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Solution I to Problem 1011.

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{n^2-n-1}{(n+1)!} &= \sum_{n=1}^{\infty} \frac{(n+1)(n-1)-n}{(n+1)!} \\
&= \sum_{n=1}^{\infty} \frac{(n-1)}{n!} - \sum_{n=1}^{\infty} \frac{n}{(n+1)!} \\
&= 0 + \sum_{n=2}^{\infty} \frac{(n-1)}{n!} - \sum_{n=1}^{\infty} \frac{n}{(n+1)!} \\
&= \sum_{n=1}^{\infty} \frac{n}{(n+1)!} - \sum_{n=1}^{\infty} \frac{n}{(n+1)!} \\
&= 0
\end{aligned}$$

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Solution II

We will use the following series

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1 \tag{1}$$

$$\sum_{n=1}^{\infty} \frac{n^k}{n!} = S_k \quad \text{with} \quad S_1 = e \quad \text{and} \quad S_2 = 2e \tag{2}$$

$$\sum_{n=1}^{\infty} \frac{n^2-n-1}{(n+1)!} = \sum_{n=1}^{\infty} \frac{(n+1)^2-2(n+1)-n}{(n+1)!}$$

$$
\begin{aligned}
&= \sum_{n=1}^{\infty} \frac{(n+1)^2}{(n+1)!} - 2\sum_{n=1}^{\infty} \frac{n+1}{(n+1)!} - \sum_{n=1}^{\infty} \frac{n}{(n+1)!} \\
&= (2e-1) - 2(e-1) - 1 \\
&= 0
\end{aligned}
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