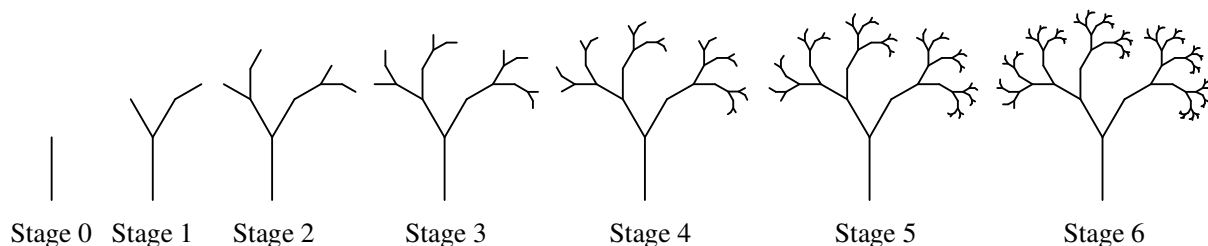


# FIBONACCI TREE FRACTAL\* (1)

**COURSE/LEVEL:** NSW Secondary High School Stage 5 Mathematics – Additional Content

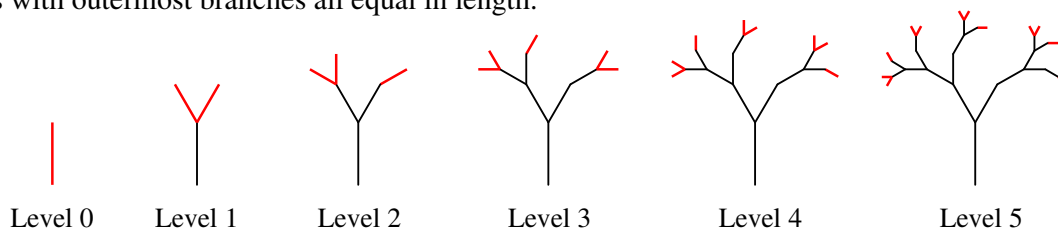


At each stage of iteration, two branches are attached at a fixed angle to each of the outermost branches. Another branch is then immediately attached to the right branch by the same angle. The length of each successive branch decreases by a fixed ratio.

The initial stage consists of a single branch of length one unit.

## Pruned Tree

The trees at each stage of iteration can be pruned by removing branches. Illustrated below are the trees with outermost branches all equal in length.



- Complete the following table for the pruned Fibonacci tree.

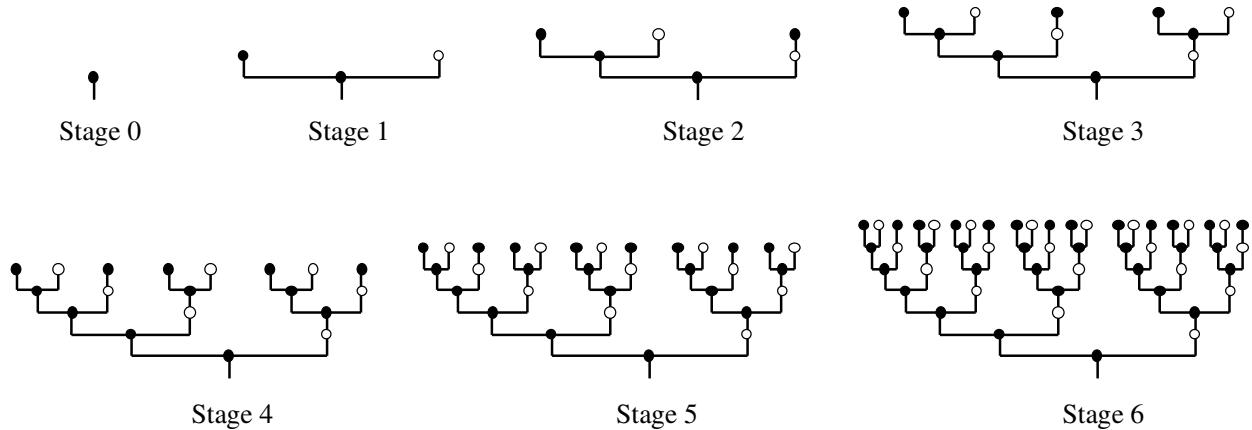
Level	Number of new branches	Total number of branches
0	1	1
1	2	3
2	3	6
3	5	11
4		
5		
6		
7		

- Let  $F_n$  denote the  $n$ th Fibonacci number where  $F_n = F_{n-1} + F_{n-2}$  and  $F_1 = F_2 = 1$ . Write expressions for the number of new branches and total number of branches for the Fibonacci tree at the  $n$ th stage of iteration.

\* Fibonacci numbers form a sequence of numbers, beginning with 1 and 1, with each number equal to the sum of the preceding two. The first few Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,...

## FIBONACCI TREE FRACTAL (2)

The Fibonacci tree below has essentially the same structure as the pruned Fibonacci tree. The tree is made up of nodes, either black or white, and branches. From one stage to the next, every black node branches into a black and a white node, while each white node branches into a black node. The initial stage consists of a single branch with a black node.



1. Complete the following table for the above Fibonacci tree.

Stage	Number of new black nodes	Number of new white nodes	Total number of new nodes	Total number of black nodes	Total number of white nodes	Total number of nodes
0	1	0	1	1	0	1
1	1	1	2	2	1	3
2	2	1	3	4	2	6
3						
4						
5						
6						

2. Let  $F_n$  denote the  $n$ th Fibonacci number where  $F_n = F_{n-1} + F_{n-2}$  and  $F_1 = F_2 = 1$ . Complete the following table for the Fibonacci tree at the  $n$ th stage of iteration.

Stage	Number of new black nodes	Number of new white nodes	Total number of new nodes	Total number of black nodes	Total number of white nodes	Total number of nodes
$n$						