

THE PIGEON HOLE PRINCIPLE

Rex H. Wu
SUNY, Brooklyn

The pigeon hole principle states that if a set of $(n + 1)$ elements were partitioned into n subsets then there is one subset containing at least two elements.

Probably the most common pre-calculus problem that applies this principle is to put 5 points in a unit square and prove that there are two points such that the distance between them is less than $\sqrt{2}/2$.

Here are two examples that use this principle.

Example 1: A bag contains 4 red socks, 5 blue socks, 6 green socks and 7 white socks. If you draw socks randomly from this bag, what is the minimum number of socks you need to draw in order to guarantee that you have a pair of socks of the same color?

The answer is 5. Since there are four colors (subsets), if we have one for each color, the fifth one will guarantee a pair.

Example 2: There are at least two people born in the same year, same month, same day, same hour, same minute and same second.

There are about 5.5 billion people in the world. Let's assume the oldest person is 150 years. There are less than 4.8 billion seconds in 150 years. If we put one person into each second, there will be 0.7 billion people left, which proves the above statement by the pigeon hole principle.

Appeared (with modification) on the *Pi Mu Epsilon Journal* 10(2), p. 117 (1995).