ERRATA & NOTES
Calculus of Variations and Functional Analysis

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- Chapter 1. As a supplementary exercise, the reader may wish to show that the Euler equation for a functional of the form

$$I(u) = \iint F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy})\, dx\, dy$$

  is

$$F_u - \frac{\partial F_{u_x}}{\partial x} - \frac{\partial F_{u_y}}{\partial y} + \frac{\partial^2 F_{u_{xx}}}{\partial x^2} + \frac{\partial^2 F_{u_{yy}}}{\partial y^2} + \frac{\partial^2 F_{u_{xy}}}{\partial x \partial y} = 0.$$

- Page 93, Example 1.14.1. The phrase "symmetry in c" refers to symmetry in the dependence on c about zero.

- Page 160, Theorem 3.0.2. The parenthetical phrase "(independent of ε)" should obviously read "(dependent only on ε)".

- Page 172, 9th line from bottom. "limit of" should read "limit point of".

- Page 177, 7th line from bottom. "Exercise ??" should read "Example 3.3.1".

- Page 214, last displayed formula on the page. Should read

$$f(\mathbf{y}) - f(\mathbf{x}) = \int_0^1 \nabla f(\mathbf{z})\big|_{\mathbf{z}=t\mathbf{y}+(1-t)\mathbf{x}} \cdot (\mathbf{y} - \mathbf{x})\, dt.$$

- Page 219, two formulas for the inner product in L^2. Delete power of 1/2 from the right-hand side.

- Page 246, 5th displayed equation from the top. Add a period to end the sentence.

- Page 250, proof of Theorem 3.13.1. Change "hence $N \neq \emptyset$" to "hence N has nonzero elements."

- Page 276, proof of Theorem 3.17.3, "It is easy to show that ...". Here we can fill in the details needed to show that a continuous linear operator acting from a normed space X into a normed space Y maps weakly Cauchy sequences into weakly Cauchy sequences. Suppose A is continuous and $\{x_n\}$ is weakly Cauchy in X. We want to show that $\{Ax_n\}$ is weakly Cauchy in Y. So we take an arbitrary continuous linear functional F given on Y and apply it to $\{Ax_n\}$. But this amounts to applying FA, which is clearly a continuous linear functional on X, to $\{x_n\}$. Since $\{FA(x_n)\}$ is a Cauchy sequence, so is $\{F(Ax_n)\}$.