Extension of Winding Function Theory for Nonuniform Air Gap in Electric Machinery

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***Abstract*—This paper extends the winding function theory for nonuniform air gap in rotating electric machinery. It shows that the winding function differs from that used in the symmetrical case, although several papers employ the uniform air-gap winding function to study electric motor performance under fault conditions. The extended theory will be particularly helpful in the study of squirrel-cage induction motors with a nonuniform air gap such as that caused by eccentricity of the rotor and stator.**

***Index Terms*—Eccentricity, electric machinery, nonuniform air gap, winding function theory.**

NOMENCLATURE

p	Number of pole pairs.
l	Length of stack.
r	Average radius of air gap.
g_0	Length of nonuniform air gap.
δ	Nonuniform air-gap coefficient
γ	Angle of nonuniform air gap.
J_x, i_x	Current density and current of winding x.
$g(\alpha)$	Distribution of air gap.
$P(\alpha)$	Distribution of inverse of air gap.
$n_x(\alpha)$	Turn function of winding x.
$N_x(\alpha)$	Winding function of winding x.
$B_x(\alpha)$	Magnetic field density of winding x.
L_{xy}	Inductance between windings x and y.
α	An arbitrary angle in stator reference frame.
γ	Angle of skewing.

I. INTRODUCTION

THE WINDING function theory used for magnetic inductances calculation of induction motor in 1965 [1] gives a solution of coupled electromagnetic equations, which are obtained using simple laws of electric circuit theory. An essential part of this theory is the calculation of the motor inductances. These inductances are evaluated using winding functions and other equations within the theory.

This theory was used to analyze two-phase induction motors in 1969 [2], linear induction motor in 1979 [3], three-phase squirrel-cage induction motor with concentric winding in 1991 [4], study of saturated induction machines in 1992 [5], and finally, synchronous reluctance motor [6]. For the first time, in 1992, this theory was used to study the performance of induction motor with different faults [7]. Since this theory is capable to take into account any winding distribution or air-gap distribution around rotor, it is capable to model and analyze the fault conditions. This capability leads to the use of winding function in internal short circuit of the stator winding, rotor broken bars, fracture of the rotor end ring, and stator and rotor eccentricity [8]–[11].

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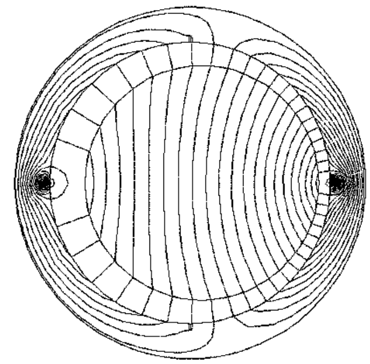

Fig. 1. Effect of nonuniform air gap upon the flux distribution of a simple winding.

This paper reviews the theory and shows that the extension of the theory for a nonuniform air gap leads to a calculated inductance which differs with that evaluated for the uniform air gap.

The modified winding function for nonsymmetrical air gap in a salient pole synchronous machine has been proposed in [12], several other papers have used this theory for induction motor analysis with nonuniform air gap while of calculated inductance is only valid for a uniform air gap. This indicates clearly that equality of $L_{12} = L_{21} = L$ is not observed in [10]. Illustrations in [10] and [11] show the case. It will be observed that by extension of the winding function theory presented in this work the above-mentioned equality is held for any air-gap distribution as expected in the linear magnetic circuit. Thus, the results presented in [10] and [11] are doubtful.

II. NONUNIFORM AIR GAP IN ELECTRIC MACHINERY

Nonuniformity of the air gap in rotating electric machinery leads to a nonuniform permeance of the air gap, if at the same time there is an unsymmetrical air gap, unbalanced magnetic pull (UMP), and applied unsymmetrical forces on the rotor occur. In such a case, air-gap flux is also nonuniform and unsymmetrical. Fig. 1 shows the flux distribution of a simple winding in the nonuniform air gap.

Many mechanical faults such as misalignment, rotor shaft bending, and even weak bearing lead to various nonuniform air gaps including stator and rotor eccentricity. To analyze these

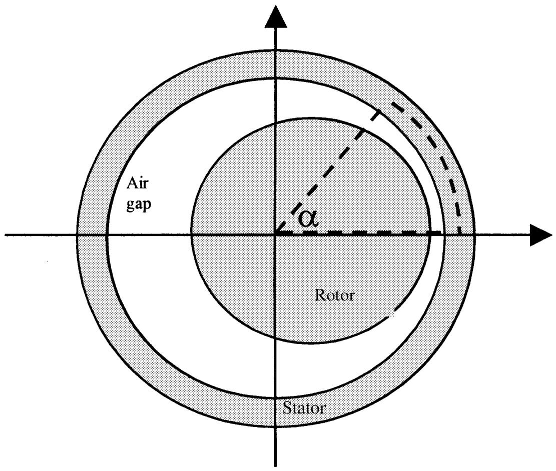

Fig. 2. Closed loop for application of the Ampere circuit law for nonuniform air gap.

cases, the air gap can be approximated by two first terms of the Fourier series as follows:

$$g(\alpha) \approx g_0 - g_0\delta\cos(\alpha - \gamma) \tag{1}$$

where δ, γ are the nonuniformity air-gap indexes that are generally functions of the rotor angular position. In order to keep $g(\alpha)$ positive, the condition $\delta < 1$ must be always held. Approximation used in (1) is valid for all rotating electric machinery with cylindrical stator and rotor. The permeance of air gap can be expanded using (1) as follows:

$$P(\alpha) = \frac{1}{g_0}\sum_{i=0}^{\infty} P_i \cos(i\alpha - i\gamma) \tag{2}$$

where

$$\begin{cases} P_0 = \frac{1}{\sqrt{1-\delta^2}} \\ P_i = \frac{2}{\sqrt{1-\delta^2}}\left(\frac{1-\sqrt{1-\delta^2}}{\delta}\right)^i, \quad i = 1, 2, \ldots. \end{cases} \tag{3}$$

It is suggested that (2) is approximated by the first p terms for the $2p$ poles winding.

III. Extension of Winding Function Theory for Arbitrary Distribution of Air Gap

Using the closed path shown in Fig. 2 and the Ampere circuit law for winding x

$$\oint \vec{H}_x(r,\alpha)\cdot d\vec{\ell} = \int_S \vec{J}_x(r,\alpha)\cdot d\vec{s}. \tag{4}$$

If $n_x(\alpha)$ represents the turn number over different position, we can write that

$$\int_S \vec{J}_x(r,\alpha)\cdot d\vec{s} = n_x(\alpha)i_x. \tag{5}$$

Since the relative permeability of iron is considerably larger than that of the air gap, the magnetomotive force (MMF) drop of the iron may be ignored. Also, the air-gap length is short and the magnetic field intensity at arbitrary angle α is independent of the radius and equal to the middle of the air gap. Thus

$$\begin{aligned}\oint \vec{H}_x(r,\alpha)\cdot d\vec{\ell} &= \int_{\text{airgap}} \vec{H}_x(\alpha)\cdot d\ell \\ &= H_x(\alpha)g(\alpha) - H_x(0)g(0).\end{aligned} \tag{6}$$

Manipulation of the above equations leads to

$$\int_{\text{airgap}} \vec{H}_x(\alpha)\cdot d\ell = n_x(\alpha)i_x \tag{7}$$

and the integration yields

$$H_x(\alpha) = \frac{n_x(\alpha)i_x + H_x(0)g(0)}{g(\alpha)}. \tag{8}$$

For $H_x(0)$ calculation, a Gaussian cylindrical-shape surface in the depth of the air gap is considered; therefore

$$\int_S \mu_0 H_x(\alpha)\cdot ds = 0. \tag{9}$$

Manipulation of the above equation leads to

$$\int_0^{2\pi} \frac{n_x(\alpha)}{g(\alpha)} i_x d\alpha + g(0)H_x(0)\int_0^{2\pi}\frac{1}{g(\alpha)}d\alpha = 0 \tag{10}$$

and (8) and (9) yield

$$H_x(\alpha) = \left(n_x(\alpha) - \frac{\langle Pn_x\rangle}{\langle P\rangle}\right)P(\alpha)i_x. \tag{11}$$

Operator $\langle f\rangle$ is defined as the mean of function f over $[0, 2\pi]$ range as follows:

$$\langle f\rangle = \frac{1}{2\pi}\int_0^{2\pi} f(\alpha)d\alpha. \tag{12}$$

Referring to (11), the winding function of winding x or $N_x(\alpha)$ reduced to

$$N_x(\alpha) = n_x(\alpha) - \frac{\langle Pn_x\rangle}{\langle P\rangle}. \tag{13}$$

This winding function is similar to the equation in [12], which has been introduced as winding function of a synchronous motor. When the air-gap distribution is uniform, (13) can be simplified as follows:

$$N_x(\alpha) = n_x(\alpha) - \langle n_x\rangle. \tag{14}$$

However, (14) is not valid for a nonuniform air gap while [10] and [11] have used this equation for a nonuniform air gap. Referring to (11), the magnetic flux density produced by winding x is

$$B_x(\alpha) = \mu_0 N_x(\alpha)P(\alpha)i_x. \tag{15}$$

Thus, the flux produced by winding x and links with the adjacent winding y is as follows:

$$\lambda_{yx}(\alpha) = \mu_0 r l i_x \int_0^{2\pi} P(\alpha)N_x(\alpha)n_y(\alpha)d\alpha. \tag{16}$$

Since the ratio of the air-gap length and the mean radius is very small, for the uniform air gap the mean radius of the air gap

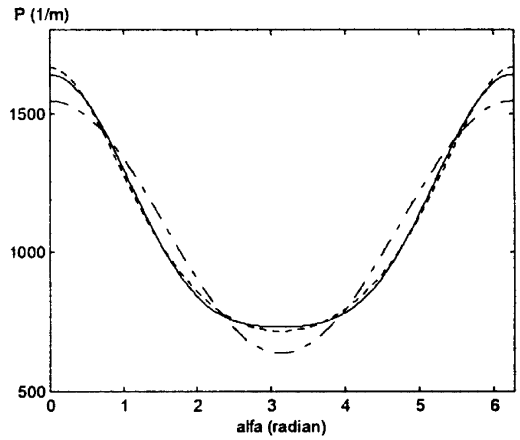

Fig. 3. Distribution of inverse of air gap. (—) Precise. (—) Two-term approximation. (...) Three-term approximation.

can be used with a good approximation. Referring to (16), the mutual inductance of winding x and y is

$$L_{yx} = \mu_0 r l \int_0^{2\pi} P(\alpha) N_x(\alpha) n_y(\alpha) d\alpha \tag{17}$$

and its final form is

$$L_{yx} = 2\pi\mu_0 r l \langle P n_x n_y \rangle - 2\pi\mu_0 r l \frac{\langle P n_x \rangle \langle P n_y \rangle}{\langle P \rangle}. \tag{18}$$

Equation (18) has cumulative property; thus, the following is always held:

$$L_{yx} = L_{xy}. \tag{19}$$

The above equation exists in the linear magnetic circuits. If the winding functions are used as reported in [10] and [11], equality (19) cannot be obtained. In spite of the existing error in [10] and [11], the simulated frequency spectrum amplifies the harmonics caused by the eccentricity. This is in agreement with the experimental results. This can be explained such that in these studies, winding function (13) has been approximated by (14), which is exaggeratory. However, variations of the frequency spectrum of the motor line current has been included and modeled. Obviously, these variations can be more accurately taken into account and the condition of $L_{xy} = L_{yx}$ will be satisfied. If the case of the rotor bars is taken into account, the inductance is evaluated as follows:

$$L'_{yx} = \int_0^l \frac{L_{yx}\left(\theta \pm \zeta \frac{\gamma}{l}\right)}{l} d\zeta. \tag{20}$$

IV. Inductance Calculation for a Simple Case

Fig. 3 shows a precise value of the reverse of the air gap for the static eccentricity with eccentricity degree of $\delta = 0.4$ and eccentricity angle $\gamma = 0$ (solid line). Two-term and three-term approximations of (2) have been shown in this figure, which denotes that the three-term approximation is more precise than that of the two-term approximation.

Fig. 4 extends the turn function of the two arbitrary windings: one on the stator and other on the rotor. Fig. 5 presents the winding function of the rotor winding in three angular positions. When the air gap is uniform, this function is only displaced along the horizontal axis with varying the rotor angular position. As shown in Fig. 5, for the nonuniform air gap, the function value varies with the rotor position. Fig. 6 shows the mutual inductance of the mentioned windings versus the rotor angular position.

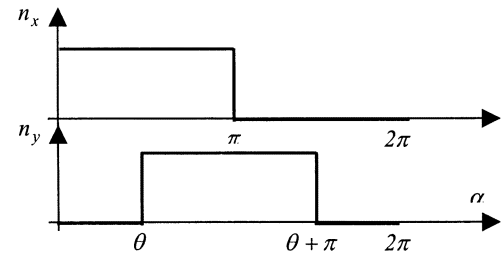

Fig. 4. Winding functions of two arbitrary loops. Up: on stator. Down: on rotor.

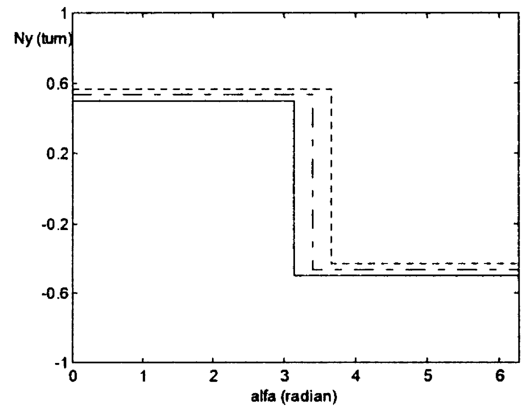

Fig. 5. Winding function y at three rotor angular positions. (—) $\theta = 0$. (—) $\theta = \pi/12$. (...) $\theta = \pi/6$.

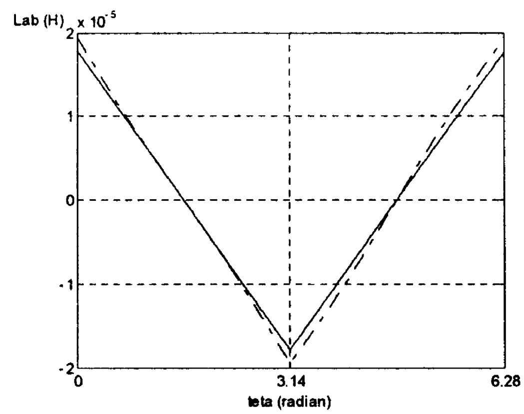

Fig. 6. Mutual inductance of winding x and y. (—) Uniform air gap. (...) Nonuniform air gap.

V. Conclusion

This paper extends the winding function theory for nonuniform air gap in rotating electric machinery. It shows that the winding function for nonuniform air gap differs with that used in the symmetrical case, while several papers employ the uniform air-gap winding function in order to study the electric motors performance. It is particularly the case in the study of squirrel-cage induction motor in the nonuniform air gap. The extension of the winding function indicated that equality of the mutual inductances is held at any air-gap distribution. Thus, there is doubt concerning the results presented in [10] and [11]. The interesting point is that in [11], incorrect inductances have been used in the analysis of the dynamic eccentricity but the extra harmonic on the line is the same harmonic found in the dynamic eccentricity, as expected.

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