1127. *Proposed by Arthur Holshouser, Charlotte, NC*

A bug starts from the origin on the plane and crawls one unit upwards to $(0, 1)$ after one minute. During the second minute, it crawls two units to the right ending at $(2, 1)$. Then during the third minute, it crawls three units upward, arriving at $(2, 4)$. It makes another right turn and crawls four units during the fourth minute. From here it continues to crawl n units during minute n and then making a 90^o, either left or right. The bug continues this until, after 16 minutes, it finds itself back at the origin. Its path does not intersect itself. What is the maximum possible area of the 16-gon traced out by its path?

Solution by **Rex H. Wu**, *Brooklyn, NY.*

Notice that a polygon thus formed has all the odd numbers as the vertical sides and even numbers as the horizontal sides. Other properties we note are (1) the polygon is cyclic; (2) the sum of the sides going up is the same as the sum of the sides going down; (3) the same is true for the sides going to the left and to the right. Intuitively, to maximize the area of the polygon, we should not make premature turn backs, i.e. 4 right, 5 up, 6 left, 7 up and 8 right. Instead, it should be 6 right.

This comes down to looking for dividing the set $\mathcal{O} = \{1, 3, 5, \ldots, 13, 15\}$ into two subsets of consecutive odd numbers such that the sum of these two subsets are equal. Note that 15 is considered to be consecutive with 1.

The same applies to the set $\mathcal{E} = \{2, 4, 6, \ldots, 14, 16\}$ with 16 considered consecutive with 2.

The sum of the members of the set $\mathcal{O}$ is 64 and the set $\mathcal{E}$ is 72.

The subsets $\mathcal{O}_1 = \{1, 3, 13, 15\}$ and $\mathcal{O}_2 = \{5, 7, 9, 11\}$, $\mathcal{E}_1 = \{2, 4, 6, 8, 16\}$ and $\mathcal{E}_2 = \{10, 12, 14\}$ are the only subsets satisfying these properties.

Thus the 16-gon sought is 1 up, 2 right, 3 up, 4 right, 5 down, 6 right, 7 down, 8 right, 9 down, 10 left, 11 down, 12 left, 13 up, 14 left, 15 up and 16 right.

The area of this 16-gon is 664 square units.

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