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Solution to *CMJ* Problem 750.

$$\frac{x}{y} = \frac{2y+1-n}{2x-1-n}$$

can be rewritten as

$$\begin{aligned} n &= \frac{(x+y)[2(y-x)+1]}{y-x} \\ &= \frac{[2x+(y-x)][2(y-x)+1]}{y-x} \\ &= \left(\frac{2x}{y-x}+1\right)[2(y-x)+1] \end{aligned}$$

for some positive integers $y > x$ and $(y-x)|2x$.

If we let $m = y - x$ with $m = 1, 2, 3, \ldots$ and let $k = \frac{2x}{y-x} + 1$, then $n = k(2m+1)$ where $k$ is a positive integer, which requires $m|2x$.

Now look at $k = \frac{2x}{m} + 1$. If $m$ is odd, then $x$ has to be a multiple of $m$, say $x = mp$. Then $k = 2p + 1$ for some $p = 1, 2, 3, \ldots$. If $m$ is even, then $2x = mp$ for $p = 1, 2, 3, \ldots$, which means $k = p + 1$.

Thus,

$$n = k(2m+1) \begin{cases} \text{for } m = \text{odd and } k = 3, 5, 7, \ldots, 2i+1, \ldots \\ \text{for } m = \text{even and } k = 2, 3, 4, \ldots, i, \ldots \end{cases}$$

allows the original expression to have positive integral values for $x$ and $y$.

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