

**1076.** Proposed by Peter A. Lindstrom, Batavia, NY.

Consider the right triangle  $ABC$ , where  $|AB| = x$ ,  $|BC| = y$  so that  $|AC| = \sqrt{x^2 + y^2}$ . Consider another right triangle  $A'B'C'$  which is congruent to  $ABC$ , where  $A = A'$  and  $B'$  is a point on  $AC$ . Extend  $BC$  and  $A'C'$  so that they intersect at  $D$ . Let  $|CD| = a$  and  $|C'D| = b$ . Express  $a$  and  $b$  in terms of  $x$  and  $y$ .

*Solution by Rex H. Wu, Brooklyn, NY.*

From the double angle formulae, we have

$$\begin{aligned}\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} \\ \frac{a+y}{x} &= \frac{\frac{2y}{x}}{1 - \frac{y^2}{x^2}} \\ \frac{a+y}{x} &= \frac{2xy}{x^2 - y^2} \\ a &= \frac{y(x^2 + y^2)}{x^2 - y^2}\end{aligned}$$

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \frac{a+y}{b + \sqrt{x^2 + y^2}} &= 2 \frac{y}{\sqrt{x^2 + y^2}} \frac{x}{\sqrt{x^2 + y^2}} \\ \frac{\frac{y(x^2 + y^2)}{x^2 - y^2} + y}{b + \sqrt{x^2 + y^2}} &= \frac{2xy}{x^2 + y^2} \\ \frac{\frac{2x^2y}{x^2 - y^2}}{b + \sqrt{x^2 + y^2}} &= \frac{2xy}{x^2 + y^2} \\ \frac{\frac{x}{x^2 - y^2}}{b + \sqrt{x^2 + y^2}} &= \frac{1}{x^2 + y^2} \\ b &= \frac{x(x^2 + y^2)}{x^2 - y^2} - \sqrt{x^2 + y^2}\end{aligned}$$

■