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Solution to Problem 1047.

Substitute $\sec\alpha = \dfrac{1}{\cos\alpha}$ and $\csc\alpha = \dfrac{1}{\sin\alpha}$ into the original expression,

$$\begin{aligned}\sec^2\alpha + \csc^2\alpha + \sec^2\alpha\csc^2\alpha &= \frac{1}{\cos^2\alpha} + \frac{1}{\sin^2\alpha} + \frac{1}{\cos^2\alpha\sin^2\alpha}\\ &= \frac{2}{\cos^2\alpha\sin^2\alpha} \geq 8.\end{aligned}$$

Equivalently, $\frac{1}{4} \geq \cos^2\alpha\sin^2\alpha$ for $0 < \alpha < \pi/2$.

It is obvious that the maximum of $\cos^2\alpha\sin^2\alpha$ is achieved when $\cos^2\alpha = \sin^2\alpha$, or $\alpha = \pi/4$. A little computation shows the maximum is indeed $\frac{1}{4}$. ■