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Solution to Problem 1021.

Let $p = \sum_{i=0}^{k} a_i \cdot 10^i$ and $p+2 = \left(\sum_{i=0}^{k} a_i \cdot 10^i\right) + 2$ be a pair of twin primes.

If $a_0 \neq 9$, then $p + 2 = \left(\sum_{i=1}^{k} a_i \cdot 10^i\right) + (a_0 + 2)$. Subsequently, $D(p) = \sum_{i=0}^{k} a_i$ and $D(p+2) = \left(\sum_{i=1}^{k} a_i\right) + (a_0+2) = \left(\sum_{i=0}^{k} a_i\right) + 2$. Obviously $D(p) \neq D(p + 2)$.

If $a_0 = 9$ but $a_1 \neq 9$, then $p + 2 = \left(\sum_{i=2}^{k} a_i \cdot 10^i\right) + (a_1 + 1) \cdot 10 + 1$. We then have $D(p) = \left(\sum_{i=1}^{k} a_i\right) + 9$ and $D(p + 2) = \left(\sum_{i=2}^{k} a_i\right) + (a_1 + 1) + 1 = \left(\sum_{i=1}^{k} a_i\right) + 2$. Again, $D(p) \neq D(p + 2)$.

If $a_0 = 9, a_1 = 9$ but $a_2 \neq 9$, then $p + 2 = \left(\sum_{i=3}^{k} a_i \cdot 10^i\right) + (a_2 + 1) \cdot 10^2 + 0 \cdot 10 + 1$. And we have $D(p) = \left(\sum_{i=2}^{k} a_i\right) + 9 + 9 = \left(\sum_{i=2}^{k} a_i\right) + 18$ and $D(p+2) = \left(\sum_{i=3}^{k} a_i\right) + (a_2+1) + 1 = \left(\sum_{i=2}^{k} a_i\right) + 2$. Again, $D(p) \neq D(p+2)$.

Then by induction, we know $D(p) \neq D(p+2)$ for any pair of twin primes.

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