1083. *Proposed by Brian Smith, Utica, MN.*

Let N denote a positive integer. Find the number of solutions of $a+b+c=N$ with $a<b<c$, where a, b and c are positive integers.

Solution by **Rex H. Wu**, *Brooklyn, NY.*

This is equivalent to looking for the number of partitions of the integer N into three distinct parts.

Define $P_3(N)$ as the number of partitions of N into three distinct parts. A simple calculation $P_3(N)$ gives $0,0,0,0,0,1,1,2,3,4,5,7,8,10,\ldots$ for $N=1,2,3,\ldots$

It is well-known that the generating function for $P_3(N)$ is $\frac{1}{(1-x)(1-x^2)(1-x^3)}$.

For our question, arrange the values of $P_3(N)$ in columns of 6 with respect to $N \pmod 6$,

0	0	0	0	0	1
1	2	3	4	5	7
8	10	12	14	16	19
21	24	27	30	33	37
40	44	48	52	56	61
65	70	75	80	85	91
		⋮			

Going down each column and using the method of finite differences, the general formulae are

$$P_3(N) = \begin{cases} 3i^2-2i, & N=1+6i \\ 3i^2-i, & N=2+6i \\ 3i^2, & N=3+6i \\ 3i^2+i, & N=4+6i \\ 3i^2+2i, & N=5+6i \\ 3i^2+3i+1, & N=6+6i \end{cases}$$

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