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Solution to Problem 1043.

Part (a) Suppose $1991 = xb^3 + yb^2 + yb + x$, where b is the base, with $0 < x < b$ and $0 \leq y < b$. Then $1991 = 11 \times 181 = (b+1)[x(b^2 - b + 1) + by]$. Just to point out the obvious, $b+1 < x(b^2 - b + 1) + by$. Then it is clear that the only solution to b is $b = 10$. Thus, 1991 cannot be written as a four-digit palindrome in a base other than 10.

Part (b) We have $2002 = 2 \times 7 \times 11 \times 13 = (b+1)[x(b^2 - b + 1) + by]$. Observe that $b^4 > 2002 > b^3$. This implies $7 \leq b \leq 12$. So in this case we may have $b = 10$ or $b = 12$. If $b = 12$, then $(13)[(12^2 - 12 + 1)x + 12y] = 2002$, or $133x + 12y = 154$, which has no positive integer solution. Therefore neither can 2002 be written as a four-digit palindrome in a base other than 10. ■