EIGENVALUES AND EIGENVECTORS

LEE G. GILMAN

1. Eigenvalues

We examine the transition matrix for various small n and r, $c = 2$. The nonzero eigenvalues for each matrix is given by

$$\frac{\binom{i}{r}}{\binom{n}{r}}$$

for i chosen appropiately so that this formula is defined. Note that $r < i \leq n$. Also note that for each matrix, there are zero eigenvalues.
Proof: Shannon is working on it.

2. Eigenvectors for non-zero eigenvalues

Of particular interest to our problem are the eigenvectors for the nonzero eigenvalues. The eigenvectors contain the binomial coefficients and are dependent on the multiplicity of the eigenvalue, not the actual eigenvalue.
i is the index of the eigenvector starting from the bottom row (we set i of the bottom row equal to zero.
x is the multiplicity of the eigenvalue λ.
For the $\left(\left(\frac{n^2+3n+2}{2}\right) - i\right)$th to the $\left(\frac{n^2+3n+2}{2}\right)$th entries in the eigenvector, the entry is given by

$$(-1)^i \binom{x-1}{i}$$

That is,

i	*eigenvector*
$\left(\frac{n^2+3n+2}{2}\right) - 1$	0
$\left(\frac{n^2+3n+2}{2}\right) - 2$	0
$\vdots$	$\vdots$
$x - 1$	$(-1)^i \binom{x-1}{x-1}$
$x - 2$	$(-1)^i \binom{x-1}{x-2}$
$\vdots$	$\vdots$
i	$(-1)^i \binom{x-1}{i}$
$\vdots$	$\vdots$
0	$\binom{x-1}{0}$

Proof: Shannon is working on it.

3. First Order Preimages

Calculations for the first order preimages are given as follows. Note that for this vector, we index the vector from the top instead of the bottom and we begin indexing at 1 instead of zero.

i	*preimage*
1	0
2	0
$\vdots$	$\vdots$
$\frac{(n-1)^2+3(n-1)+2}{2} - (x-2)$	$(-1)^i \left(\lambda r \frac{1}{n-x}\right)^{-1} \binom{x-2}{x-2}$
$\frac{(n-1)^2+3(n-1)+2}{2} - (x-3)$	$(-1)^i \left(\lambda r \frac{1}{n-x}\right)^{-1} \binom{x-2}{x-3}$
$\vdots$	$\vdots$
$\frac{(n-1)^2+3(n-1)+2}{2} - (x-t), 2 \leq t \leq x$	$(-1)^i \left(\lambda r \frac{1}{n-x}\right)^{-1} \binom{x-2}{x-t}$
$\vdots$	$\vdots$
$\frac{(n-1)^2+3(n-1)+2}{2}$	$(-1)^i \left(\lambda r \frac{1}{n-x}\right)^{-1} \binom{x-2}{0}$
$\frac{(n-1)^2+3(n-1)+2}{2} + 1$	0
$\vdots$	$\vdots$
$\frac{(n-1)^2+3(n-1)+2}{2} + (n-x) + 1$	0
$\vdots$	*Free*
$\frac{n^2+3n+2}{2}$	*Variables*

The general form for the $\left(\frac{(n-1)^2+3(n-1)+2}{2} - (x-t)\right)$th to the $\left(\frac{(n-1)^2+3(n-1)+2}{2}\right)$th entries in the vector are given by:

$$(-1)^i \left(\lambda r \frac{1}{n-x}\right)^{-1} \binom{x-2}{x-t}$$

so that $2 \leq t \leq x$.

4. Second Order Preimages

The second order preimages appear to be of the form:

i	*preimage*
1	0
2	0
$\vdots$	$\vdots$
$\frac{(n-2)^2+3(n-2)+2}{2} - (x-3)$	$(-1)^i \left(\lambda c \sum_{z=1}^{n-r-1} \frac{(z+1)^2}{z(z+2)}\right)^{-1} \binom{x-3}{x-3}$
$\frac{(n-2)^2+3(n-2)+2}{2} - (x-4)$	$(-1)^i \left(\lambda c \sum_{z=1}^{n-r-1} \frac{(z+1)^2}{z(z+2)}\right)^{-1} \binom{x-3}{x-4}$
$\vdots$	$\vdots$
$\frac{(n-2)^2+3(n-2)+2}{2} - (x-t), 3 \leq t \leq x$	$(-1)^i \left(\lambda c \sum_{z=1}^{n-r-1} \frac{(z+1)^2}{z(z+2)}\right)^{-1} \binom{x-3}{x-t}$ $\vdots$
$\vdots$	
$\frac{(n-2)^2+3(n-2)+2}{2}$	$(-1)^i \left(\lambda c \sum_{z=1}^{n-r-1} \frac{(z+1)^2}{z(z+2)}\right)^{-1} \binom{x-3}{0}$
$\frac{(n-2)^2+3(n-2)+2}{2} + 1$	$Zeros$
$\vdots$	$Free$
$\frac{n^2+3n+2}{2}$	$Variables$

The general form for the $\left(\frac{(n-2)^2+3(n-2)+2}{2} - (x-3)\right)$th to the $\left(\frac{(n-2)^2+3(n-2)+2}{2}\right)$th entries in the vector are given by:

$$(-1)^i \left(\lambda c \sum_{z=1}^{n-r-1} \frac{(z+1)^2}{z(z+2)}\right)^{-1} \binom{x-3}{x-t}$$

so that $3 \leq t \leq x$.
c is unknown. It may either be a constant, a variable, or some value dependent on n and/or r.

7872 University Station, Department of Mathematical Sciences, Clemson University, Clemson, SC, 29632-7872

E-mail address: `lgilman@clemson.edu`