**1112.** *Proposed by Harry Sedinger, St. Bonaventure University, St. Bonaventure, NY*

Without using a calculator or computer, determine whether or not the positive integer $9753^{2468} + 3579^{8642} + 9357^{2468} + 9573^{8642}$ is a perfect square.

*Solution by* **Rex H. Wu**, *Brooklyn, NY.*

Let $\mathcal{A} = 9753^{2468} + 3579^{8642} + 9357^{2468} + 9573^{8642}$.

Since the square of any integer must end with the digit $0, 1, 4, 5, 6$, or $9$ (i.e. $n^2 \equiv 0, 1, 4, 5, 6$ or $9 \pmod{10}$), if $\mathcal{A}$ does not end with one of these digits, then it is not a perfect square.

Look at $9753^{2468}$. The last digit is 3. Simple calculation shows $3^0 \equiv 1 \pmod{10}$, $3^1 \equiv 3 \pmod{10}$, $3^2 \equiv 9 \pmod{10}$, $3^3 \equiv 7 \pmod{10}$. And the cycle repeats after this, $3^4 \equiv 1 \pmod{10}$. Since the exponent $2468 \equiv 0 \pmod{4}$, the last digit of $9753^{2468}$ is 1, $9753^{2468} \equiv 1 \pmod{10}$.

For $3579^{8642}$, the unit digit is 9 which has a cycle of 2. Then $3579^{8642} \equiv 1 \pmod{10}$.

Similarly, $9357^{2468} \equiv 1 \pmod{10}$ and $9573^{8642} \equiv 9 \pmod{10}$.

Since $\mathcal{A} = 9753^{2468} + 3579^{8642} + 9357^{2468} + 9573^{8642} \equiv 1 + 1 + 1 + 9 \equiv 2 \pmod{10}$, $\mathcal{A}$ cannot be a square.

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