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Solution to Problem 1052.

$$\begin{array}{r} D\ O\ D\ G\ E \\ \underline{G\ R\ E\ A\ T} \\ E\ D\ I\ T\ O\ R \end{array}$$

It is seen immediately that $E = 1$. Since $D + G \equiv D \pmod{10}$, then $G = 9$. Therefore we have the following system of equations in modulo 10

$$\begin{cases} 1 + T & \equiv & R \\ 9 + A + \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} & \equiv & O \\ D + 1 + \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} & \equiv & T \\ O + R + \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} & \equiv & I \\ D + 9 + 1 & \equiv & D \end{cases}$$

Since $T < 9$ and $A \neq 1$, the above system can be refined to

$$\begin{cases} 1 + T & = & R \\ 9 + A & = & 10 + O \\ D + 2 & = & T \\ O + R & = & 10 + I \end{cases}$$

Here I assume $D \neq 8$ because if $D = 8$, then $T = 0$ and $R = 1$, contradicts $E = 1$.

The above system of equations gives $R = 5, 6, 7$ or 8; $T = 4, 5, 6$ or 7 and $D = 2, 3, 4$ or 5.

Now it is a matter of trial and error. But first note that $O \neq 8$ because this forces $A = D = 9$.

If $D = 2$, then $T = 4$ and $R = 5$. Subsequently $O = 6$ or 7, for which $I = 1$ or 2 respectively. Neither one gives a solution because $I = E = 1$ or $I = D = 2$.

If $D = 3$, then $T = 5$ and $R = 6$. Note that $O \neq 4$. Otherwise, $A = T = 5$. Subsequently $O = 7$ and $I = 3$. This is not a solution since $I = D = 3$.

If $D = 4$, then $T = 6$ and $R = 7$. It follows that $O = 3$ or 5. If $O = 3$, then $A = D =4$. And $O = 5$ implies $A = T = 6$.

Finally, if $D = 5$, then $T = 7$ and $R = 8$. Then $O = 2, 3$ or 4. If $O = 2$, then $I = 0$ and $A = 3$. We have a solution. Let's look at the remaining cases. $O = 3$ implies $I = E = 1$. If $O = 4$, then $A = D = 5$.

Therefore the only solution is $(A, D, E, G, I, O, R, T) = (3, 5, 1, 9, 0, 2, 8, 7)$.

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