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Solution to Problem 1007.

$$PEANUT = BUTTER + BANANA$$

$$\begin{aligned}
P &\equiv 2B + \{0,1\} \quad (\text{mod}10) && (1)\\
E &\equiv U + A + \{0,1\} \quad (\text{mod}10) && (2)\\
A &\equiv T + N + \{0,1\} \quad (\text{mod}10) && (3)\\
N &\equiv T + A + \{0,1\} \quad (\text{mod}10) && (4)\\
U &\equiv E + N + \{0,1\} \quad (\text{mod}10) && (5)\\
T &\equiv R + A \quad (\text{mod}10) && (6)
\end{aligned}$$

Adding (3) and (4) together, we get

$$\begin{aligned}
A + N &\equiv 2T + A + N + \{0,2\}\\
0 &\equiv 2T + \{0,2\}\\
T &\equiv 4 \text{ or } 5
\end{aligned}$$

If $T = 4$, then $10 + U = E + N + \{0,1\}$ and $10 + N = T + A + \{0,1\}$. We can infer that $A \geq 5$ and $N \leq 4$. We can also conclude $10 + T = R + A$ and $R \geq 5$ because $R + A \geq 14$. Then (2) becomes $E \equiv U + A$ and (5) becomes $U \equiv E + N + 1$. Adding these two together gives $N + A + 1 \equiv 0$. From (3), we know $A \equiv N + 5$. Substitute A into the previous equation yields $2N + 6 \equiv 0$, or $N = 2$. But then $A = 7$ and $R = 7$.

If $T = 5$, then $N \geq 6$, $A \leq 4$ and $A + 5 \equiv N$. Then (2) becomes $A + U + 1 \equiv E$ and $N + E \equiv U$. Adding these two to get $N + A + 1 \equiv 0$, which in turn implies $2A + 6 \equiv 0$ or $A = 2$. It follows that $R = 3$ and $N = 7$. Then $E = 1$ and $U = 8$. And finally, $B = 4$ and $P = 9$.

Therefore, $912785 = 485513 + 427272$.

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