**1118.** *Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, MA*
Prove that $\sin 9^\circ + \cos 9^\circ = \frac{1}{2}\sqrt{3+\sqrt{5}}$.

*Solution by* **Rex H. Wu**, *Brooklyn, NY.*

Take the square of both sides of $\sin 9^\circ + \cos 9^\circ = \frac{1}{2}\sqrt{3+\sqrt{5}}$, we can transform the question to

$$\begin{aligned}
\sin 9^\circ + \cos 9^\circ &= \frac{1}{2}\sqrt{3+\sqrt{5}} \\
(\sin 9^\circ + \cos 9^\circ)^2 &= \left(\frac{1}{2}\sqrt{3+\sqrt{5}}\right)^2 \\
\sin^2 9^\circ + \cos^2 9^\circ + 2\sin 9^\circ \cos 9^\circ &= \frac{1}{4}(3+\sqrt{5}) \\
1 + 2\sin 9^\circ \cos 9^\circ &= \frac{1}{4}(3+\sqrt{5}) \\
2\sin 9^\circ \cos 9^\circ &= \frac{\sqrt{5}}{4} - \frac{1}{4} \\
\sin 18^\circ &= \frac{\sqrt{5}-1}{4}
\end{aligned}$$

This is a classic problem in trigonometry. Notice that $36^\circ$ and $54^\circ$ are complementary. Then $\sin 36^\circ = \cos 54^\circ$.

$$\begin{aligned}
\sin 36^\circ &= \cos 54^\circ \\
2\sin 18^\circ \cos 18^\circ &= 4\cos^3 18^\circ - 3\cos 18^\circ \\
2\sin 18^\circ &= 4\cos^2 18^\circ - 3 \\
2\sin 18^\circ &= 4(1-\sin^2 18^\circ) - 3 \\
2\sin 18^\circ &= 1 - 4\sin^2 18^\circ \\
4\sin^2 18^\circ + 2\sin 18^\circ - 1 &= 0
\end{aligned}$$

Let $x = \sin 18^\circ$. Note that $x$ is positive. Then the above equation is transformed into $4x^2 + 2x - 1 = 0$. Solving for $x$ and only taking the positive root to get $x = \sin 18^\circ = (\sqrt{5}-1)/4$, which proves the problem.

■