1115. *Proposed by Nick Hobson, London, England*

Is the number 256128000000000000000000001 prime or composite? No calculators allowed, of course.

Solution by **Rex H. Wu**, *Brooklyn, NY.*

Let $\mathcal{A} = 256128000000000000000000001$.

$$\begin{aligned} 256128000000000000000000001 &= 2^8 \times 10^{24} + 2^7 \times 10^{21} + 1 \\ &= 2001 \times 2^7 \times 10^{21} + 1 \end{aligned}$$

Next, if we can decompose 2001 into two numbers m and n such that $2001 = m + n$, $m = pq$ and $n = p + q$ with $m, n, p, q \in \mathbb{N}$, then we can factor 256128000000000000000000001, namely, in the form $\mathcal{A} = m \times 2^7 \times 10^{21} + n \times 2^7 \times 10^{21} + 1 = (p \times 2^7 \times 10^{21} + 1)(q \times 2^7 \times 10^{21} + 1)$.

This is to say if $2001 = m + n = pq + p + q = p(q + 1) + q$ has a solution in $\mathbb{N}$, then $\mathcal{A}$ is composite . Rearranging this expression, we have $p = (2001 - q)/(q + 1)$. By trial and error, $(p, q) = (1, 1000), (6, 285), (10, 181), (12, 153), (21, 90), (25, 76), (76, 25), (90, 21), (153, 12), (181, 10), (285, 6)$ and $(1000, 1)$ are solutions.

Take $(p, q) = (1000, 1)$. Then $256128000000000000000000001 = (1000 \times 2^7 \times 10^{21} + 1)(2^7 \times 10^{21} + 1)$.

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