1092. *Proposed by the editors.*

A relation R on a set A is a set of ordered pairs of member of A. That is R is a relation on A if $R \subset A \times A$. A relation R is called *transitive* if

$$\forall x, y, z \in A, xRy \text{ and } yRz \Rightarrow xRz.$$

Let $A = \{1, 2, 3\}$. There are $2^9 = 512$ relations on A. How many of these relations are transitive?

Solution by **Rex H. Wu**, *Brooklyn, NY.*

There is no easy way for computing the number of transitive relations. Klaška gave a formula in [**1**] that is dependent on posets.

$$T(n) = \sum_{k=1}^{n} \left(\sum_{s=0}^{k} \binom{n}{s} S(n-s, k-s) \right) P(k)$$

where $\binom{m}{n} = m!/n!(m-n)!$, $S(m,n)$ is the Stirling number of the second kind and $P(k)$ is the number of partially ordered sets in the set A of k elements.

Let $N_n(k) = \sum_{s=0}^{k} \binom{n}{s} S(n-s, k-s)$. Then $N_3(1) = 1$, $N_3(2) = 6$ and $N_3(3) = 8$. We also know $P(1) = 1$, $P(2) = 3$ and $P(3) = 19$. By the above formula, $T(3) = 1 \times 1 + 6 \times 3 + 8 \times 19 = 171$.

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References

1. J. Klaška. Transitivity and Partial Order. *Mathematica Bohemica* 122(1997), 75-82.
2. G. Pfeiffer. Counting Transitive Relations, *Journal of Integer Sequences* 7(2004), 11pp.