1126. *Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, MA*

Find a rational function $f(x)$ with integer coefficients such that

$$\cos\theta = f(\sin\theta - \cos\theta)$$

is an identity or prove that no identity of this form exists.

Solution by **Rex H. Wu**, *Brooklyn, NY.*

There is no such an identity exist.

Suppose $\cos\theta = x$, then $\sin\theta = \sqrt{1-x^2}$, for $x \in [-1, 1]$. Let $g(x) = \sin\theta - \cos\theta = \sqrt{1-x^2} - x$.

Let's further suppose there is a function f such that $f(\sin\theta - \cos\theta) = \cos\theta$. In terms of x, $f(g(x)) = f(\sqrt{1-x^2} - x) = x$. This means f is the inverse function of g. We need to show f is not a rational function with integer coefficients.

To look for f is the same as solving for y in $x = \sqrt{1-y^2} - y$, which gives $y = f(x) = \frac{1}{2}(\sqrt{2-x^2} - x)$ for $x \in [-1, \sqrt{2}]$. Clearly, this is not a rational function.

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