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Solution to Problem 1049.

It is obvious that $F_n^3 + F_{n+1}^3 + F_{n+2}^3 \neq F_{n+2}[2F_n + 2F_{n+1} + F_nF_{n+1}]$ unless $F_n = F_{n+1} = 1$.

Since $F_{n+2} = F_n + F_{n+1}$, a simple substitution gives,

$$\begin{aligned} F_n^3 + F_{n+1}^3 + F_{n+2}^3 &= F_n^3 + F_{n+1}^3 + (F_n + F_{n+1})^3 \\ &= 2F_n^3 + 2F_{n+1}^3 + 3F_n^2F_{n+1} + 3F_nF_{n+1}^2 \\ &= (F_n + F_{n+1})[2F_n^2 + 2F_{n+1}^2 + F_nF_{n+1}] \\ &= F_{n+2}[2F_n^2 + 2F_{n+1}^2 + F_nF_{n+1}] \end{aligned}$$

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