1122. *Proposed by Arthur Holshouser, Charlotte, NC*

A. $(S, \cdot)$ is a binary operator that satisfies $(xy)x = y$ for all $x, y \in S$. This is what Donald Knuth calls a grope.

   (a) Is it provable that for all $a, b \in S$, there exists a unique $x \in S$ such that $xa = b$?

   (b) Is it provable that for all $a, b \in S$, there exists a unique $x \in S$ such that $ax = b$?

B. Now suppose that $(\overline{S}, \cdot)$ satisfies $(xy)y = x$, for all $x, y \in \overline{S}$. Answer (a) and (b)

Solution by **Rex H. Wu**, *Brooklyn, NY.*

I will solve part A.(b) first. For all $a, b \in S$, there is a $x \in S$ such that $ax = b$. Suppose there is a $y \in S$ and $ay = b$. Then $ax = b = ay$. By the property of $(S, \cdot)$, $(ax)a = x = ba = (ay)a = y$. Therefore, this x is unique.

For part A.(a), we start with $xa = b$. Multiple x on the right, $(xa)x = a = bx$. This would reduce the problem to case A.(b). x is unique and $x = ab$.

For part B.(a), let $xa = b$ and $ya = b$. Then $xa = b = ya$. $(xa)a = xba = (ya)a = y$. So x is unique.

As to part B.(b), the same argument applies. Multiply x on the right of $ax = b$ to get $(ax)x = bx = a$. This is case B.(a).

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