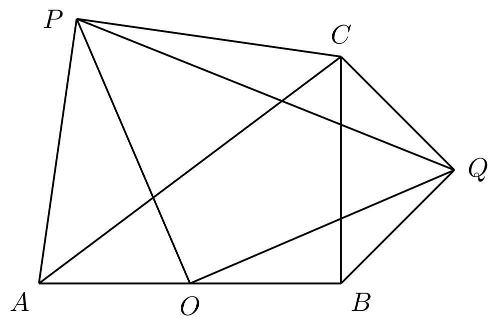
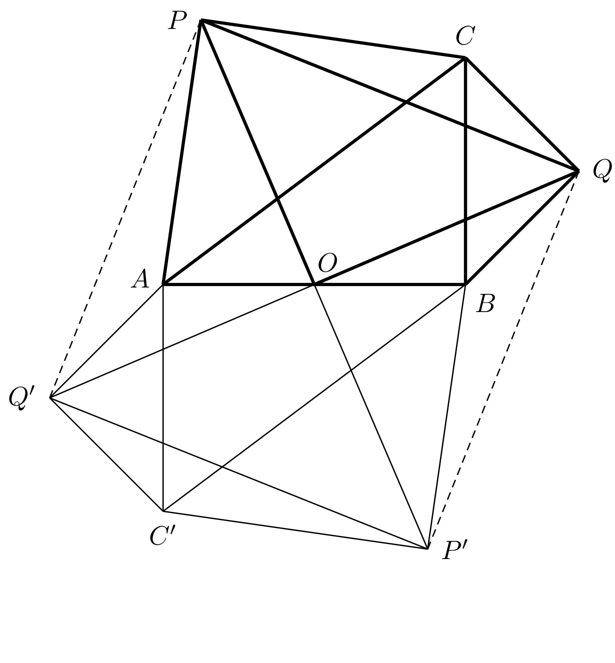
1084. *Proposed by Ayoub B. Ayoub, Pennsylvania State University, Abington College, Abington, PA.*

Given triangle ABC, isosceles right triangles ACP and CBQ are constructed, external to $\triangle ABC$, with right angles at P and Q. Prove that if O is the midpoint of AB, then $\angle POQ$ is also a right angle and $\triangle POQ$ is isosceles.



Solution by **Rex H. Wu**, *Brooklyn, NY.*

Rotate the entire figure 180° around point O. Now points P, C and Q become P', C' and Q' respectively. Connect points P' and Q, P and Q'.



Since $PO = P'O$ and $QO = Q'O$, the quadrilateral $PQP'Q'$ is a parallelogram. Also note that $ACBC'$ is a parallelogram.

$\angle Q'C'P' = \angle AC'B + 90°$. $\angle QBP' = 360° - 90° - \angle CBC' = 360° - 90° - (180° - \angle AC'B) = \angle AC'B + 90°$. A $90°$ rotation clockwise around point P' would superimpose $\triangle Q'C'P'$ onto $\triangle QBP'$. Therefore, $Q'P' = QP'$ and $\angle QP'Q' = 90°$.

Therefore, quadrilateral $PQP'Q'$ is a square. Consequently, $\triangle POQ$, formed by the diagonals of $PQP'Q'$, is a right triangle.

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