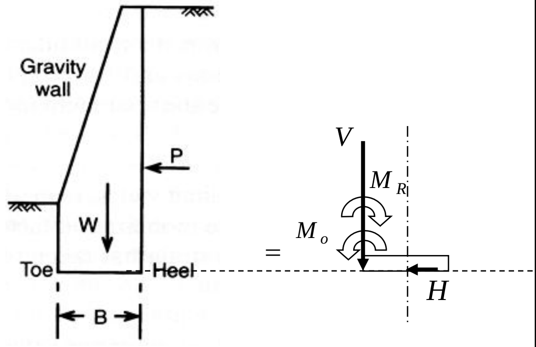
$$p_{min} = \frac{V}{B}(1 \pm \frac{6e}{B}) = 0; \Rightarrow e = \frac{B}{6}$$

Valid for *e*>*B*/6 only

Calculation of factor of safety against overturning:

$$F_{overturning} = F = \frac{M_R}{M_o}$$

F>2 in Hong Kong



Calculation of factor of safety against sliding:

$$F_{sliding} = F = \frac{\sum R}{\sum H} = \frac{B(c + \bar{\sigma}_n \tan\delta)}{H} = \frac{Bc + V\tan\delta}{H} \overset{c=0}{=} \frac{V\tan\delta}{H}$$

F>1.4 in Hong Kong

Example 6.4

Details of a cantilever retaining wall are shown in Figure 6.19, the water table being below the base of the wall. The unit weight of the backfill is 17 kN/m^3 and a surcharge pressure of 10 kN/m^2 acts on the surface. Characteristic values of the shear strength parameters for the backfill are $c' = 0$ and $\phi' = 36°$. The angle of friction between the base and the foundation soil is 27° (i.e. $0.75\phi'$). Is the design of the wall satisfactory according to (a) the traditional approach and (b) the limit state (EC7) approach?

The position of the base reaction is determined by calculating the moments of all forces about the toe of the wall, the unit weight of concrete being taken as 23.5 kN/m^3. The active thrust is calculated on the vertical plane through the heel of the wall, thus $\delta = 0$ and the Rankine value of K_a is appropriate.

Solution:

(a) For $\phi' = 36°$ and $\delta = 0$, $K_a = 0.26$.

$$F_{overturning} = F = \frac{M_R}{M_o} = \frac{M_V}{M_H} = \frac{397.3}{153.8} = 2.58 > 2.0 \quad ok$$

$$l = \frac{\sum M}{V} = \frac{243.5}{212.3} = 1.15m; \quad e = \frac{B}{2} - l = 1.5 - 1.15 = 0.35m < \frac{B}{6} = 0.5m$$

$$p = \frac{V}{B}(1 \pm \frac{6e}{B}) = \frac{212.3}{3.0}(1 \pm \frac{6 \times 0.35}{3.0}) = \begin{cases} p_{max} = 120kN/m^2 \\ p_{min} = 21kN/m^2 \end{cases}$$

$$F_{sliding} = F = \frac{\sum R}{\sum H} \overset{c=0}{=} \frac{V\tan\delta}{H} = \frac{212.3\tan 27^o}{78.4} = 1.38$$

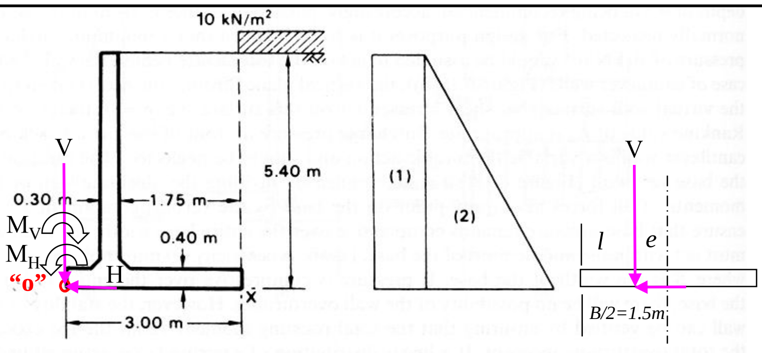


Figure 6.19 Example 6.4.

Table 6.3

"o"

(per m)	Force (kN)		Arm (m)	Moment (kN m)
(1)	$0.26 \times 10 \times 5.40$	$= 14.0$	2.70	37.9
(2)	$\frac{1}{2} \times 0.26 \times 17 \times 5.40^2$	$= 64.4$	1.80	115.9
		$H = 78.4$		$M_H = 153.8$
(Stem)	$5.00 \times 0.30 \times 23.5$	$= 35.3$	1.10	38.8
(Base)	$0.40 \times 3.00 \times 23.5$	$= 28.2$	1.50	42.3
(Soil)	$5.00 \times 1.75 \times 17$	$= 148.8$	2.125	316.2
		$V = 212.3$		$M_V = 397.3$
				153.8
				$\Sigma M = 243.5$

(b) Limit state design approach

Case C: Factors of 1.25, 1.60 and 1.4 for ϕ', c', and c_u.

Load factor 1.3 for variable load, and 1.0 for permanent load

$$\phi_d' = \tan^{-1}(\tan 36/1.25) = 30^o; \quad K_a = 0.33; \quad \delta_d = 0.75 \times 30 = 22.5^o$$

$$H = (0.33 \times 10 \times 5.40 \times 1.30) + \left(\frac{1}{2} \times 0.33 \times 17 \times 5.40^2\right)$$

$$= 23.2 + 81.8 = 105.0\,\text{kN}$$

$$M_H = (23.2 \times 2.70) + (81.8 \times 1.80) = 209.9\,\text{kN m}$$

$$V = 212.3\,\text{kN (as before)}$$

$$M_V = 397.3\,\text{kN m (as before)}$$

$$\Sigma M = 187.4\,\text{kN m}$$

$$M_H = 209.0 kNm/m < M_V = 397.3 kNm/m \quad ok$$

$$l = \frac{\sum M}{V} = \frac{187.4}{212.3} = 0.88m; \quad e = \frac{B}{2} - l = 1.5 - 0.88 = 0.62m > \frac{B}{6} = 0.5m$$

$$p = \frac{V}{B}(1 \pm \frac{6e}{B}) = \frac{212.3}{3.0}(1 \pm \frac{6 \times 0.66}{3.0}) = \begin{cases} p_{max} = 159 kN/m^2 \\ p_{min} = -17 kN/m^2 \end{cases}$$

$$V \tan\delta_d = 212.3 \tan 22.5 = 88.0 kN/m < H = 105.0 kN/m \quad NO\,ok$$

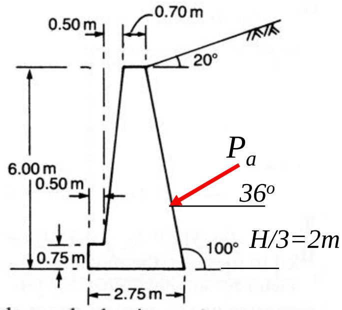
Example 6.5

Details of a gravity-retaining wall are shown in Figure 6.20, the unit weight of the wall material being 23.5 kN/m^3. The unit weight of the backfill is 18 kN/m^3 and *design* values of the shear strength parameters are $c' = 0$ and $\phi' = 33°$. The value of δ between wall and backfill and between wall and foundation soil is 26°. The pressure on the foundation soil should not exceed 250 kN/m^2. Is the design of the wall satisfactory?

Solution:

As the back of the wall and the soil surface are both inclined, the value of K_a will be calculated from Equation 6.17. The values of the angles in this equation are $\alpha = 100°, \beta = 20°, \phi = 33°$ and $\delta = 26°$. Thus,

$$K_a = \left(\frac{\sin 67°/\sin 100°}{\sqrt{\sin 125°} + \sqrt{\sin 58° \sin 13° / \sin 80°}} \right)^2 = 0.48$$



$$P_a = \frac{1}{2} \times 0.48 \times 18 \times 6^2 = 155.5\ \text{kN/m}$$

acting at ⅓ height and at 26° above the normal, or 36° above the horizontal. Moments are considered about the toe of the wall, the calculations being set out in Table 6.4.

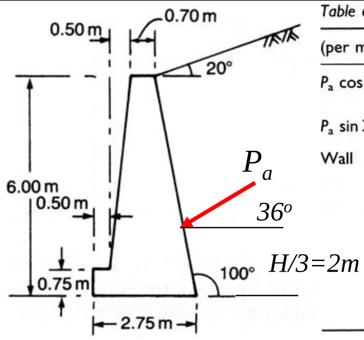


Table 6.4

(per m)	Force (kN)	Arm (m)	Moment (kN m)
$P_a \cos 36°$	= 125.8 H = 125.8	2.00	251.6 M_H = 251.6
$P_a \sin 36°$	= 91.4	2.40	219.4
Wall	$\frac{1}{2} \times 1.05 \times 6 \times 23.5$ = 74.0	2.05	151.7
	$0.70 \times 6 \times 23.5$ = 98.7	1.35	133.2
	$\frac{1}{2} \times 0.50 \times 5.25 \times 23.5$ = 30.8	0.83	25.6
	$1.00 \times 0.75 \times 23.5$ = 17.6 V = 312.5	0.50	8.8 M_V = 538.7 251.6 ΣM = 287.1

$$F_{overturning} = F = \frac{M_R}{M_o} = \frac{M_V}{M_H} = \frac{538.7}{251.6} = 2.14 > 2.0 \quad ok$$

$$l = \frac{\sum M}{V} = \frac{287.1}{312.5} = 0.92m; \quad e = \frac{B}{2} - l = 1.375 - 0.92 = 0.455m < \frac{2.75}{6} = 0.458m$$

$$p = \frac{V}{B}(1 \pm \frac{6e}{B}) = \frac{312.5}{2.75}(1 \pm \frac{6 \times 0.455}{2.75}) = \begin{cases} p_{max} = 226 kN/m^2 \\ p_{min} = 1 kN/m^2 \end{cases}$$

$$F_{sliding} = F = \frac{\sum R_{c=0}}{\sum H} = \frac{V \tan\delta}{H} = \frac{312.5 \tan 26^o}{125.8} = 1.21 < 1.40 \quad no\,ok$$

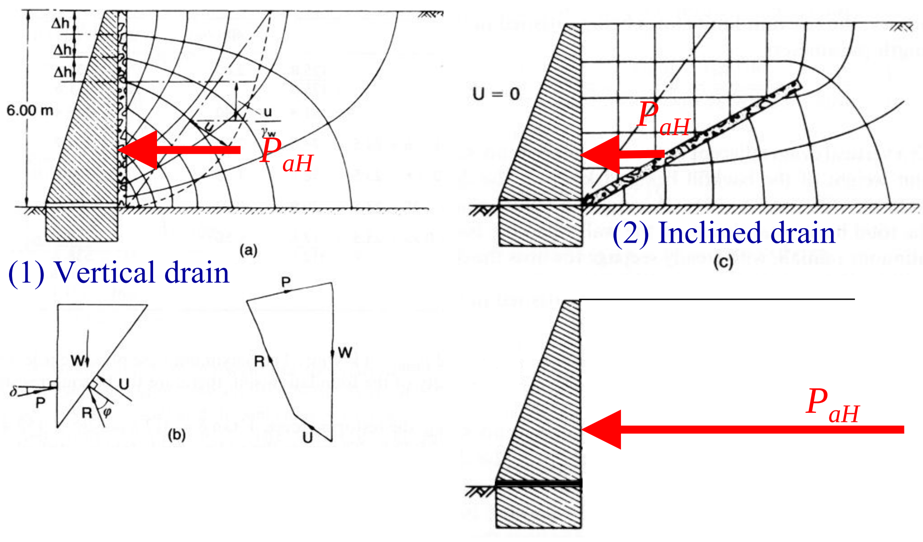
Example 6.6

Details of a retaining structure, with a vertical drain adjacent to the back surface, are shown in Figure 6.21(a), the saturated unit weight of the backfill being 20 kN/m^3. The design parameters for the backfill are $c' = 0$, $\phi' = 38°$ and $\delta = 15°$. Assuming a failure plane at 55° to the horizontal, determine the total horizontal thrust on the wall when the backfill becomes fully saturated due to continuous rainfall, with steady seepage towards the drain. Determine also the thrust on the wall (a) if the vertical drain were replaced by an inclined drain below the failure plane and (b) if there were no drainage system behind the wall.

Solution:

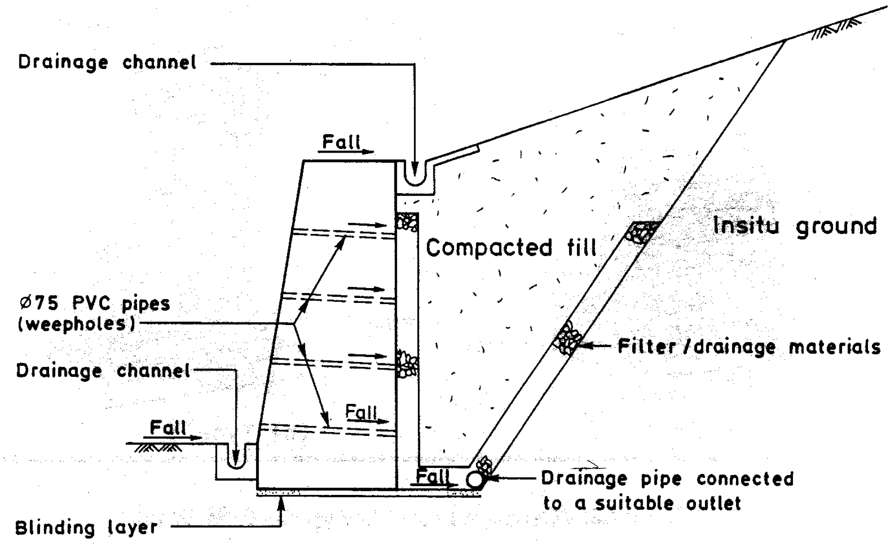
Comparison of thrusts/forces for 3 cases: (1) vertical drain, (2) inclined drain and (3) no drain

	Vertical drain	Inclined drain	No drain
Horizontal force P_{aH}	105 kN/m	76kN/m	215kN/m

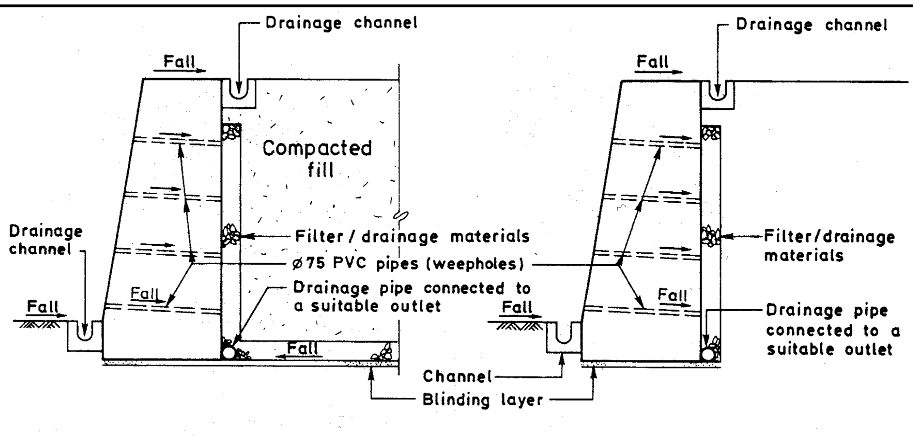


(3) No drain

Design, construction and maintain of good drainage !



(a) Preferred Drainage Scheme A



(b) Preferred Drainage Scheme B

(c) Drainage Scheme C

Notes :

(1) For the preferred drainage scheme A, the extent of the inclined drain is dependent on the design groundwater level behind the retaining wall. To intercept infiltration, the inclined drain should be installed to a level of at least two-thirds of the height of the wall.

(2) The filter/drainage layers may be omitted if a free-draining granular backfill is used. However, a drainage pipe should be provided to discharge water safely.

(3) The vertical and horizontal filter/drainage layers may be replaced by suitable prefabricated drainage composites.

(4) For a retaining wall with level backfill, the top 1.5 m layer of the fill should be a suitable material of relatively low permeability. For sloping backfill, the same provision should be made for a vertical thickness of at least 3 m (see Section 3.7).

PROBLEMS

6.1 The backfill behind a retaining wall, located above the water table, consists of a sand of unit weight $17\,kN/m^3$. The height of the wall is 6 m and the surface of the backfill is horizontal. Determine the total active thrust on the wall according to the Rankine theory if $c' = 0$ and $\phi' = 37°$. If the wall is prevented from yielding, what is the approximate value of the thrust on the wall?

6.2 Plot the distribution of active pressure on the wall surface shown in Figure 6.37. Calculate the total thrust on the wall (active + hydrostatic) and determine its point of application. Assume $\delta = 0$ and $c_w = 0$.

6.3 A line of sheet piling is driven 4 m into a firm clay and retains, on one side, a 3 m depth of fill on top of the clay. Water table level is at the surface of the clay. The unit weight of the fill is $18\,kN/m^3$ and the saturated unit weight of the clay is $20\,kN/m^3$. Calculate the active and passive pressures at the lower end of the sheet piling (a) if $c_u = 50\,kN/m^2$, $c_w = 25\,kN/m^2$ and $\phi_u = \delta = 0$ and (b) if $c' = 10\,kN/m^2$, $c_w = 5\,kN/m^2$, $\phi' = 26°$ and $\delta = 13°$, for the clay.

6.4 Details of a reinforced concrete cantilever retaining wall are shown in Figure 6.38, the unit weight of concrete being $23.5\,kN/m^3$. Due to inadequate drainage the water table has risen to the level indicated. Above the water table the unit weight of the retained soil is $17\,kN/m^3$ and below the water table the saturated unit weight is $20\,kN/m^3$. Characteristic values of the shear strength parameters are $c' = 0$ and $\phi' = 38°$. The angle of friction between the base of the wall and the foundation soil is 25°. (a) Using the traditional approach, determine the maximum and minimum pressures under the base and the factor of safety against sliding. (b) Using the limit state approach, check whether or not the overturning and sliding limit states have been satisfied.

6.6 The section through a gravity retaining wall is shown in Figure 6.39, the unit weight of the wall material being $23.5\,kN/m^3$. The unit weight of the backfill is $19\,kN/m^3$ and design values of the shear strength parameters are $c' = 0$ and $\phi' = 36°$. The value of δ between wall and backfill and between base and foundation soil is 25°. The ultimate bearing capacity of the foundation soil is $250\,kN/m^2$. Use the limit state method to determine if the design of the wall is satisfactory with respect to the overturning, bearing resistance and sliding limit states.

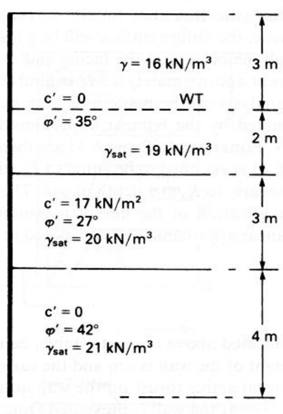


Figure 6.37

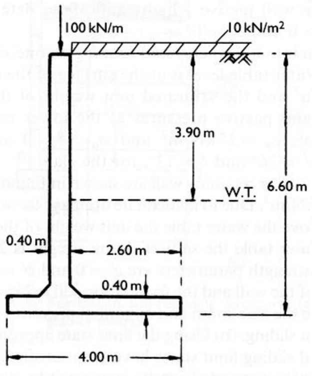


Figure 6.38

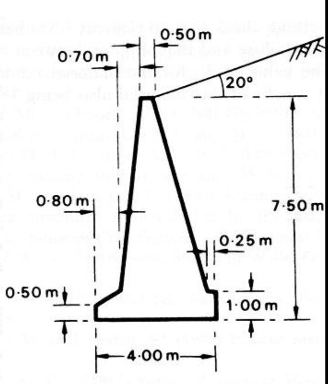


Figure 6.39