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Solution to Problem 1058.

Let $|PA| = \alpha$, $|PB| = \beta$, $|PC| = \gamma$, $|PD| = q$, $|PF| = p$, $|PE| = r$, $|DB| = x - a$, $|EC| = y - b$ and $|FA| = z - c$.

From Pythagorean's theorem, we have the following

$$\begin{aligned} \alpha^2 - q^2 &= a^2 \\ \beta^2 - r^2 &= b^2 \\ \gamma^2 - p^2 &= c^2 \end{aligned}$$

$$\begin{aligned} \beta^2 - q^2 &= (x-a)^2 \\ \gamma^2 - r^2 &= (y-b)^2 \\ \alpha^2 - p^2 &= (z-c)^2. \end{aligned}$$

(1) Adding the first group of three equations and the second group shows $a^2 + b^2 + c^2 = (x-a)^2 + (y-b)^2 + (z-c)^2 = \alpha^2 + \beta^2 + \gamma^2 - p^2 - q^2 - r^2$.

(2) If $\triangle ABC$ is equilateral, then $x = y = z$ with perimeter $3x$.

$$\begin{aligned} a^2 + b^2 + c^2 &= (x-a)^2 + (y-b)^2 + (z-c)^2 \\ &= (x-a)^2 + (x-b)^2 + (x-c)^2 \\ &= 3x^2 - 2ax - 2bx - 2cx + a^2 + b^2 + c^2 \end{aligned}$$

$$\begin{aligned} 2x(a+b+c) &= 3x^2 \\ a+b+c &= \tfrac{3x}{2} \end{aligned}$$

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Problem 1058 figure.