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Solution to Problem 1036.

$$
\begin{aligned}
f(x) &= \prod_{i=0}^{\infty}(1+x^{2^i}) \\
&= (1+x)(1+x^2)(1+x^{2^2})(1+x^{2^4})\cdots \\
&= (1+x+x^2+x^3)(1+x^4)(1+x^8)\cdots \\
&= (1+x+x^2+x^3+x^4+x^5+x^6+x^7)(1+x^8)\cdots \\
&= 1+x+x^2+x^3+x^4+\cdots+x^i+\cdots
\end{aligned}
$$

For $|x|<1$, $1+x+x^2+x^3+x^4+\cdots+x^i+\cdots = \dfrac{1}{1-x}$. Therefore,

$$
\begin{aligned}
\int_0^c f(x)dx = \pi &= \int_0^c \frac{dx}{1-x} \\
\pi &= -\ln(1-x)\Big|_0^c \\
\pi &= -\ln(1-c)+\ln(1) \\
\pi &= -\ln(1-c) \\
c &= 1-e^{-\pi}
\end{aligned}
$$

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