1121. *Proposed by Gus Mavrigian, Youngstown, OH*
Suppose angles α and β satisfy $(\cos\beta - \cos\alpha) \neq 0$ and

$$\frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \sqrt{3}.$$

Show that $\sin(3\alpha) + \sin(3\beta) = 0$.

Solution by **Rex H. Wu**, *Brooklyn, NY.*

From the sum/difference identities, we know

$$\begin{aligned}\sin\alpha + \sin\beta &= 2\cos\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\\ \cos\alpha - \cos\beta &= 2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\end{aligned}$$

Then

$$\frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \frac{2\cos\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}}{2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}} = \cot\frac{\alpha-\beta}{2} = \sqrt{3}$$

The last part of the equation gives $\frac{\alpha-\beta}{2} = \frac{\pi}{6}$ or $3\alpha = \pi + 3\beta$.

$$\begin{aligned}\sin(3\alpha) + \sin(3\beta) &= \sin(\pi + 3\beta) + \sin(3\beta)\\ &= -\sin(3\beta) + \sin(3\beta)\\ &= 0.\end{aligned}$$

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