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Solution to Problem 1022.

Theorem 1 *For any integer $n \geq 3$, $n! \neq m^2$ for any integer m.*

Proof: Bertrand's postulate tells us that for any integer $q > 2$, there is a prime number p such that $q < p < 2q$.

If n is even, then there is a prime p such that $\frac{n}{2} < p < n$. Note that $2p > n$.

If n is odd, then there is a prime p such that $\frac{n+1}{2} < p < n+1$ and again, $2p > n$.

In any case, if $n! = p_0^{e_0} p_1^{e_1} p_2^{e_2} \cdots p_i^{e_i}$, there is a prime p_j with $j \leq i$ such that $2p_j > n$ and therefore $e_j = 1$. And this proves $n! \neq m^2$.

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