

1116. *Proposed by Marcin Kuczma, University of Warsaw, Warsaw, Poland*

Call a number “nice” if it can be represented as the sum of fourth powers of three positive integers. Let $q[1] = 3$, $q[2] = 18$, $q[3] = 33$, \dots be the increasing sequence of all nice numbers - and let $q[q[4]]$ be nice and lucky and happy for you!

Solution by Rex H. Wu, Brooklyn, NY.

I am not sure what is being sought here. If $q[q[4]]$ is sought, here is the solution.

Let $(a, b, c) = a^4 + b^4 + c^4$ for $a, b, c \in \mathbb{Z}^+$ and $a \leq b \leq c$. Then $q[4] = (2, 2, 2) = 48$. $q[q[4]] = q[48] = 2002$.

Unfortunately, there is no easy formula for $q[n]$. The list of $q[n]$ is as follow.

n	(a, b, c)	$q[n]$
1	(1,1,1)	3
2	(1,1,2)	18
3	(1,2,2)	33
4	(2,2,2)	48
5	(1,1,3)	83
6	(1,2,3)	98
7	(2,2,3)	113
8	(1,3,3)	163
9	(2,3,3)	178
10	(3,3,3)	243
11	(1,1,4)	258
12	(1,2,4)	273
13	(2,2,4)	288
14	(1,3,4)	338
15	(2,3,4)	353
16	(3,3,4)	418
17	(1,4,4)	513
18	(2,4,4)	528
19	(3,4,4)	593
20	(1,1,5)	627
21	(1,2,5)	642
22	(2,2,5)	657
23	(1,3,5)	707
24	(2,3,5)	722
25	(4,4,4)	768
26	(3,3,5)	787
27	(1,4,5)	882
28	(2,4,5)	897
29	(3,4,5)	962
30	(4,4,5)	1137

n	(a, b, c)	$q[n]$
31	(1,5,5)	1251
32	(2,5,5)	1266
33	(1,1,6)	1298
34	(1,2,6)	1313
35	(2,2,6)	1328
36	(3,5,5)	1331
37	(1,3,6)	1378
38	(2,3,6)	1393
39	(3,3,6)	1458
40	(4,5,5)	1506
41	(1,4,6)	1553
42	(2,4,6)	1568
43	(3,4,6)	1633
44	(4,4,6)	1808
45	(5,5,5)	1875
46	(1,5,6)	1922
47	(2,5,6)	1937
48	(3,5,6)	2002

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