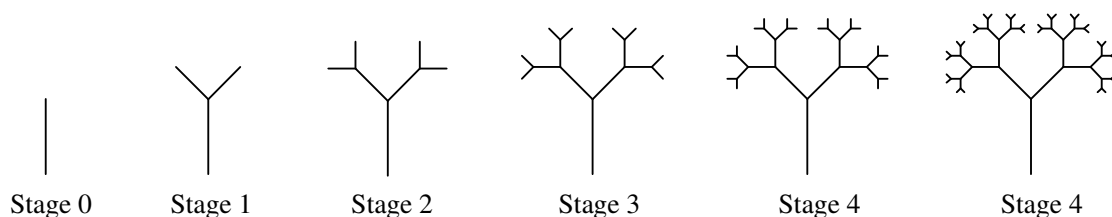


## FRACTAL TREE

**COURSE/LEVEL:** NSW Secondary High School Stage 5 Mathematics – Additional Content

At each stage of iteration, two branches are attached at a fixed angle to each of the outermost branches. The length of each successive branch decreases by a fixed ratio. The initial stage consists of a single branch of length one unit.

In the diagram below, the angle between successive branches is  $45^\circ$  while the ratio of the length of any branch to its preceding branch is  $\frac{1}{2}$ .



1. Complete the table below.

| Stage | Number of new branches | Total number of branches | Length of each new branch | Total length of all branches |
|-------|------------------------|--------------------------|---------------------------|------------------------------|
| 0     | 1                      | 1                        | 1                         | 1                            |
| 1     | 2                      | 3                        | $\frac{1}{2}$             | 2                            |
| 2     | 4                      | 7                        | $\frac{1}{4}$             | 3                            |
| 3     |                        |                          |                           |                              |
| 4     |                        |                          |                           |                              |
| $n$   |                        |                          |                           |                              |

2. Complete the following table.

|                            | Stage 0 | Stage 1              | Stage 2       | Stage 3               | Stage 4        | Stage 5 | Stage 6 |
|----------------------------|---------|----------------------|---------------|-----------------------|----------------|---------|---------|
| Increase in height of tree | 1       | $\frac{\sqrt{2}}{4}$ | $\frac{1}{4}$ | $\frac{\sqrt{2}}{16}$ | $\frac{1}{16}$ |         |         |

3. If  $h$  is the height of the fractal tree, show that  $h = 1 + \frac{\sqrt{2}+1}{4} \left( 1 + \frac{1}{4} + \frac{1}{16} + \dots \right)$
4. Using the formula  $1 + r + r^2 + \dots = \frac{1}{1-r}$  (where  $-1 < r < 1$ ), find the simplest expression for  $h$ .
5. If  $w$  is the width of the fractal tree, explain why  $w = 2(h-1)$  and hence find the simplest expression for  $w$ .