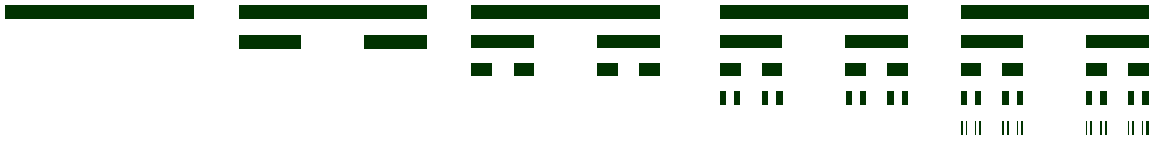
CANTOR DUST

COURSE/LEVEL: *NSW Secondary High School Stage 5 Mathematics – Additional Content*

At every stage of iteration, the middle third of each line interval is removed. The process starts with one line segment and continues indefinitely. The Cantor dust is actually the set of line segments that are left. The diagram below illustrates the first few stages in the process.



Stage 0 Stage 1 Stage 2 Stage 3 Stage 4

1. Complete the table below.

Stage n	Number of *new* intervals	Length of each new interval	Total length of new intervals	Total length of all intervals
0	1	1	1	1
1	2	$\frac{1}{3}$	$\frac{2}{3}$	$1+\frac{2}{3}$
2	2^2	$\left(\frac{1}{3}\right)^2$	$\left(\frac{2}{3}\right)^2$	$1+\frac{2}{3}+\left(\frac{2}{3}\right)^2$
3				
4				
n				

2. Using the formula $1+r+r^2+r^3+\ldots+r^n=\frac{r^{n+1}-1}{r-1}$, find an expression for the total length of intervals at the nth stage of iteration.

3. Use the formula $S=1+r+r^2+\ldots=\frac{1}{1-r}$ (where $-1<r<1$), to find an expression for the total length of intervals at the nth stage of iteration, where $n\to\infty$.