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Solution to Problem 1027.

Let $P(X_i)$ denote the probability of the ith jar is green.

All 5 balls are red = the product of the probability of each jar such that the ball is red.

$$\begin{aligned} P(X=0) &= \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &= \frac{1512}{10000} \end{aligned}$$

One ball is green = $P(X_1) + P(X_2) + P(X_3) + P(X_4) + P(X_5)$.

$$\begin{aligned} P(X=1) &= \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &= \frac{3714}{10000} \end{aligned}$$

Two balls are green = $P(X_{1,2}) + P(X_{1,3}) + P(X_{1,4}) + P(X_{1,5}) + P(X_{2,3}) + P(X_{2,4}) + P(X_{2,5}) + P(X_{3,4}) + P(X_{3,5}) + P(X_{4,5})$

$$\begin{aligned} P(X=2) &= \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\ &+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \end{aligned}$$

$$
\begin{aligned}
&+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&= \frac{3274}{10000}
\end{aligned}
$$

Three balls are green $= P(X_{1,2,3})+P(X_{1,2,4})+P(X_{1,2,5})+P(X_{1,3,4})+P(X_{1,3,5})+P(X_{1,4,5})+P(X_{2,3,4})+P(X_{2,3,5})+P(X_{2,4,5})+P(X_{3,4,5})$

$$
\begin{aligned}
P(X=3) &= \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&+ \left(\frac{9}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)\\
&= \frac{1274}{10000}
\end{aligned}
$$

Four balls are green $= P(X_{1,2,3,4}) + P(X_{1,2,3,5}) + P(X_{1,2,4,5}) + P(X_{1,3,4,5}) + P(X_{2,3,4,5})$

$$
P(X=4) = \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right)
$$

$$
\begin{aligned}
&+ \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) \\
&+ \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{7}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right) \\
&+ \left(\frac{1}{10}\right)\left(\frac{8}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right) \\
&+ \left(\frac{9}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right) \\
&= \frac{214}{10000}
\end{aligned}
$$

All five are green $= P(X_{1,2,3,4,5})$

$$
\begin{aligned}
P(X=5) &= \left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\left(\frac{5}{10}\right) \\
&= \frac{12}{10000}
\end{aligned}
$$

Exactly two days of rain $= P(X=2) = \frac{3274}{10000} = \frac{1637}{5000}$.

At least two days of rain $= 1 - P(X=0) - P(X=1) = 1 - \frac{1512}{10000} - \frac{3714}{10000} = \frac{2387}{5000}$.

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