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Solution to Problem 1046.

*Proof:* Suppose $S_1 = \{x_1, x_2, \ldots, x_n\}$ and $S_k = \{x_1x_2\cdots x_k, x_1x_2\cdots x_{k+1}, \ldots, x_{n-k+1}x_{n-k+2}\cdots x_n\} = \{x|x = x_{j1}x_{j2}\cdots x_{jk}\}$. Let's call $P_k$ the product of all the elements of $S_k$. Then, by symmetry in the set $S_k$, $P_k = (x_1x_2\cdots x_n)^e$, for some exponent $e$. Because there are $\binom{n}{k}$ elements in $S_k$ and each element has $k$ $x_i$'s, then the sum of all the exponents of the elements in $S_k$ is $k\binom{n}{k}$. Since this sum of the exponents is equally distributed among $n$ members, the exponent of each element is $\frac{k}{n}\binom{n}{k}$. Or simply, $e = \frac{k}{n}\binom{n}{k}$. Therefore,

$$G_k = P_k^{\frac{1}{\binom{n}{k}}} = \left[(x_1x_2\cdots x_n)^{\frac{k}{n}\binom{n}{k}}\right]^{\frac{1}{\binom{n}{k}}} = (x_1x_2\cdots x_n)^{\frac{k}{n}}.$$

For $S_1$, there are $n$ elements. So $G_1 = (x_1x_2\cdots x_n)^{\frac{1}{n}}$ and subsequently, $(G_1)^k = (x_1x_2\cdots x_n)^{\frac{k}{n}}$, which shows $G_k = (G_1)^k$.

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