1071. *Proposed by Peter A. Lindstrom, Batavia, NY.*
In a beginning calculus course, one finds two Mean Value Theorems:

MVT for the Derivative: If a function $f \in C[a,b]$ and $f \in D(a,b)$, then there exists at least one point $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

MVT for the Definite Integral: If a function $f \in C[a,b]$, then there exists at least one point $d \in (a,b)$ such that $\int_a^b f(x)dx = f(d)(b-a)$.

Does there exist a non-constant function f such that $c = d$ in these two theorems for any given a and b, where $a < b$?

Solution by **Rex H. Wu**, *Brooklyn, NY.*

The answer is yes, there is such a function.

Suppose $g(x) \neq t$ where t is a constant, and $g \in [a,b]$. Let's say $\int_a^b g(x)dx = G(x)|_a^b = G(b) - G(a)$. We also further assume that there is a $c_0 \in (a,b)$ such that

$$g'(c_0) = \frac{g(b) - g(a)}{b - a}$$

and

$$\int_a^b g(x)dx = g(c_0)(b-a) = G(b) - G(a)$$

From the above two equalities, we have

$$(b-a) = \frac{g(b) - g(a)}{g'(c_o)} = \frac{G(b) - G(a)}{g(c_0)}$$

An obvious solution is when $g(x) = g'(x) = \int g(x)dx$. This condition is satisfied by the function $g(x) = e^x$.

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